Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

The Elliptical Dome. A survey of constructive techniques to stabilize a sopisticated structure

F. Escrig[1], A. Cobreros[2], J.P. Valcárcel[3]
[1]*Prof. of School of Architecture of Sevilla, Avda. Reina Mercedes, 2. 41012. Sevilla*
Email: felix@arqui4.us.es
[2]*Assistant Prof. of Architecture of Sevilla, Avda. Reina Mercedes, 2. 41012. Sevilla*
[3]*Prof. of School of Architecture of La Coruña*

Abstract

The Renaissance and Baroque architecture is very well studied and nevertheless the most complicated structures that architects then used have a lack of information and research. Elliptical domes were built with profusion, but their particular way of working is not well studied yet. Even in the actuality the mathematical analysis ignore systematically these forms. This paper tries to introduce for a first time, an attempt to know something more about these structures.

1. Introduction

The geometry of the ellipse is very well know from Euclides. Their use to solve the design of Roman Buildings as amphitheatres are well known. But they never was traced as true ellipses and roman architects made approximate lines by means of arches of circle (Fig. 1) as Romanesque, Byzantine, Renaissance and Baroque architects did later.

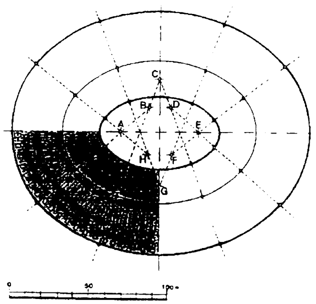


Figure 1. Oval trace of Coloseum in Rome

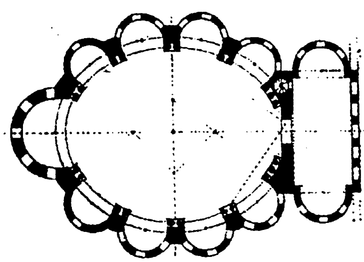


Figure 2. St. Gereon in Colone

The first domed building in a near elliptical plan was St. Gereon in Colone (Fig. 2). And we have samples in the ottoman architecture like Zeyrek Kilise Camil, (Fig. 3). The true ellipse was used only in rare occasions because the difficulty of drawing them and not because ignorance. Durero in 1525 (Fig. 4) and Serlio in 1545 (Fig. 5) or De L'Orne in 1561 explained the trace very well. But all them proposed alternatives to draw ellipses by means of equivalent circular arches (Fig. 7 and 8).

