The effect of steel reinforcing in the static analysis of historical vaulted buildings under horizontal actions
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Abstract

Different systems of reinforcements can be arranged in order to improve the seismic strength of the historical masonry buildings. The most common one consists in the insertion into the original structure of suitable horizontal devices as steel ties passing through the masonry walls at the floor levels with anchor plates at the heads. The aim of this paper is to perform a complete analysis of this kind of building when the floors are made up by masonry crossed vaults. The study, in line with previous paper, will be performed in the framework of the well known limit analysis theory by using the unilateral no tension model of the masonry material.

1 Introduction

The structural behaviour of the historical masonry buildings, mainly for their repair and maintenance, is subject of increasing interest. Particularly the actions of the horizontal forces due to the earthquakes are very dangerous for the historical masonry structures: it is urgent therefore the demand of simple and rational methods to control their lateral strength and to calculate possible reinforcement.

Since the traditional linear elastic analysis very seldom can be useful, a fundamental starting point is to characterize the masonry material whose response to the applied actions has an unilateral nature: infact masonry can support also high compressive stresses but only feable tensile ones. Therefore according to the assumptions that J. Heyman expressed in his pioneering studies
[1], [2], we state that:
- masonry has no tensile strength
- masonry has infinite compressive strength
- masonry is rigid in compression
- sliding cannot occur

The resistant structure of a typical historical building is made by a double order of multistorey plane walls connected with floors made up by wooden beams or crossed masonry vaults. Such a building is extremely vulnerable to seismic actions and its collapse occurs as a rule with the out of plane failure of some external walls. The most common system of reinforcing the original structure, used also in the past to improve the seismic strength of the building, is therefore the insertion of steel ties passing through the walls at the floor levels with anchor plates at the heads. This reinforcement prevents the out of plane collapse of the external walls and consequently the full inplane strength of all the walls can be exploited.

Aim of the paper is to perform a complete analysis of the lateral strength of this kind of building. The study, in line with previous papers, will be performed in the framework of the well known limit analysis theory by using the unilateral no tension model for masonry, while the steel ties will be supposed without any compression strength. We will analyze first the inplane strength under horizontal forces of a reinforced masonry wall, which is the main resistant element of the building. Then the lateral strength of the tridimensional structure of the whole building will be studied. Finally a simplified method will be described in order to control the ability of the floors to support the horizontal inplane stresses which arise at the ultimate state of the structure.

The object of our study is a regular multistorey masonry building (fig. 1). The horizontal seismic action on the building strongly depends on the weights distribution and can be represented by the global statical forces \( \lambda \sigma_j W_j \) acting at each j-th floor, growing with the multiplier \( \lambda \) and directed along one of the two principal direction \( x \) or \( y \). The term \( W_j \) is the floor weight while the factor \( \sigma_j \) represents the distribution law of the forces along the height, in such a way that the global shear at the building toe is \( \lambda \sum W_j \).

The building has a typical regular floor plan (fig. 2), characterized by the
presence of doors and windows which interrupt the continuity of the walls. Every floor is made up by a system of masonry crossed vaults supported by the walls (fig 3).

The main resistant structure of the building is a double order of orthogonal walls placed respectively along the x and y directions. Every plane multistorey wall has a regular array of openings in such a way that it can be considered as an arrangement of a number of piers linked by masonry panels. Moreover the system is reinforced by steel ties passing through the wall at the floor levels and blocked with anchor plate at the heads (Fig 5).

According to the different directions of the seismic actions, the walls can be considered as “neutral” or “active” according to whether they are subjected to out of plane or inplane horizontal forces, i.e. whether they are set along the seismic action direction or orthogonally to it. As the neutral walls are not much resistant to orthogonal forces, the horizontal loads due to the masses of these walls and to the masses of the portions of floors supported from them, move from the neutral to the active walls by the setting up of a suitable horizontal arch effect acting at the floor levels in the neutral walls (fig 4). Because of the small displacements at the springings, the thrusts in the horizontal arch will be minimum and the arch can be approximatively considered as a three pin structures with the three hinges as in fig 4. We can observe that when the arch is contiguous to an external active wall, the arch thrust can be supported only if a steel tie is present in the neutral wall.

