Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

The Structural Safety and Acceptability of Bell Towers.

AR.Selby & JM Wilson
University of Durham, DH1 3LE, a.r.selby@durham.ac.uk

Abstract

An exciting element of the Millennium celebrations in the United Kingdom is the re-instatement or renewal of peals of church bells in towers throughout the country to welcome in the year 2000. A sum of £3M is allocated towards this work. Many towers will need to be assessed structurally, to ensure their stability under the large dynamic forces arising from the full circle swing of heavy bells. To date the choice of critical stress or deflection parameter, and of a safe limiting value is unresolved. The majority of traditional bell towers are constructed from sandwich form masonry walls, with indeterminate stiffness and mass properties, resting on spread footing foundations of unknown size and depth, and on ill-defined soils.

A survey of 19 active bell towers throughout the North of England provides a database of towers each of which is in satisfactory condition with respect to whole-circle bell ringing. Of particular interest is the tower of St Matthew's, Newcastle, in which the bells were initially hung, and rung, at the original designed height near to the top of the tower; however, the tower swayed so fiercely that it was designated unsafe in that condition, and the bells were re-hung lower down the tower, where they are now rung without ill effects.

The 19 towers were assessed structurally, using parameters of varying complexity. First, the towers were ranked based simply on tenor bell mass and tower proportions. Secondly, the maximum ratio of observed sway displacement to overall tower height was calculated for each tower. Next, the peak accelerations in the ringing chambers were compared. Finally, St Matthew's tower was analysed by finite element, with various stress parameters being calculated for the two cases of the tenor bell in its original position and in its present position lower down the tower.

The conclusion from the study was that the preferred parameter for evaluation of acceptability of a bell tower is the ratio of maximum sway/tower height. A limiting acceptable level of 30 x 10^{-6} is proposed for square topped towers. A tower topped with a spire may be competent to tolerate a slightly higher value of this ratio, because of its greater flexibility. The peak acceleration at ringing chamber should also be limited to $50mm/s^2$.

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

Introduction.

There are many church bell towers around the United Kingdom which once carried a peal of bells, numbering between three and 12, with each bell weighing between 100kg and 1500kg; the bells would have been rung in methods or rounds, see Wilson [1], with each bell swinging in a full vertical circle. Other towers contain bells which are fixed, and are sounded only by swinging a clapper to strike the rim of the stationary bell. The constructional design of the tower and bell frame evolved over many centuries through the guilds of masons, see Towers and Bells Handbook [2]. Each bell has a headstock supported by bearings on the bell frame, and a rope wheel and rope to enable ringing. The bells are generally arranged within a tower so that the heaviest bells swung nearest to the tower axis so as to cause least tower torsion, and also so that the ropes in the bell chamber hang in a circle ranging progressively from the heaviest bell through to the lightest. Nearly all traditional bell towers were constructed before structural analysis was available. Consequently, there are no agreed standards for limiting the forces from a swinging bell onto the tower.

The horizontal force from a heavy bell swinging through a full vertical circle can be evaluated as a function of time using elliptic integrals to model the behaviour of a compound pendulum, Wilson [3]. An example trace is shown in Figure 1. To quantify the force function for a particular bell, it is necessary to estimate the bell mass, the bell moment of inertia about its centroid, and the eccentricity of the bell centroid from the bearings axis. The mass of a bell is normally recorded both in the bell tower and also by the foundry. The latter two parameters can be evaluated by simple measurements following the proposals of Heyman and Threlfall [4]. This technique has been used many times on bells in the North of England, e.g. Evans [5] and Lund *et al*[6].

