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Persian vaulted architecture: morphology and equilibrium of vaults under static and dynamic loads

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Abstract

Persia has created an original monumental vaulted architecture, whose morphology replies to spiritual and technical imperatives, sometimes contradictory. The analysis of calculations to finite elements undertaken on several models of vaults, under dynamic and static loads, allows us to specify the evolution of techniques of vaulting of the Persian architecture in the course of centuries, and to test the empirical solution pertinence used.

1 Specificity of the Persian vaulted architecture

Pre-Islamic Persia has created an original monumental vaulted architecture. Its architectural forms reply essentially to two imperatives: a technical imperative and a spiritual imperative. From a technical point of view, the Iranian geographical space being often arid and lacking of wood, builders have perfected vault techniques especially in bricks, cooked or unfired and more rarely, in stones built without using centerings. These vaults had furthermore to guarantee as far as possible a suitable resistance to frequent earthquakes in the region. From a spiritual point of view, symbols linked to the space have determined an original architectural morphology.

In the tradition of antic Persia, the square is the symbol of the terrestrial world, it represents the four cardinal points corresponding to the four directions of terrestrial space, the four elements of the matter: the earth, the air, the water and the fire, the four seasons, etc... The dome, a "flawless" geometrical structure, possessing only one remarkable point: its summit, is the symbol of the heavens. Builders by superposing these two volumes, have had to solve the technical problem of the continuity between the circular plan of the dome and the squared plan of the pedestal of support without having recourse to beams, not possessing adequate materials. The Persian solution is that a zone from transition comprising four angle squinches (little vaults in form of 1/2 cones to the intrados) spanning the dihedres formed by the top of walls, at 45°, connected laterally by parts of masonries, overcoming walls warping gradually between the base and the key of squinches and often pierced by an opening (Figure 3).

An other original architectural form created in Persia is that the "ivan"or monumental porch. It concerns a room in which one of the sides is entirely opened to the exterior. It is a space of transition that indicates the path of the material world to the spiritual world. The monumental ivans were first constructed in the form of barrel vault in semicircular arch or elliptic arch by leaving one tympanum opened. By continuation the ivans were created in the form half-domes sectioned following a meridian vertical plan (Figure 1c). This half-dome structure constitutes an original stability problem. The stability of a dome depends largely on the continuity of its average surface from the summit to the supports, horizontal sections of the dome constituting rings subjected to compression stress in the upper part, or sometimes, subjected to traction stress at the base. In the case of a half-dome, the rings are not continuous; there can not be circular horizontal chaining.

These archetypal forms of Persian architecture have become, as time goes by, major elements of the Persian national identity, beyond religious and political upheavals. The ancient symbolic has been integrated in the Islamic tradition. These two architectural forms, often associated, have been required on all architectural programs.

The moslem builders have kept ancient architectural forms by evolving their techniques. Geometrical forms of constructions become more precise and more complex, thanks to progress in mathematics (geometry especially), semicircular or elliptic arches are replaced by ogee arches. The essential innovation is however that of the replacement of conical squinches in the transition zone of domes by arch ribs (Figures 1b, 4).

2 Purpose of the research

The purpose of the research is to analyze the evolution of techniques of vaulting of the Persian architecture of the 3rd century to the 15th century, to test the pertinence of empirical solutions used and to take notice of the most vulnerable zones in the structures.

There are many questions one can ask concerning the equilibrium of Persian vaults.

- What-is the role of angle conical squinches in the transfer of loads to supports? Is it that of an arch of unload carrying the weight of the dome to its springs, or does assure the continuity of the shell to the walls?

- What-is the role of the parallelepipedic underpin of masonry wrapping the zone of squinches on the extrados of a Sasanian dome? Was this as has suggested Andre Godard [1], a simple work platform for workers constructing