2 The collapse multiplier of the reinforced wall

The strength of a single wall to the horizontal inplane actions is the first question to examine because the seismic resistance of the whole building from this strongly depends. Since we can sketch a seismic loading condition on the building as an action of fixed dead loads coupled with gradually increasing horizontal loads, we can suppose an analogous loading condition for a single active wall. More in detail we can state that at a stage of loading the following forces act on the generic i-th pier of the wall (fig 5):

- the vertical dead loads $G_{ij}$ applied at a generic j-th storey representing the weights of a piece of pier and the floor sustained by it
- the horizontal seismic loads $\lambda \sigma_i G_{ij}$ including a first share due to the inertial forces corresponding to the weights $G_{ij}$ and a second one due to the forces that are transferred by the neutral contiguous walls.
We can emphasize that the resultant of all the the vertical loads $G_{ij}$ acting on all the active walls at a generic floor coincides with the global weight $W_j$ of the same floor, while the resultant of the seismic loads $\lambda \sigma_j G'_{ij}$ at a generic floor coincides with the global seismic force $\lambda \sigma_j W_j$ applied at the same floor.

The lateral strength of a wall essentially depends by the ability of its piers to resist to the overturning action of the horizontal forces. In fact the failure of a single pier occurs with a rotation mechanism around its toe and its lateral strength is due to the uplifting action of the weights that occurs during this rotation. When the wall is not tie-reinforced, its failure can occur with a sidesway mechanism involving only a part of its piers as panels are able to support only compressive forces (fig. 6).

The first relevant effect of reinforcing is then the prevention of an anticipated collapse of the weakest piers before the whole wall is engaged in a global sidesway mechanism involving the overturning of all the piers. Infact the steel ties permit the redistribution of the seismic load as they provide a tensile resistant horizontal connection between the piers. When the panels height is negligible respect to the storey height, i.e. when the horizontal connections among the piers are monodimensional architraves, the failure sidesway mechanism is characterized by equal toe rotation of all the piers (fig. 7) and the collapse multiplier $\lambda_0$ can be calculated, according to the virtual work principle, equating the active work of the thrusts $\lambda \sigma_j G'_{ij}$ with the resistant one of the
weights $G_{ij}$:

$$\lambda_0 = \frac{\Sigma M_s^i}{\Sigma M_r^i}$$

where $i$ is the pier index, $M_s^i$ is the stabilizing moment around the toe of all the vertical forces $G_{ij}$ acting on the pier, $M_r^i$ is the overturning moment of all the horizontal forces $\sigma_i G_{ij}$. The stress in the steel ties can be easily evaluated with the procedure shown in [3].

On the contrary, when the panels heigth is not negligible, the sidesway mechanism requires the development of plastic strains in the ties and new other fractures in the piers as shown in fig. 8 (see also [4]). The strength increment is remarkable because in the virtual works balance we add the resisting plastic work of the tensile stresses in the ties. A procedure to relate the rotations of the pier pieces, the ties elongations, the horizontal displacements of the thrusts and the upliftings of the weights is described in [4] in order to calculate the collapse multiplier.

### 3 The collapse multiplier of the tridimensional building

According to the various kinds of connections between the walls, the seismic behaviour of a masonry building can follow different schemes. If there is lack of connections between the walls, i.e. when the floors are made up by wooden beams simply supported by the walls, the strength of the whole building is identified with the inplane strength of its weakest wall. If, on the contrary, the walls are suitably connected by the floors, there is full participation between all of them and the strength of the tridimensional structure of the building can be fully exploited (see also [5]). In this paragraph we suppose that rigid horizontal diaphragms connect the walls at the heigh of the floors; in the next one we will show a simple procedure to verify if the crossed vaulted floors of the building effectively can satisfy the above mentioned hypothesis.