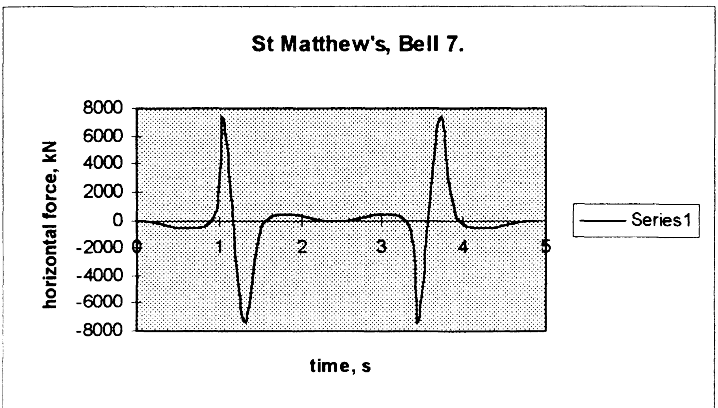

Figure 1. Horizontal force as a function of time, St Matthew's, Bell 7.

Measurements of the transient sway behaviour of a tower during tolling of a heavy bell have been made by several investigators including Lund *et al*[6] and Wilson *et al* [7], with progressively improved equipment and techniques, so that a comprehensive database of tower behaviour has been assembled. Each of 19 towers has been surveyed, the bell forces estimated, and the sway response recorded in detail. Natural frequency, mode shape damping factor, and maximum transverse amplitude (in terms of displacement, velocity and acceleration) have been deduced from the records of tower response. The mode shape was separated into components of rocking on the foundation, shear sway and bending as a vertical cantilever beam. The measurements were conducted using high sensitivity, low frequency, accelerometers or velocity transducers. Figure 2 shows a typical time based record of transient acceleration as a function of time.

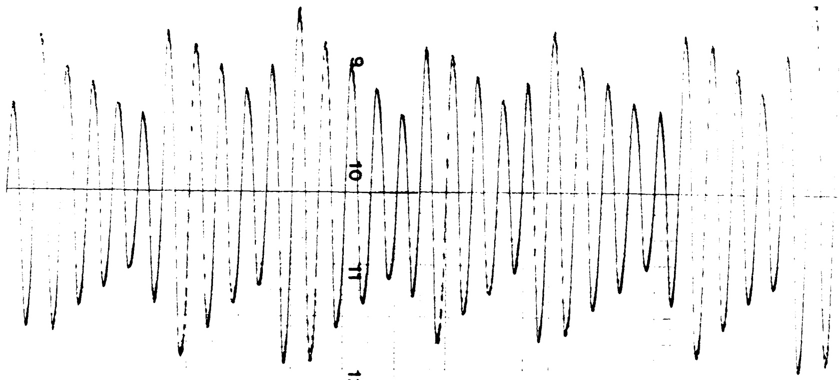

Figure 2. Example trace of tower acceleration against time.

Given the geometric dimensions of a tower, and a description of a bell force as a function of time, it should be a straightforward computational exercise to calculate an eigenvalue solution for natural frequencies and mode shapes, and a transient response during bell ringing. However, the uncertainties of the stiffness of the sandwich wall construction, the condition of the mortar joints, the mass of the walls, the depth and stiffness of the foundations, and the degree of structural connection to the church nave create difficulties in an analysis.

In recent publications, Wilson *et al*[7], Wilson and Selby[8] a combined measurement-and-analysis approach has been described from which stiffness, mass and damping properties of a tower responding to the forces from a tolling bell may be derived. Ranges of typical values of these parameters have been identified, Selby and Wilson[9], and a number of towers have been analysed successfully using finite elements. Tall, relatively slender towers can be represented effectively using Timoshenko beam elements incorporating shear deformation, while short stocky towers may need 3D brick elements.

Transactions on the Built Environment vol 26, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

This background of measured tower behaviour and developed computational methods is now used to evaluate a range of parameters for determining the structural acceptability of re-commissioned towers, taking particular account of the tower of St Matthew's.

Parameters for structural acceptability.

Structural acceptability can be considered in terms of either a serviceability limit state or an ultimate limit state. The limit to serviceability is excessive deflection causing discomfort or anxiety to people in the tower or church during ringing, or to render accurate change-ringing very difficult. Ultimate limit state conditions will here be grouped as those which cause an unacceptably low factor of safety against structural damage.

Criteria for evaluation of a number of possible parameters will be either that the 19 towers are acceptable, although St Matthew's was unacceptable with bells higher in the tower, or a small number of stress limits.