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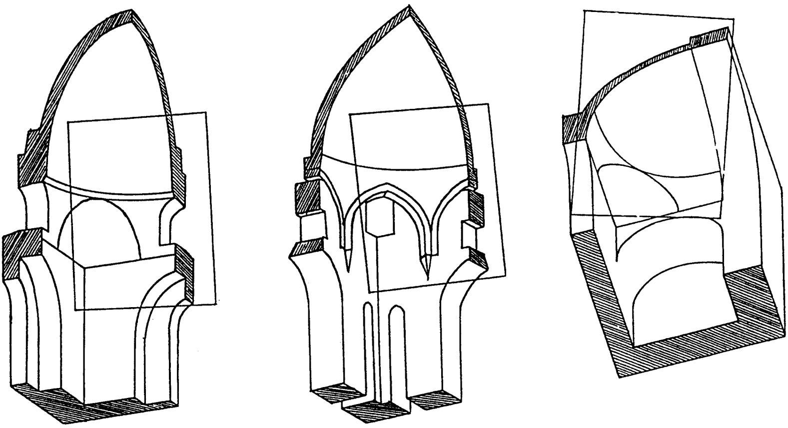
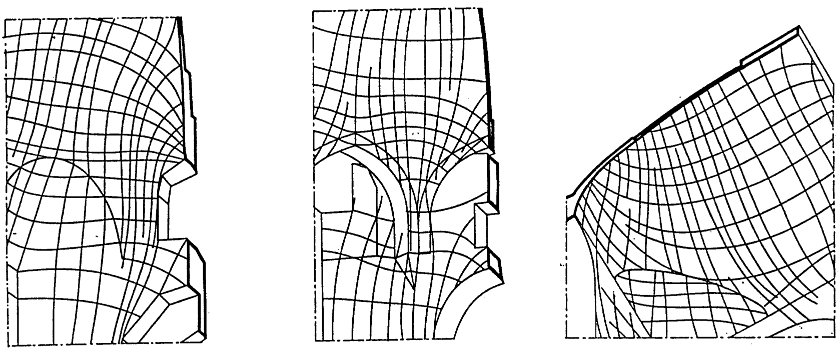


1.a: Dome of palace at Sarwistan.
height : 22.4 m.
ϕ intérior : 12.3 m.
height of wall : 9 m.

1.b: Dome Nizam-ol-Molk Friday mosque, Isfahan
height : 26.55 m.
ϕ intérior : 16 m.
height of wall : 10 m.

1.c: Ivan of palace at Sarwistan.
height : 13.34 m.
ϕ intérior : 10.1 m.
height of wall : 6.8 m.

Figure 1: Morphology of vaults studied.



2.a. 2.b. 2.c.

Figure 2: Isostatic Lines on intrados of vaults under dead-loadings.

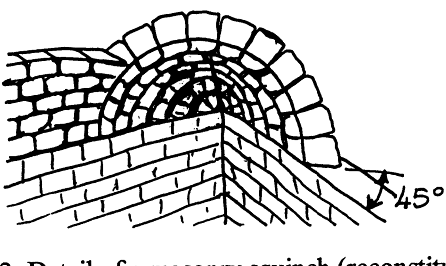


Figure 3: Detail of a masonry squinch (reconstituting).

the upper part of the dome, or perhaps an overload on squinches to avoid the appearance of prohibitory stresses at the key of these squinches?

- The ivan built in the form of a half-dome resting on a half-square by the intermediary of two squinches constitutes a structure of form totally illogical when considered from the point of view of static plan. What-is the state of equilibrium of a dome cut in two by a vertical symmetry plan?

- The problem of the equilibrium of vaults under dynamic loads has as yet barely been studied. A priori the morphology of domes seems to present a good resistance to dynamic efforts, although this remains to be demonstrated, as well as for the ivans?

3 Methodology

Geometrical models, meshwork of structures

A vault of masonry constitutes in first approximation a continuous milieu. The method of finite elements allows, by the technique of discretisation, to calculate such structures of complex three dimensional geometric forms.

The analysis of structures is undertaken by mathematical simulations, under the form of calculations to finite elements of several types of vaults under dynamic and static loads. The specificity of the masonry material, and the geometrical structure complexity oblige to develop a methodology specifically adapted to these structures. Initially, structures were discretised by Marguerre's hybrid quadrilateral elements. Results of calculations not being satisfactory, the choice of the discretisation of structures in masonry has focused on elements of type hexaehedron (to 8 nodes) and tetraehedron (to 4 nodes). In inferior parts of vaults the differences of structure thickness are important, being no longer are assimilable to slender shells.

The meshworks always comprise two elements in the thickness of the structure, so as to know variations of state of stress in the vaults or walls thickness. Except for a few simplifications, of geometrical order, the 3D meshwork has been undertaken by respecting scrupulously archaeological structure layouts.

Hypotheses of calculation

Calculations are undertaken with the help of the program SAMCEF. Having no rheologic studies of materials of structures (various masonries) allowing to define term in term the matrix of Hooke defining the relations between stress and deformation, we have adopted the hypothesis of a isotrop material in linear elasticity.