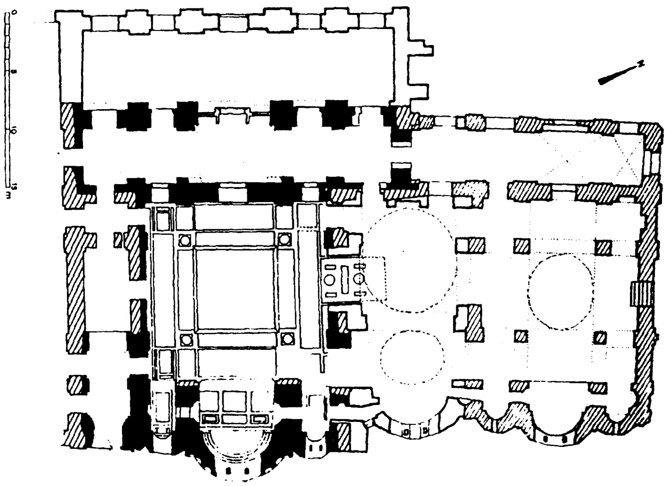


Figure 3. Zeirek Kilise Camii

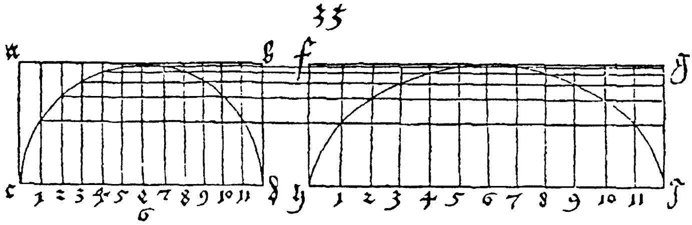


Figure 4. Durero trace of ellipse

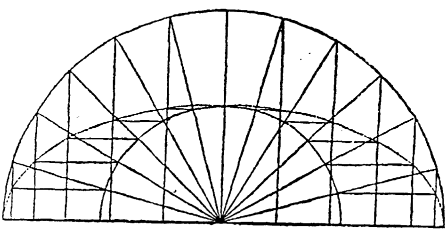


Figure 5. Serlio trace of ellipse

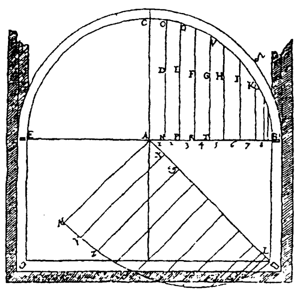


Figure 6. L'Orme trace of ellipse

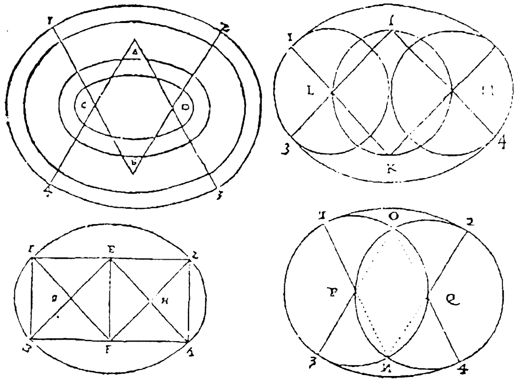


Figure 7. Serlio traces of different ovals

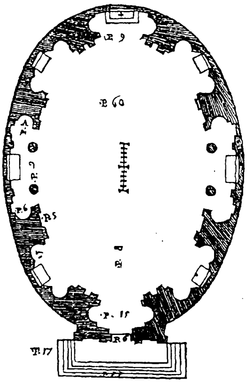


Figure 8. Serlio Oval Room. Book V

With the basis of an elliptical plan we can generate domes with different geometry. They can be ellipsoids with three different axis lengths or with only two different lengths. The dome can be constructed with meridians and parallels or by the revolution of plane elliptical arches. It is evident that each of these different possibilities gives different form of working.

If we take as a reference the shell of revolution with elliptical directrix, domes with oval plan varie the curvature non only in the meridian planes but also in the hoops. In structural analysis this means that they have not only normal stresses but this includes shear stresses.

2. Elliptical domes in the Renaissance and Baroque

During XVI and XVII Centuries a lot of oval and elliptical domes were built. Along this period the oblong geometry was used to cover rectangular unit naves. Later we will find in the German Rococo different combination of forms. The table 1 shows the main features till 1650. Most of them are Italian designs built in brick. But also have others as many spanish proposals built in stone. The table reveals that the relationship between the main axis goes from 1.27 till 1.5 and that height is 1 :1 with respect to shortest axis. An other characteristic is that all them are built as meridians and parallels at least in a formal aspect.

In some cases, like St. Andreas do (Fig. 9) the dome is supported directly by pendentifs, in other they lean on a perforated drum, and even in others the dome is perforated with oculi at its lower part, like in Ste. Anne (Fig. 10).

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

TABLE I

Architect	Building	Date	Dimen.	Prop.	Place	Fig.
Quijano	Capilla Catedral	1540			Murcia	12
Vignola	S. Andrea	1550	10x7	1,35	Rome	9
Vignola	S. Anne	1565	16x11	1,50	Rome	10
Volterra	S. Giacomo	1592	27x19	1,40	Rome	13
H. Ruiz II	Sala Capitular	1592	15x10	1,50	Seville	16
Vittozzi	S. María	1596	30x20	1,50	Vicoforte	14
H. Ruiz II	Catedral	1599	14x11	1,27	Córdoba	20
P. Sánchez	S. Hermenegildo	1616	24x16	1,5	Córdoba	22
P. Sánchez	S. Antonio Portugués	1624	16x12	1,33	Madrid	21
S. Plaza	Convento Bernardas	1626	25x17,5	1,43	Alcalá	23
Borromini	S. Carlo	1638	25x19	1,31	Rome	24
Ponce	N.S. Desamparados	1652	21x14	1,5	Valencia	25
Bermini	S. Andrea	1658	26x17	1,5	Rome	26

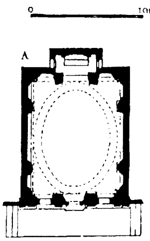


Fig. 9

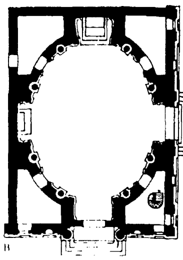


Fig. 10

Figure 9.Vignola. St. Andrea. Rome
Figure 10. Vignola. Ste. Anne. Rome.