1. Some simple geometrical considerations.
The first evaluation was based on an attempt to identify simple measurements which might give a coarse acceptability parameter. They were the ratios of tenor bell mass to tower mass, and of tower height to width, and the product of these two ratios.

i) Mass of tenor bell divided by total tower mass, or m_b/m_t. This ratio is a crude measure of the significance of the peak bell force on the tower. The results of calculations for the 19 churches were not helpful, in that the ratio was largest for the smallest towers with spires, St Mary's in Shincliffe, and St Cuthbert's in Benfieldside, see Table 1. This result was obtained because the variation of tower size was far greater than the variation in bell mass. Overall, the ratio ranged from 0.0002 to 0.002.

ii) Height to width ratio of the towers, h/b. This ratio gives an indication of the flexibility of the towers. In fact, the results indicated that the four churches with spires ranged from 5.7 to 8.4, while the remaining 15 square topped towers were tightly grouped around a mean of 3.2. Of itself this parameter is insufficient to give guidance towards the most responsive towers.

iii) The product of the previous two parameters, $(h \times m_b)/(b \times m_t)$. Superficially, this non-dimensional parameter might be expected to identify the most flexible tower with the most significant bell force, but it too failed to identify the towers considered to be most lively or most at risk. Values lay between 0.0009 and 0.0058; see Figure 3.

Table 1 Tower characteristics.

Church	Tower height h, m	Tenor bell mass kg	bell mass/ tower mass	height / width	$\frac{m_b \times h}{m_t \times b}$	ringing chamber accln m/s^2	disp/ ht x10^{-6}
St Oswald, Durham	24.8	650	0.0006	3.4	0.002	19	15
St Mary, Schincliffe	30.4	210	0.0002	5.7	0.001	4	2
St Cuthberts Ch.le.st	43.5	966	0.0007	6.0	0.003	42	29
All Saints Lanchester	19.7	431	0.0008	3.3	0.003	18	17
Christchurch Consett	16.2	821	0.0012	1.9	0.002	44	25
Newcastle Cathedral	32.8	1913	0.0009	3.0	0.003	7	7
StMichael Houghton	21.4	610	0.0009	2.9	0.003	13	5
St Nicholas Durham	43.5	516	0.0006	8.4	0.005	3	14
St Margaret Tanfield	16.5	660	0.0017	2.7	0.004	23	6
St Andrew Roker	20.5	1154	0.0022	2.6	0.006	2	0.1
St Matthew Newcastle	28	1102	0.0010	3.9	0.004	44	30
StAndrewBpAuckland	23.2	608	0.0006	3.2	0.002	9	5
St John Shildon	18.5	829	0.0014	2.7	0.004	6	2
St Michael Heightn	16.2	781	0.0019	2.7	0.005	14	5
St Edmund Sedgefield	25.4	533	0.0006	3.5	0.002	14	6.4
StBrandon Brancepeth	20	693	0.0012	3.6	0.004	10	11
St Cuthbert Benfldsde	33.8	642	0.0006	6.8	0.004	20	4
Durham Cathedral	66	1425	0.0002	4.9	0.001	21	5
St Mary Richmond	24	559	0.0007	4.1	0.002	10	5

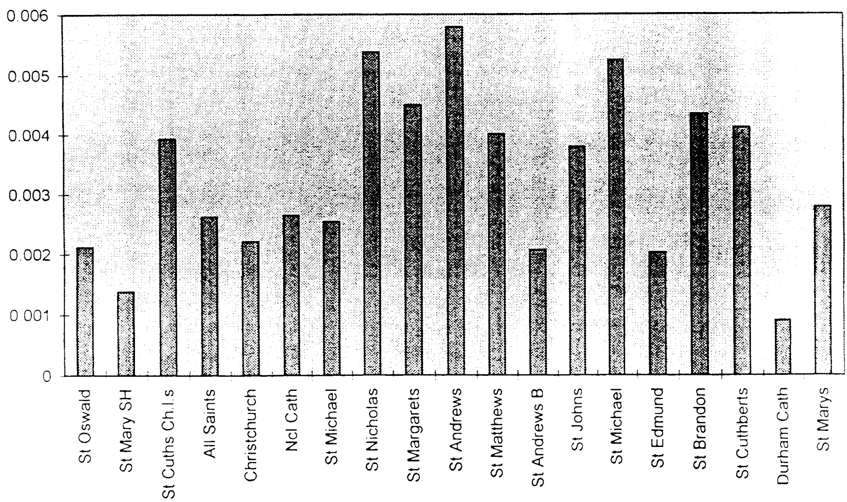

Figure 3 Product of the ratios (bell mass/tower mass)*(height/width)

