Structural analysis of Roman honorary arches: Constantine’s Arch
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ABSTRACT

The structural analysis of Constantine’s Arch, admirable example of a Roman honorary arch with three vaults, has been performed by a three dimensional finite element model with the aim of developing these methodologies as a part of the structural diagnosis and repair of this kind of monument. The static response under gravity loads and the dynamic response to seismic actions, characteristic of the Roman site, are examined, from which the structural behavior of the Arch comes out. Furthermore, the validity limits of the static analysis are pointed out on the basis of comparisons with the results obtained by applying the smeared cracking model. A more accurate identification of the model’s characteristics will be possible once the results of the current experiments are available.

INTRODUCTION

Honorary arches are amongst the most significant examples of Roman architecture: their typical massive forms have permitted these structures to resist better than others over the centuries. Moreover the increase of chemical and mechanical agents of degradation, linked to the evolution of the urban area, may damage such architectural works so that a complete knowledge of the state of maintenance of these monuments is needed, which must be available before every act of restoration. These studies have an interdisciplinary disposition in that they involve historical, artistic, chemical, physical and structural aspects. These latter not only concern the experimental work, to be performed on the materials and eventually on the entire monument, but also the modeling of the structural system; in particular, the experimental results form the support needed for the identification of the model to which is allotted the task of predicting the structural behavior in the presence of exceptional conditions, such as the seismic actions.

In the present paper the problem of structural modeling and identification of the Roman honorary arches is tackled, with the objective of
establishing the most suitable method of achieving reliable results within acceptable computational efforts. Furthermore, some characteristics of the static and dynamic behavior of the structures under consideration are recognized and related to their shape; on this subject a distinction has to be made between arches having a single vault and arches having three vaults.

The analysis of Constantine’s Arch in Rome created the opportunity to obtain knowledge about the structural behavior of three vault honorary arches and to verify the feasibility of this analysis. Constantine’s Arch is, together with the Arch of Septimius Severus, the archetype of three vault honorary arches [1]; the monument, which underwent some restoration in the 1950’s [2], has recently been the subject of experimental and theoretical analyses with the aim to evaluate the stress state in the monument [3]. Artistic restoration is currently taking place, concerning the reliefs and the friezes sculptured on the blocks and panels, as well as experimental testings, preliminary to an eventual structural restoration [4]. In particular, experimental analyses are being carried out with the aim of achieving a better understanding of the mechanical characteristics of the materials constituting the arch, of the stress state at some points, of the foundations and of the dynamic characteristics of the structure. These results form an element of guidance and comparison for the structural modeling.

STRUCTURAL ANALYSIS OF CONSTANTINE’S ARCH

Constantine’s Arch in Rome rises on the ancient Triumphal Road, between the Colosseum and the Imperial Fora; it consists of three barrel vaults surmounted by an attic, and, because of its size, is the most imposing honorary arch of the Roman Age (26 m length, 10 m width and 21 m high). The three barrel vaults are placed on four pillars, built with blocks of marble; on each facade and in correspondence to the pillars, there are four columns, set on raised plinths and connected at their top to the cornice at the level from which the attic rises. Of the eight marble columns which support the Trajan statues of the Dacians, one appears to have been replaced during restoration works in 1720 [2]. The attic rooms can be accessed by a staircase placed higher than the lateral vault on the west side. The construction is founded on a composition of brickworks of the age of Nero, having high stiffness, as it has been found in a recent soil survey.

During the restoration in 1955, in addition to the artistic repairs of the bas-reliefs, the plugging of deteriorated parts and the waterproofing of the coverings, steel gudgeons were introduced inside the four columns in the corners and connected in pairs at the top by tie rods.

The barrel vault in the middle of the arch spans 6.6 m and is set at 8.3 m from the ground level, whereas the lateral ones span 3.4 m and are placed at 5.8 m high. The eight pillars, being approximately the same size (3.5×6.5 m), are wholly made from blocks of marble with no significant cavities inside. No precise information is available
about the volume included between the vaults and the attic, which is probably filled with tufa conglomerate. The attic, made up in brickwork externally covered by marble blocks, holds three intercommunicating rooms, each barrel-vaulted on 3 m span. Finally the eight columns have a diameter of 0.9 m and are 8.7 m length.

The shape of the monument requires a three dimensional analysis by means of a finite element code [5] by using isoparametric eight node brick elements. Since the structure is nearly characterized by two vertical planes of symmetry corresponding to the transversal and the longitudinal middle planes, a model of a quarter of the Arch has been considered and analyzed, which involves 2021 elements, 2979 nodes and 8199 dofs (figure 1). The model reflects the actual geometry with sufficient accuracy and, as far as possible, has been obtained by applying finite elements of a fairly cubic shape and of comparable size. The section of the pillars has been discretized in 20 elements and the columns have been modeled by considering 13 segments, each consisting of 4 elements. In modeling the vaults in the attic the ribs between adjoining rooms have been considered, whereas the presence of the staircase room on the west side has been neglected.

The points at the base of the structure have been considered restrained since the soil under the Arch is markedly stiff, because of the presence of ancient brickwork, and the weight of the building is relevant, so avoiding tractions at the base and sliding movements prevented by the friction. The validity limits of these last assumptions, which allow to leave out the unilateral boundary
conditions from the model, have been verified *a posteriori* on the basis of the obtained results.