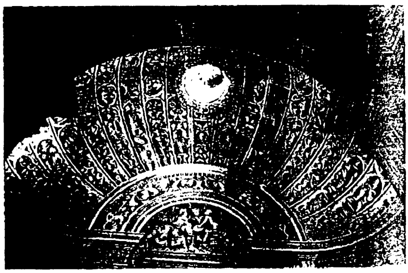


Figure 12. Quijano. Chapel of the Junterones. Murcia Catedral

3. Constructive systems. Vandelvira notebook

The Vandelvira Notebook shows a complete guide of cutting patterns and traces of these domes (Fig. 11). Vandelvira begins with the revolution of an arch of ellipse twisting around the short axis in plan that concludes in a torical geometry when the ellipse is a circle. The chapel of Murcia Cathedral is build with this system. It is called the "Murcia Dome" (Fig. 12). The next pattern by twisting a ellipse around the long axis goes like a half melon. The other in Fig. 11 are solved by meridian and parallels as usual.

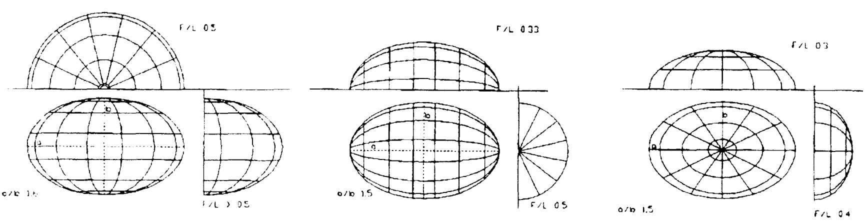
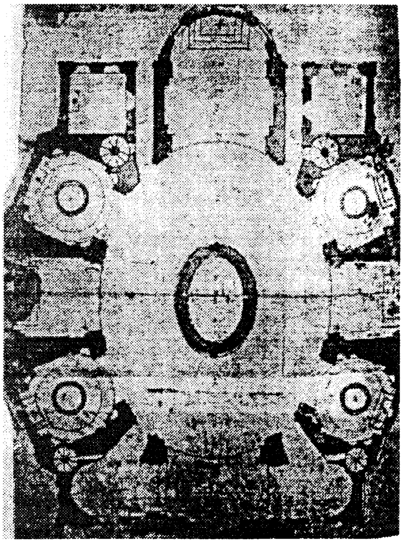


Figure 11. Vandelvira traces for oval domes. a) Murcia dome. b) Melon dome. c) Meridianas and paralles dome

4. Structural analysis of the dome of revolution with horizontal axis

The building of these domes presents some questions because stresses in each meridians are not the same nor the stresses in each loop. We know in practice that these domes are very flat at their crown an suffers great bending moments an also cracks in the lowest part. They are greater in the short axis, as we can see in some examples as S. Giacomo (Fig. 13) and Vicoforte di Mondovi (Fig. 14 and 15).



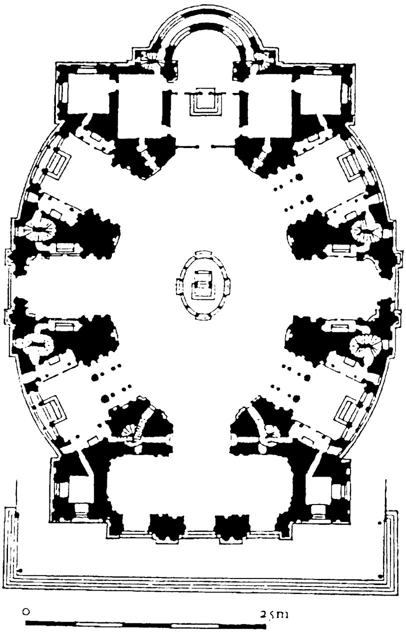


Figure 13.Volterra. St. Giacomo
Figure 14. Vittozzi. St. Maria in Vicoforte

In this case we studied briefly the case of Capitular Room Dome in The Cathedral of Sevilla by Hernan Ruiz II. We plan our analysis with spatial model of bars in which we include also diagonal elements to count with the stiffness of the coffers and the nerves. For our proposal it is enough to consider only dead loads (Fig. 16).