In order to analyze the global collapse of the building, we define

$$D_j = [ u_j, v_j, \phi_j ]$$

as the displacement vector of the $j$-th floor, linearly increasing with the floor heigh $z_j$. Adopting a reference system having the origin in the gravity center $C$ of the floor (fig. 2), the components $u_j, v_j, \phi_j$ respectively represent the two translations along the axis directions $x$ and $y$ and the floor torsional rotation around $C$. Besides we assume that the global seismic action on the building is represented by horizontal forces applied at the centers of gravity $C$ of the floors and having the $y$ direction. So analogously to (2) we define

$$\lambda \sigma_j Q_j = \lambda \sigma_j W_j [ 0, 1, 0 ]$$

where $\lambda \sigma_j Q_j$ is the horizontal seismic action on the $j$-th floor.
as the load vector applied at the j-th floor having global weight \( W_j \), we can assume that the distribution factor \( \sigma_j \) increases linearly with the floor height \( z_j \):

\[
\sigma_j = z_j \sum W_j / \sum W_j z_j
\]  

(4)

In the balance of the virtual work principle the seismic forces are the active ones; so the global active work \( W_a \) at the collapse condition is

\[
W_a = \lambda_0 \sum \sigma_j Q_j D_j = \lambda_0 \Psi W_1 v_1
\]  

(5)

where \( \Psi \) is easily evaluable because of the simplifying hypothesis (4). At the same collapse condition the global resisting work \( W_r \) is evaluable taking into consideration the contribution of every single wall: in fact in each of them the corresponding contribution is the sum of the uplifting work of the weights acting on the piers and the plastic one of the tensile stresses in the ties. These works obviously develop in the unknown walls which effectively collapse.

If we take in consideration a collapsed active wall having the index \( k \), the corresponding resisting work is equal to the active one of a distribution of horizontal thrusts \( \lambda_0 \sigma_i G'_{ijk} \) or \( \lambda_0 \sigma_i G'_{ijk} \) according to the wall displacements at the collapse are positive or negative, \( \lambda_0 \) is the positive (negative) collapse multiplier of the wall as defined in 2. If on the contrary the wall is a neutral one, we can take in account a distribution of horizontal thrusts \( \lambda_0 \sigma_i G_{ijk} \).

In order to evaluate \( W_r \) in a compact form we define

\[
S_j = \sum \lambda_0 \sigma_i G'_{ij} \quad (S_j = \sum \lambda_0 \sigma_i G'_{ij})
\]  

(6)

in an active wall and analogously in a neutral one, the positive (negative) “storey” limit thrust of the wall; if \( d_j \) are the corresponding “storey” displacements, the global resisting work \( W_r \) is:

\[
W_r = \sum S_j d_j \alpha(d_j) + \sum S_j d_j \beta(d_j)
\]  

(7)

where \( \alpha = 1, \beta = 0 \) [\( \alpha = 0, \beta = 1 \)] if \( d_j > 0 \) [\( d_j < 0 \)]. Moreover if we define

\[
d_j = [d_{j1}, \ldots, d_{jk}]
\]  

(8)

as the displacement vector of all the walls at the j-th floor, we have

\[
d_j = C D_j
\]  

(9)

where \( C \) is the compatibility matrix that by effect of the supposed floor rigidity links the inplane displacements of the walls to the global displacements of the floors. Placing the (9) in the (7) and taking into account the linearity of the
displacements $D_j$ along the building height, we can write:

$$W_i = \Phi_1(D_1) u_1 + \Phi_2(D_1) v_1 + \Phi_3(D_1) \phi_1 \quad (10)$$

where $\Phi_1(D_1), \Phi_2(D_1), \Phi_3(D_1)$ are suitable piecewise constant functions; in fact if corresponding to two distinct floor displacements $D_1$ and $D_1'$, the displacements of all the walls have the same signs, the storey limit thrusts activated are also the same and consequently $\Phi_i(D_1') = \Phi_i(D_1'')$ for $i = 1, 2, 3$.