2. Peak acceleration at ringing chamber level.

Magnitudes of vibration likely to cause concern to occupants of buildings are discussed critically in BS 6472, [10], although the tolerance of bell-ringers is not mentioned specifically! During bell ringing, it is standard practice that no-one is allowed in the belfry, above the ringing chamber (except during tower assessment, with careful safety procedures). The ringing chamber is generally at a fairly low level in the tower or may even be at ground level, so the vibration observed by the ringers is small However, in St Matthew's the ringing chamber is close to the bell chamber; see Figure 4.

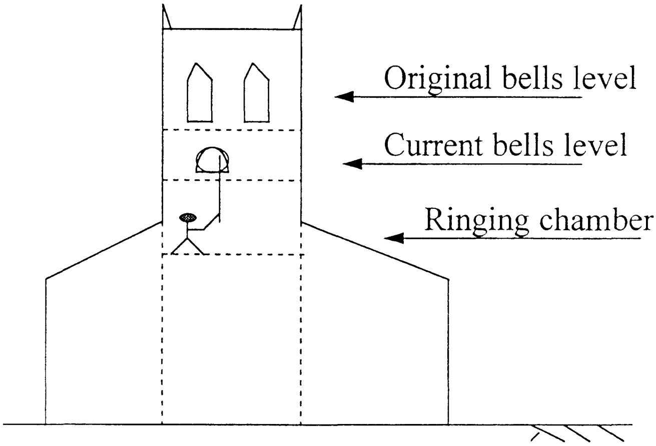

Figure 4. Schematic elevation of St Matthew's. Not to scale

By chance, the ringing chamber highest up any of the 19 towers (by proportion) was St Matthew's. The peak horizontal acceleration recorded in St Matthew's ringing chamber was $\underline{43\text{mm/s}^2}$, at a frequency of 1.5Hz.

The threshold of human perception of vibration of this frequency is probably about 5mm/s^2, rms, while the condition for low adverse comment is some 30 mm/s^2 rms, or $\underline{42\text{mm/s}^2}$ peak, according to BS 6472 [10], (working on a 1 hour duration of ringing and a location equivalent to an office environment to give a base curve multiplying factor of 8).

This close correspondence might be seen as a remarkable coincidence, or conversely it might be deduced that vibrations perceived in the ringing chamber were considered to be excessive when the bells were higher in the tower (at some 1.5 x 42 or 63 mm/s^2), but acceptable at the level recommended by the British Standard.

Christchurch Consett, another tower considered to be 'lively' and with a relatively high-level ringing chamber, showed peak acceleration of 44 mm/s^2. Conversely, the tower and spire showing by far the greatest tip movement, St Cuthbert's Chester-le-Street, had a relatively low-level ringing chamber where peak acceleration experienced was 36mm/s^2. In none of the other towers did accelerations at ringing-chamber level exceed 25mm/s^2.

Thus it might be deduced that the acceptability of a tower is primarily a function of the perception of the ringers to horizontal vibration at *ringing chamber level.* This is not related to structural integrity, but if bell-ringers feel that vibration is excessive, they may decide not to use the tower, and a serviceability limit has been reached. An inexpensive solution is to site the ringers lower down the tower with longer ropes. At a practical level, a single measurement of the peak horizontal acceleration of the ringing chamber floor can be made using a simple vibro-meter. Alternatively, peak acceleration can be estimated by FE analysis.

3. Ratio of maximum displacement to height.

There is some attraction in the parameter of maximum sway displacement divided by height, d/h, in that it relates broadly to human tolerance (people tolerate greater sway in very tall buildings) and in addition it relates to shear strain, which may be critical structurally.