Fig. 17 shows the model and Fig. 18 the variation of stresses, where the two lower loops are in tension. Fig. 18 shows displacements that coincide clearly with fotogrametic measures done in the real dome.

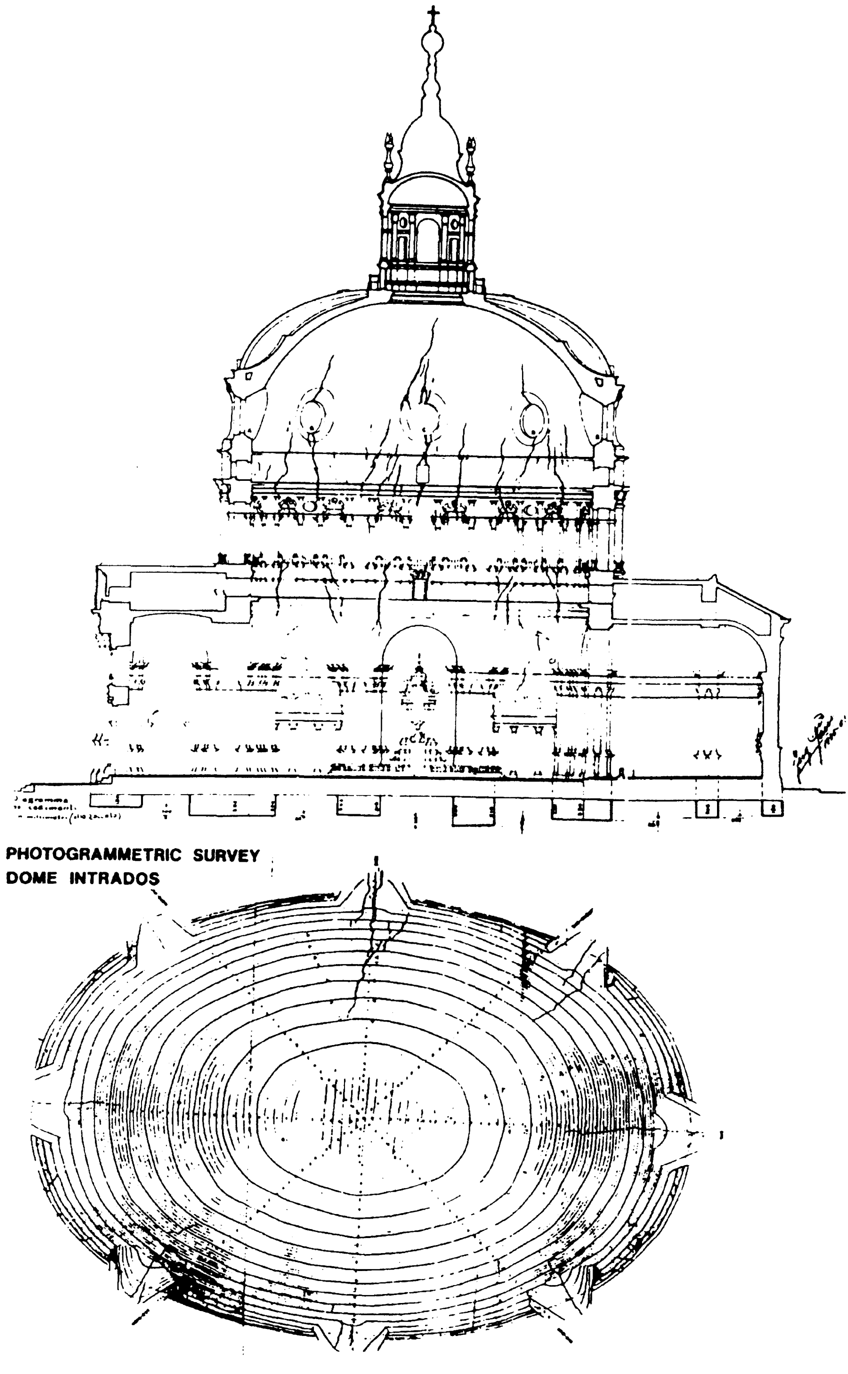


Figure 15. Crack patterns in the dome of Ste. Maria in Vicoforte di Mondovi

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

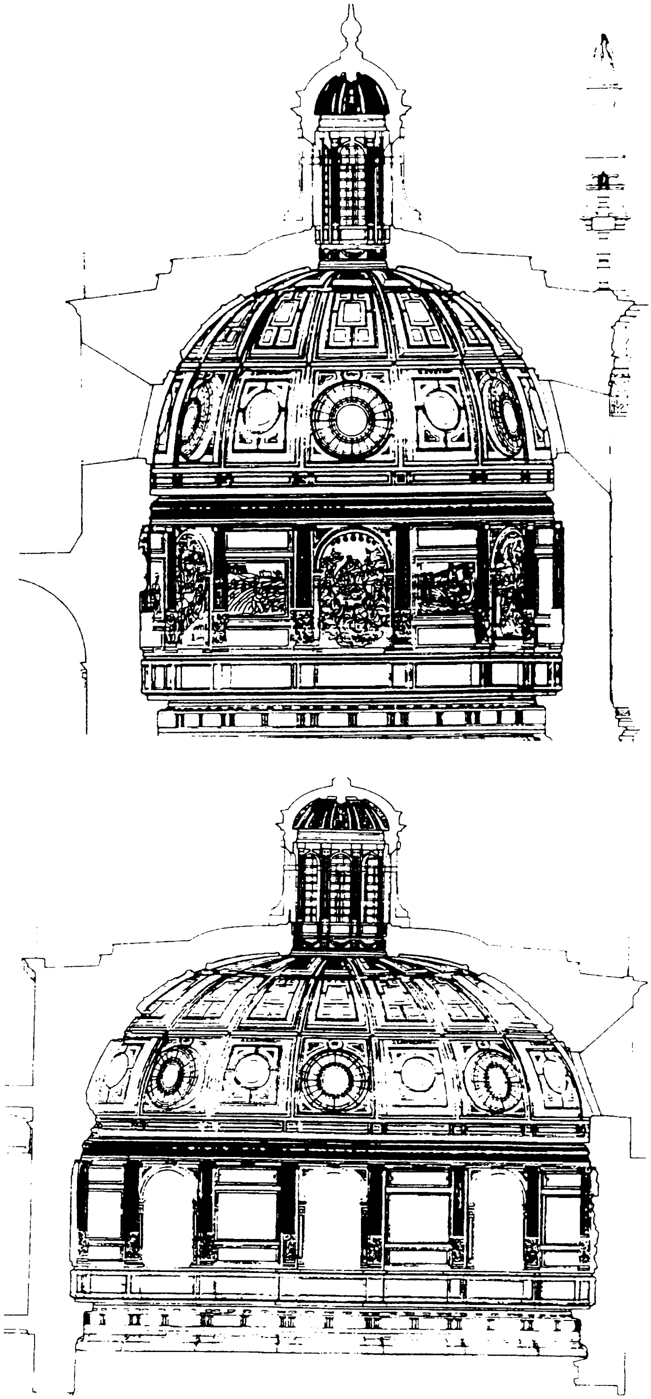


Figure 16. Hernan Ruiz II. Capitular Room in the Catedral of Seville