Post-processing of the tensors of stresses

The tensors of stresses in each node, developed by the SAMCEF program, are processed to obtain maps where structures are represented in perspective parallel by their meshwork, and the state of stress by 3 vectors representing the 3 main stresses in each node of the meshwork. The real vector length being difficult to perceive because of the variable projection angle, the former are colored according to a "rainbow" type of scale that goes from vivid red (maximal tension) to the deep violet (maximal compression).

4 Analysis of results

Calculations under dynamic loads are currently undertaken, but in this publication, only results of calculations under static loads are presented.

The palace of Sarwistan built in the 5th century, of which the layout was drawn by M. Siroux [4] in 1973, comprises a main room on squared plan covered by a dome of great height with meridian elliptic resting on four squinches (Figure 1a).

Dimensions of this room (height and span of the dome, thickness of support walls) being far greater than those of the other parts of the palace, we have isolated it from the rest of the structure to undertake the mechanical study of its structure.

The mosque of Friday of Isfahan possesses a sanctuary called south dome or dome of Nizam-ol-Molk erected between 1072/1092. This sanctuary was originally an isolated structure constituting a mosque-kiosk. The dome, with meridian in pointed arch, rests on 8 pillars and a wall by the intermediary of 8 arcs in rib (Figures 1b, 4). Proportions of this dome are close to those of the dome of Sarwistan. These two constructions erected during an interval of 6 centuries, quite the same as to their dimensions, constitute architectural spaces of similar form.

The permanence of this morphology during 6 centuries, reveals the permanence of symbols linked to the architectural space.

The dome of the 11th century differs however profoundly from that of 5th century as for techniques of vaulting. It uses vaults of ogival form instead of vaults of elliptic form, and especially the 4 squinches of connection in form of 1/2 elliptic cones of Sarwistan (Figure 1a) are replaced by 4 arcs of masonry spanning the dihedres formed by the junction of walls, accompanied by 4 other identical arcs situated in the plan of walls, prolonging the former and framing 4 windows (Figure 4c).

Calculations under static dead-load of the ***dome of Sarwistan***, show that the effect of transverse arc (to 45° of walls) played by the squinch is very weak (Figures 2a, 5):

at the base of the squinch (on the intrados), maximal stresses vary from

σ = -0.25 to -0.35 MPa.

Squinches insure an excellent continuity between walls of the pedestal and the dome itself. The observation of main stresses developed in the structure from top to bottom leads to a certain number of remarks. The successive overthicknesses on the extrados of the dome limit the regular increase of stresses of compression the long of meridians:

σ = -0.04 MPa to the key to σ = -0.3 MPa at the top of the walls,

and orient main stresses in close directions to the vertical. The increase of stresses is more important in the walls, the section of loads distribution being constant along 9 meters of high; stresses at the ground and all along the piedroits of the doors reach the values in the order:

σ = -0.6 MPa.

Notable tension stresses appear at the intrados of the key of openings arches (σ = +0.15 MPa).

In the ***dome of Nizam-ol-Molk***, loads transmitted at the top of the walls are concentrated in 8 zones which correspond to the piedroits of the arches (Figures 2b, 6):

from σ = -0.5 MPa to σ = -0.6 MPa

at the mutual support of the two arches. By observing developed main stresses from top to bottom of the structure, one observes that in the dome the presence of arches is particularly apparent due to the asymmetry of main stresses directed to the parallels that present a maximum at the right of piedroits of arcs and a minimum at the right of their keys, especially on the extrados. Stresses vary from

σ = -0.05 MPa to σ = -0.25 MPa

to the extrados above the first overthickness of masonry.

In the walls, the essential of the weight of the dome is transmitted to 8 pillars situated under piedroits of arches where stresses reach

σ = -0.8 MPa at the ground.

Stresses of tension, horizontal directions appear:

- under the key of arches: σ = +0.17 MPa,
- under the key of door arcs: σ = +0.28 MPa,
- at the top of the dihedres of join of walls: σ = +0.3 MPa.

5 Conclusion

The technique consisting to base a dome on 8 arches appears in the beginning of 10th century, it changed profoundly the construction of masonry domes in Islamic countries, in several different areas:

- By suppressing squinches and the mass of masonry that embodied them, it has allowed to lighten notably the structure.
- From a geometrical point of view and consequently from an esthetic point of view, it allows a more progressive passage between the square and the circle, the zone of 8 arcs comprises 4 symmetry plans instead of 2 for the zone of squinches.
- Thrusts being directed on 8 precise zones, one can make large openings in the walls much more easily.
- The enclosure of the space previously occupied by squinches is going to give place to geometrical compositions of bonds of masonry first, then decorations of plaster or faïence increasingly complex, called "stalactites", that will become one of the essential characteristics of the Persian architecture.
- The greatest concentrations of efforts lead to a less homogeneous functioning of the masonry. Relatively speaking, we observe the evolution from a continued structure toward a frame which ressembles somewhat the passage from Romanesque structures to Gothic structures in the architectural art of the western world.