The d/h values for the 19 towers can be plotted on a histogram, as in Figure 5.

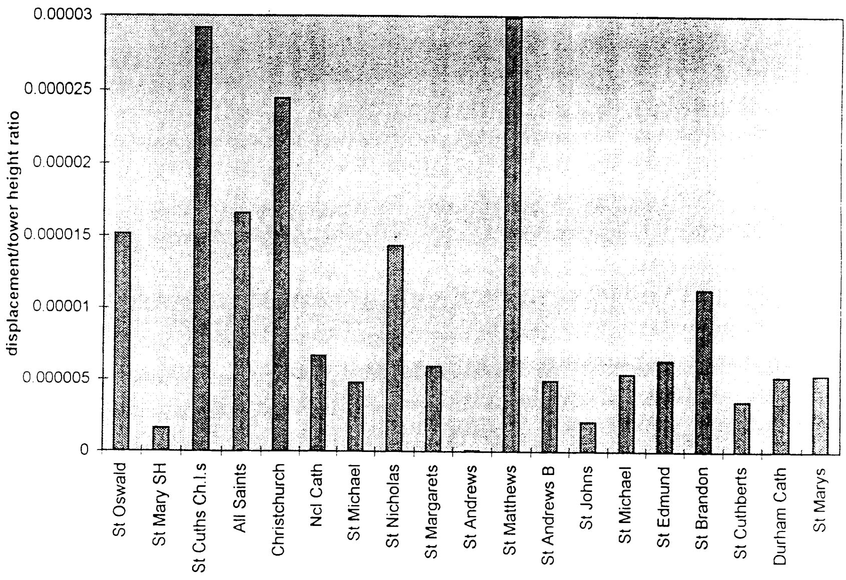

Figure 5 Histogram of values of the ratio of maximum sway to tower height.

St Matthew's, St Cuthbert's Chester-le-Street and Christchurch are all in evidence, with the largest value being 30×10^{-6}. The remaining tower values ranged from 17×10^{-6} to almost zero. Thus the parameter is appropriate with respect to human tolerance, albeit by indirect means.

With respect to global shear strain it is difficult to determine a permissible value, and the ratio in question is not an exact measure of shear deformation alone. However, a value of some 200×10^{-6} might be deduced from discussion by Hendry [11] on brickwork, and noting the softer mortars of old buildings. Thus the broad estimates of shear strain are well below a permissible value.

It seems that a limiting value of the d/h parameter may offer a control over tolerable vibrations for the bell-ringers (a serviceability criterion), and may also ensure an adequate safety factor against failure in shear (an ultimate limit state), by reference to the undamaged conditions of the examples of towers from the database.

The estimation of a tower sway due to a swinging bell can be made using established FE models, with selection of effective stiffness and mass based upon experience of previous analyses, and with bell forces calculated as described previously. Alternatively it may be measured directly or indirectly.

4. Stresses induced during ringing.

As described previously, the procedure necessary to achieve a reliable computational analysis of a tower during ringing requires detailed site measurements in combination with FE analysis firstly for an eigenvalue solution, and secondly for a transient response computation. After such an FE model has been derived and verified, it is possible to compute various stress components at many locations around the tower at the moment in time when the largest transient sway occurs.

This procedure was followed through for the St Matthew's tower, using a 3D brick element FE model, firstly for the bells in their present location, and secondly for the bells in their original, higher level location. A view of the FE half-tower model is shown in Figure 6.

i) Vertical stress due to bending and rocking deformations.
Each tower responds in its own way with different components of rocking, shearing and bending. One form of violation of the ultimate limit state would be if the vertical tensile stress in one edge of the tower exceeded the static vertical compressive stress due to the tower self weight. Detailed perusal of the vertical stress field showed a maximum tensile stress of 8.5×10^3 Pa (12×10^{3} Pa for bells in the higher belfry), which can be compared with the self weight stress in

the same location of 600 x 10^3 Pa. The safety factor for this form of behaviour is huge.