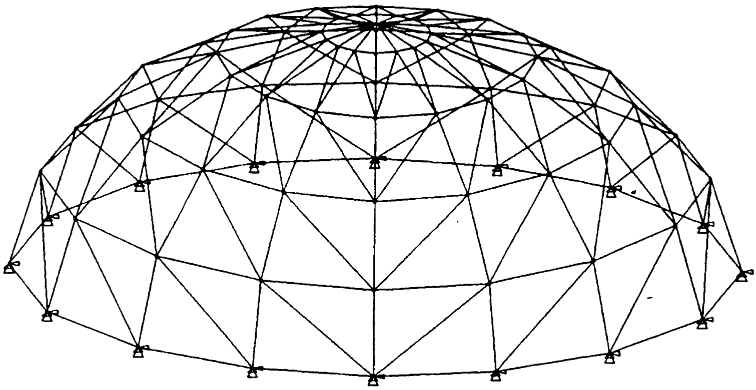


Figure 17. Matematical Model of the Capitular Room of Catedral of Sevilla

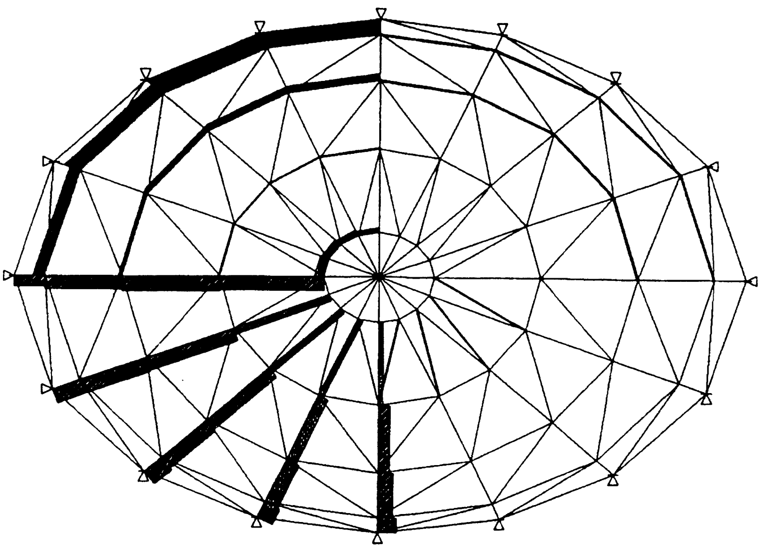
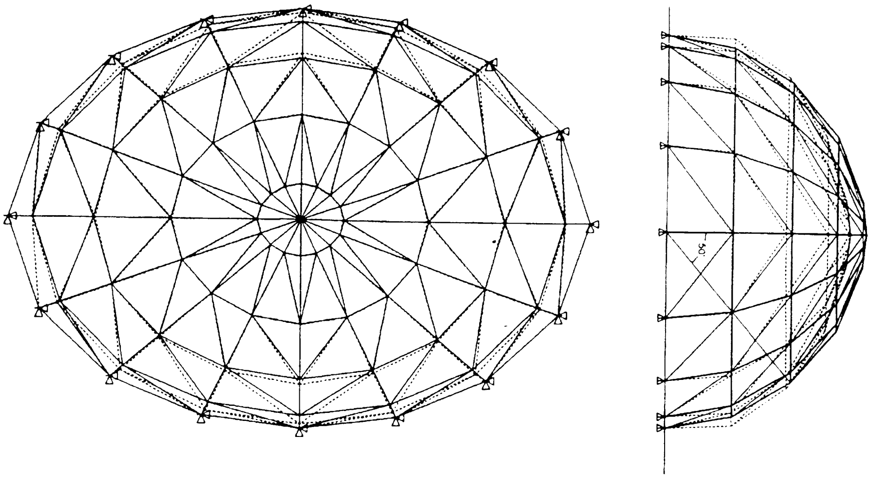


Figure 18. Variation of Stresses

5. Conclusions

Although the elliptical geometry is very problematic because its lack of uniformity in stresses, the difficulty of hoop the parallels, and great displacements, it has been an usual solution to cover rectangular areas and as