For any tridimensional mechanism characterized by the corresponding vector displacement of the first floor $D_1$, the associated kinematical multiplier $\lambda_k$ is, taking into account the (5) and (10):

$$\lambda_k(D_1) = \frac{[\Phi_1(D_1) u_1 + \Phi_2(D_1) v_1 + \Phi_3(D_1) \phi_1]}{\Psi_1 v_1} \quad (11)$$

The kinematical multiplier $\lambda_k$ depends on the chosen global mechanism, thus the collapse multiplier $\lambda_0$ is the upper lower bound of the kinematical multipliers in the set of the global mechanism $M$, i.e.:

$$\lambda_0 = \inf_{\lambda_k} \lambda_k(D_1) \quad (12)$$

If we consider a displacement vector $D_1^*$ having unitary active work:

$$\Psi_1 v_1^* = 1 \quad (13)$$

taking into account the linearity of $\lambda_k$ respect to $u_1$, $\phi_1$ and setting

$$\lambda_k^*(D_1^*) = \frac{[\Phi_1(D_1^*) u_1 + \Phi_2(D_1^*) v_1^* + \Phi_3(D_1^*) \phi_1]}{\Psi_1 v_1^*} \quad (14)$$

the collapse multiplier $\lambda_0$ is the upper lower bound of the kinematical multiplier in the set of the global mechanism $M^*$ respecting the (13):

$$\lambda_0 = \inf_{M^*} \lambda_k^*(D_1^*) \quad \text{ i.e. } (15)$$

$$\lambda_0 = \inf \lambda_k^* (u_1, \phi_1) \quad (16)$$

The surface representing the function $\lambda_k^*(u_1, \phi_1)$ has a polyedrical shape; therefore the upper lower bound of $\lambda_k^*(u_1, \phi_1)$ can be searched in the “vertex” set $V$ of the global mechanisms $D_1^*$, i.e. between that values of $u_1, \phi_1$ to which a vertex is associated. In order to characterize this set we remember that every nontranslational plane displacement $D$ is a rotation around a suitable point $\Omega$ having coordinates $x_\Omega, y_\Omega$. Restricting our attention firstly to this kind of displacements, the function $\lambda_k^*(u_1, \phi_1)$, coupled to the condition (13), can be expressed as a function of the coordinates $x_\Omega, y_\Omega$ of the rotation center and the (16) can be written again:
\[ \lambda_0 = \inf \lambda_k^* (x_\Omega, y_\Omega) \]  \hspace{1cm} (17)

When the rotation center \( \Omega \) moves in a panel of the frame shaped by the walls in the building plan (fig. 9), the coefficients \( \Phi_i \) in (14) remain constant: therefore the corresponding point \( P \) in the space \( \lambda_k^* \), \( u_1, \phi_1 \) moves along a face of a poliedrical surface (fig. 10); therefore when \( \Omega \) moves along a wall trace the corresponding point \( P \) moves along an edge. As the minimum of a poliedrical surface must be positioned at an intersection between two or more edge, we can conclude that the upper lower bound of the function \( \lambda_k^* (u_1, \phi_1) \) is reached when \( \Omega \) is placed at one of the intersections of the wall traces in the building plan (pointed in fig. 9).

If on the contrary the displacements \( D_i \) is translational, the function (14) is obviously minimum if the translation is purely along the x direction. In conclusion the minimum of the kinematical multipliers \( \lambda_k^* \) can be obtained looking this minimum only between the rotational mechanisms with rotation center located at the intersection of two walls and the translational mechanisms along the direction of the seismic action. The effective collapse mechanism will be translational if all the active walls have comparable strength while it will be rotational if one of them is much stronger then the others; in such a case this wall don’t collapse and the rotation occurs around to one of the its points of intersection with the neutral walls.

4 The inplane stresses in the building floors

The method shown in the previous paragraph to evaluate the collapse multiplier of the tridimensional building is based on the hypothesis of inplane rigidity of the floors. In order to verify if the stress distribution in the floors is consistent with this hypothesis, we firstly examine the case in which the collapse mechanism is purely translational. In such a case only the active walls collapse and the essential function of the floors is to transfer suitable horizontal stresses from the weakest to the strongest walls.