ii) Shear stress in the side wall.
The computed side wall shear stresses, τ_{xy}, were found to be nowhere in excess of 1000Pa. This might be compared with a value of some 200 x 10^3 Pa deduced from Hendry, [11]. Consequently they were insignificant.

iii) von Mises stresses.
A preferable estimate of significant shear stress in a 3D analysis is to consider the von Mises stresses. Around the tower base the most severe von Mises stresses were computed to be 8 x 10^3 Pa (or 11 x 10^3 Pa for the higher bell position). Again, these can be compared with a limiting shear stress of some 200 x 10^3 Pa, which results in a very large safety factor of the order of 20.

These detailed computations of peak transient stresses indicate that the levels of stress induced by bell ringing into a bell tower which is in sound condition, are extremely small and offer no risk of structural distress.

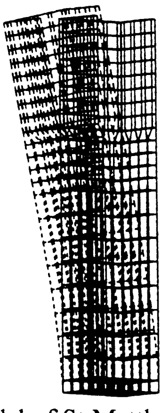

Figure 6 FE half-model of St Matthew's tower.

Conclusions

Bell towers have been considered with respect to their serviceability and structural integrity during full circle bell ringing. Although details are available for 19 active towers in the North of England for comparison, knowledge of the bell mass and tower dimensions is insufficient in itself to deduce whether a different tower would meet acceptability criteria.

There seems to be strong correlation between horizontal vibration at ringing chamber level, and personal acceptance. Where peak acceleration exceeds some 50 mm/s^2, then ringers may be reluctant to use the tower.

The ratio of peak sway displacement to tower height corresponds to personal acceptance of vibration, and also ensures that shear strain is controlled which gives some control on the structural competence of the tower. A value of 30 x 10^6 should prove acceptable, while a value of 45 x 10^6 would be excessive.

Detailed FE analysis of the sensitive tower, St Matthew's in Newcastle, revealed that the magnitudes of stresses induced by bell ringing were very small indeed and posed no risk to a tower structure in sound condition.

If doubt exists as to the structural integrity of the bell frame and tower, perhaps due to severe existing cracking, or loose joints between frame members or frame to tower, then a practical assessment of any progressive deterioration can be made simply by measuring the natural frequency of free tower vibration, periodically e.g. annually. If damage is occurring, then the natural frequency will decrease, indicating that remedial work is necessary.

References.

1. Wilson, W.G. *The Art and Science of Change Ringing,* Faber & Faber, London. 1965
2. Towers and Belfries Committee, *The Towers and Bells Handbook*, Central Council of Church Bell Ringers, Guildford
3. Wilson, J.M. Periodic Forces on Bell Towers Arising from Bell Ringing, *Internal report, Durham University*, 1988
4. Heyman, J. and Threlfall, B.D. Inertia Forces Due to Bell Ringing, *Int. J. of Mech. Sci.* Vol. 18, pp 161-164. 1976
5. Evans NC The Response of Bell Towers to Bell Ringing, *Internal Report, Durham University* 1990
6. Lund, J., Selby, A.R. & Wilson, J.M. Dynamics of Bell-Towers - a Survey in Northeast England. *STREMA 4* ed Brebbia, CA & Leftheris, B CMP p 45-52, 1995
7. Wilson, J.M., Selby, A.R. & Ross, S.E. The Dynamic Behaviour of Some Bell Towers During Bell Ringing *STREMA 3* ed Brebbia, C.A. & Frewer, RJB CMP p491-500, 1993
8. Wilson, J.M. and Selby, A.R., Durham Cathedral Tower Vibrations During Bell-Ringing in *Engineering a Cathedral* ed Jackson, MJ, T.Telford p77-100, 1993
9. Selby, AR & Wilson, J.M. Dynamic Behaviour of Masonry Church Bell Towers *ASCE Structures Congress 96, Stability Analysis of Masonry Structures*, Chicago, 1996
10. British Standard *BS6472 Guide to evaluation of human exposure to vibration in buildings (1Hz to 80Hz),* 1992. BSI London
11. Hendry AW *Structural Brickwork* 1981. Macmillan press.