Neuman did in the XVIII century to substitute barrel vaults in basilical plans by combination of sectioned elliptic domes with very light materials. The lack of knowledge of this field lead us to propose these short observations to stimulate wider research on the subject.

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509



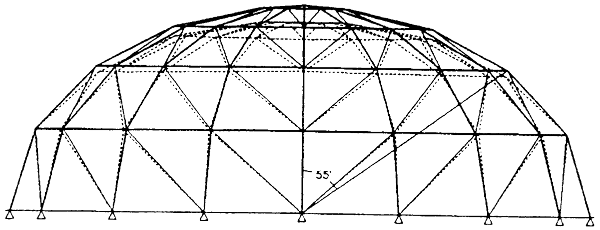


Figure 19.
Displacements

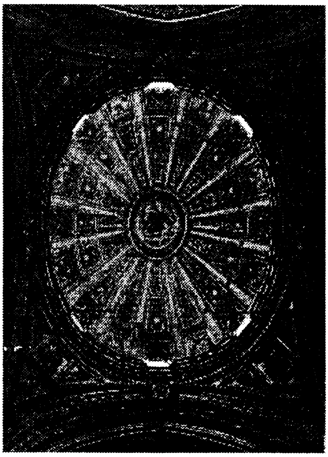


Figure 20. H. Ruiz II.
Córdoba Catedral.