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Calculations undertaken on the ivan of Sarwistan (Figure 2c) show that these structures constituted in half-domes function as inclined vaults, in barrel, whose height decreases from the key to the bottom wall.

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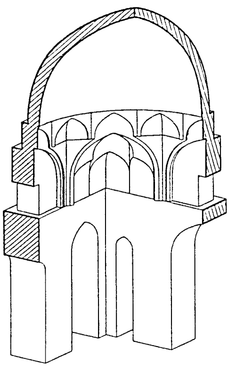
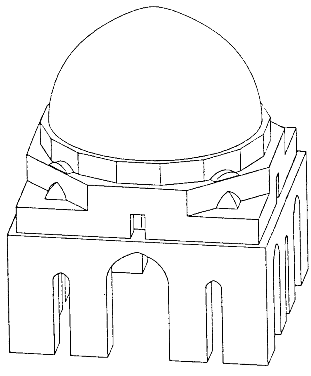
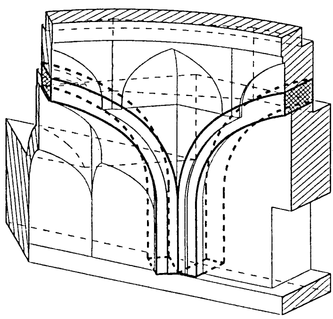


4.a: Intrados.



4.b: Extrados.



4.c: Zone of transition between cupola and pedestal showing the place of the arcs in rib.

Figure 4: Dome of Nizam-ol-Molk, Friday mosque, Isfahan.