In order to verify if the stress transfer can occur we firstly define:

\[ T_{jk} = (\lambda_0 - \lambda_{0k}) \sum \sigma_j G'_{ijk} \]  \hspace{1cm} (18)
as the unbalanced storey thrust acting on the j-th floor of the k-th wall; \( \lambda_0 \) is the collapse multiplier of the whole building while \( \lambda_{0k} \) is the collapse multiplier of the single wall. The difference \( (\lambda_0 - \lambda_{0k}) \) is positive for some walls defined "weak" while is negative for all the others defined "strong". The transfer of the unbalanced thrusts from the weak walls to the strong ones is possible if a suitable ideal truss take shape in the planes of the floors (fig. 11). The compression rods of the truss are the diagonal arches of the crossed vaults while the tension rods are the steel ties located within the neutral walls. The truss is loaded by the unbalanced thrusts acting along the seismic force direction and its equilibrium is assured because it is equivalent to the rigid diaphragm assumed in the calculation of the global collapse multiplier.

The hypothesis of inplane floor rigidity is satisfied if under the action of the \( T_{jk} \) the tensile stresses in the ties remain within the elastic limits and the arches do not collapse by effect of the over thrust that develop at there springings. In order to control if this effectively occur, we fix our attention on the stress transfer between two adjacent active walls: the possibility that this transfer occurs can be verified studying with the limit design techniques the structural scheme of fig. 12. The force \( F_{jl} \) is the shear that must pass from an active k-th wall to the following k+1-th at the j-th floor and is easily evaluable from the inplane distribution of the thrusts \( T_{jk} \):

\[
F_{jl} = \sum_{k=1}^{l} T_{jk}
\]

(19)

In the scheme of fig. 12 the transfer of \( F \) to the ground can occur if its value does not exceed the limit one \( F_0 \) depending on the limit values of the compression and tension loads in the rods. If we denote with the index \( i \) each elementary truss of fig. 12 and define \( F_{0i} \) the corresponding limit load, we have:

\[
F_0 = \sum F_{0i} ; \quad F_{0i} = \min \left[ C_{0i} \sin \alpha_i, T_{0i} \tan \alpha_i \right]
\]

(20)

where \( T_{0i} \) and \( C_{0i} \) are respectively the limit axial loads of the tension and the compression rods of the corresponding \( i \)-th elementary truss. The limit tension \( T_0 \) is equal to the yielding load of the corresponding steel tie minus the tension due to the transferring of the seismic actions from the neutral to the active walls (see 1.) and the one due, if it is an external tie, to stand up to the thrusts of the floor arches under the vertical loads. The limit compression \( C_0 \) is equal to
overthrust that the corresponding arch can support without collapse.

The calculus of $C_0$ can be approximatively carried out assuming that under the vertical loads at the extrados midspan of the arch (position A of fig. 13) an hinge forms because of some small displacements at the springings; on the contrary when the arch collapse under the action of the overthrust $C_0$ an other different hinge must form at the intrados midspan (position B of fig. 13). Therefore the limit overthrust $C_0$ is the difference between the springing thrusts calculated firstly locating the midspan hinge at the intrados and then at the extrados: with the meaning of the symbols of fig. 13 we have approximately:

$$C_0 = W \left[ \frac{(1-d)}{(h+s)} \right] \left[ \frac{s}{h} \right]$$

If the condition $F_{ji} < F_{0ji}$ is satisfied everywhere the collapse multiplier $\lambda_0$ calculated under the hypothesis of inplane rigidity of the floors is correct; on the contrary if for instance $F_{ji} > F_{0ji}$ at one floor at least, the collapse multiplier will be certainly lower; it can be however calculated as the lower between the two distinct ones corresponding to the simultaneous translational collapse of the first $l$ walls and of the other $N_w - l$, with $N_w$ number of active walls.

If under the hypothesis of inplane floor rigidity the collapse mechanism of the building is rotational around an axis located at the crossing of an active $k$-th wall and a neutral one, this means that also the neutral walls collapse; but in this case the corresponding steel ties yield and the above described inplane stress transfer from the weakest active walls to the strongest uncollapsed one cannot occur. The effective collapse multiplier of the building can then be calculated as the lower between the two distinct ones corresponding to the two smaller structures formed respectively by the first $k-1$ active walls and by the last $N_w - k$.

References