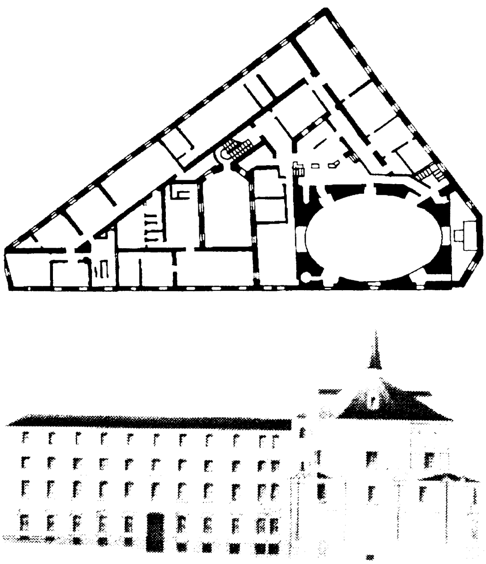


Figure 21. P. Sánchez.
S. Antonio de los Portugueses

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

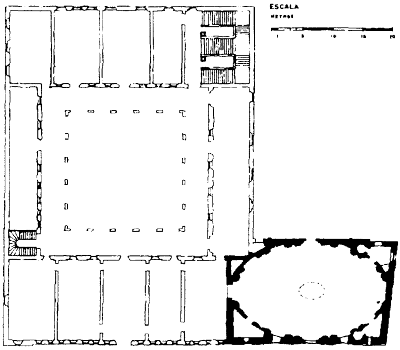


Figure 22. P. Sánchez. S. Hermenegildo

Figure 23. P. Sánchez. S. Antonio

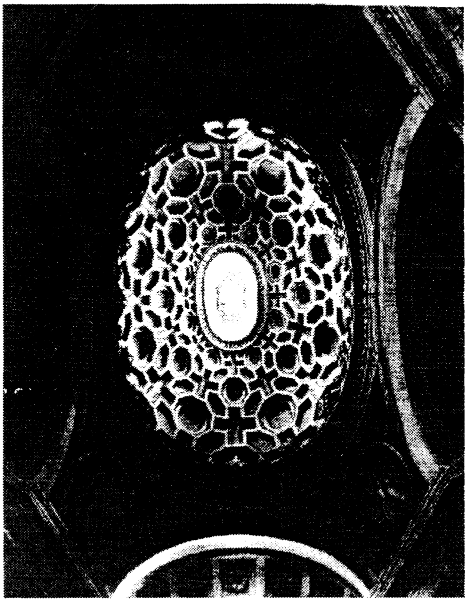


Figure 24. Borromini. S. Carlo

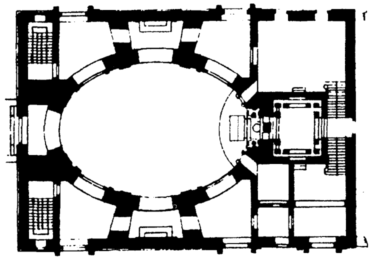


Figure 25. Ponce. N.S. Desamparados

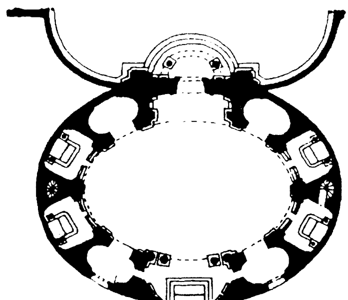


Figure 26. Bernini. S. Andrea

References

A. MORALES. "Hernan Ruiz El Joven". Akal Madrid. 1996

J.C. PALACIOS. "Trazas y Cortes de Cantería durante el Renacimiento Español". Ministerio de Cultura. Madrid. 1990.

J. COURBON. "Estructuras Laminares". E.T.A. Barcelona, 1981.

J.M. GENTIL. "La Traza Oval y la Sala Capitular de la Catedral de Sevilla". Cuatro edificios sevillanos. Colegio Arquitectos Sevilla, 1996.

W. LOTZ. "Architecture in Italy". Yale University Press, 1995.