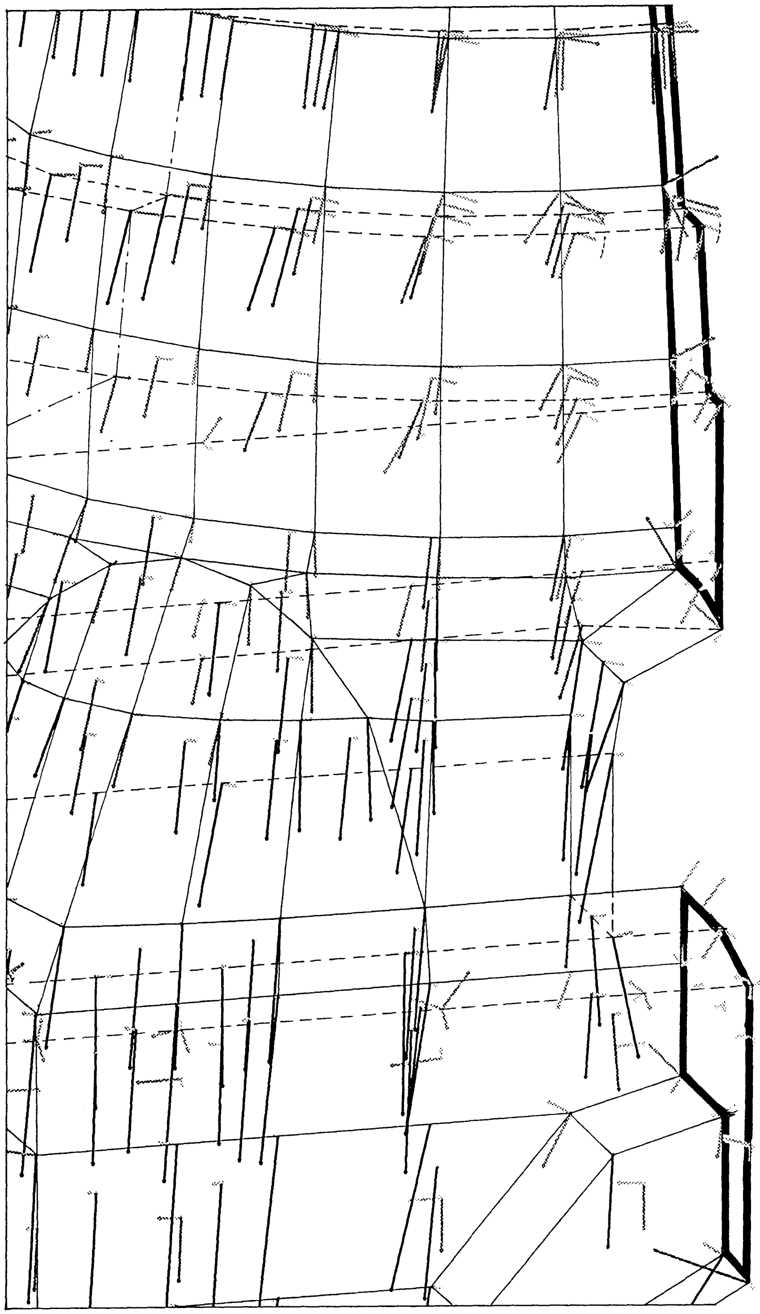


Figure 5: State of stress in squinches zone, dome of palace at Sarwistan.

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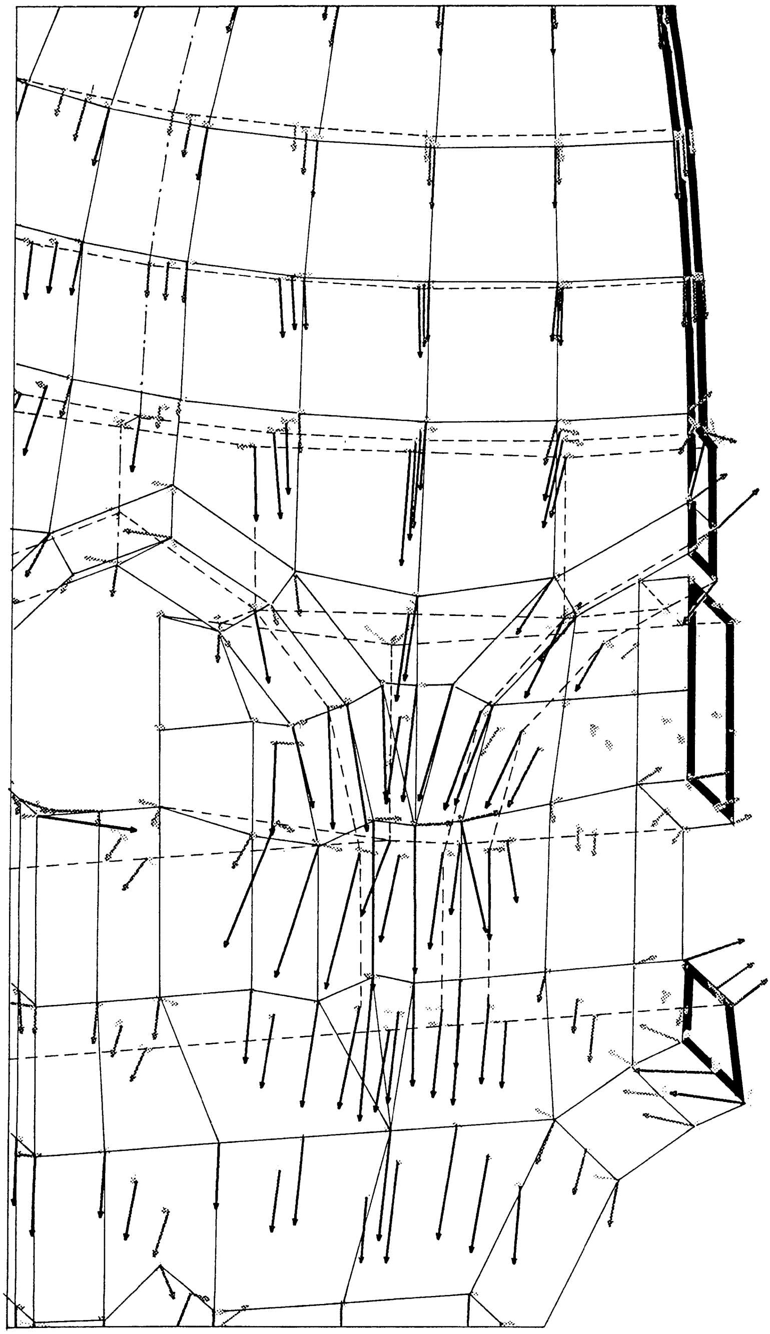

Figure 6: State of stress in pointed arches zone, dome of Nizam-ol-Molk, Friday mosque, Isfahan.