Since the mechanical characteristics of the materials and their mapping inside of the construction are not yet available, a single material has been considered that is homogeneous, elastic and isotropic (E=10000 MPa, ν=0.2, ρ=2600 kg/m³). If such an assumption may be acceptable in evaluating the stress state in the Arch due to static actions (gravity loads), on the other hand the dynamic response could differ from the effective one because of the different distribution of the materials in the height; particularly useful on this subject would be the measurements of the dynamic response to the environmental actions (such as the underground) which should give the natural frequencies of the Arch.

The numerical model has been applied to evaluate the structural response under gravity loads and to dynamic actions induced by earthquakes. Since the blocks of marble, the tufa conglomerate and the brickwork do not exhibit a linear isotropic elastic behavior, because of the limited resistance to tensile stresses and the breaks due to the combination of blocks, it has been deemed opportune to analyze the Arch under gravity loads in the hypothesis of homogeneous material having brittle-elastic response to tensile stresses. This solution and the linear one represent two situations which may be considered as limiting cases of the effective response.

Structural response under gravity loads

The obtained results related to gravity loads, the dominant static action for this kind of structure, give an insight into the static behavior of the three vault honorary arches and in particular of the Constantine’s Arch. Due to its stiffness, the more representative data concern the stress states (see figure 2 where the normal stress acting on the horizontal plane is shown).

From the analysis it has come out that the stress state in the vaults agrees only in part with that characteristic in the barrel vaults. In fact, as the longitudinal section in figure 2b shows, the span of the lateral vaults is not relevant if compared to the height of the upper volume; this causes the weight of this volume to be directed on the pillars by means of shearing stresses rather than by uniaxial stresses, which are dominant in arches. More critical is the situation in the central vault, because of both the greater span length and the lesser height of the layer between the barrel vault and the setting of the attic; in the vault impost there results a maximum for the compressive stress of about -0.75 MPa, while in the crown the tensile stress gets the maximum values of 0.12 MPa.

The resultant of the vertical loads is nearly equally shared by the four pillars (the two central pillars support 53% of the total gravity load). The central pillars are subjected to very low horizontal forces because the drifts of the lateral and central vaults are nearly opposite each other; on the contrary the drift of the lateral vaults induces on the lateral pillars a horizontal force which is about 0.07
of the vertical one at the foundation plane and determining extra compressive stresses due to bending (maximum compressive stress -1.2 MPa). Since the construction was built using overlapping blocks and the model has been developed by assuming a linear elastic material, it is needed to verify the absence of slidings between the blocks on the horizontal planes, condition which is checked if the ratio $|\tau|/\sigma_z$ between the shearing stress and the normal stress on the horizontal plane is lesser than the friction coefficient; in this case the above condition is verified since the maximum for $|\tau|/\sigma_z$, which is attained at the base of the lateral pillars, is 0.25, a value lesser than the friction coefficient between marble blocks.

![Figure 2: Vertical stress distribution induced by gravity loads: a) transversal middle section; b) longitudinal middle section.](image)

The compression at the base of each column, due both to their self-weight (38%) and to the shortening of the pillars, is nearly equivalent to 380 kN and corresponds to 3% of the resultant of the vertical loads; the mean vertical stress at their base is -0.6 MPa. Furthermore the different shortening of the pillars with respect to the columns causes a low flexure of the columns themselves.

The triaxial stress states in the arch have been compared with the ones presented in [3]. The results here obtained, compared with those of the two-dimensional finite element model, provide a more detailed description of the stress state, with particular reference to the parts of the model where triaxial stress states take place (volumes near the keystones of the barrel vaults, the connections of the columns with the arch, etc.) and in isolated elements (columns). From
the comparison of the results obtained by flat-jack tests with the theoretical ones, a good agreement is achieved with reference to the vertical stresses in the pillars, whilst lower values of the theoretical stresses are obtained at the impost of the central vault.

**Brittle elastic analysis under gravity loads**

It is well known that the assumption of linear isotropic elastic response for the materials constituting the Arch is only a simplifying hypothesis; in fact such materials may exhibit anisotropy, low resistance to tensile stresses (characteristic of brittle materials), and unilateral behavior (due to the presence discontinuities between the blocks). Moreover, because of the large size of the blocks, which is not negligible with respect to the pillar size, and of the spatial location and orientation of the blocks themselves, the possibility of identifying suitable constitutive equations seems excluded.

In order to take into account the above mentioned effects, the smeared cracking model has been applied in the structural analysis of the Arch under gravity loads. The model, which is implemented in several finite element codes, has been applied here by considering an elastic isotropic response and a low resistance to tensile stresses (0.1 MPa). In this application, despite the limited volume in which cracking takes place, the convergence of the incremental-iterative procedure of solution used by the code [5] has proved problematic, given the high number of elements in the model. In order to achieve the convergence a procedure has been developed which iteratively updates the elements in which the material cracks starting from the ones deduced from the elastic solution; once the convergence to the solution has been obtained, the number of cracked elements (shown in dark grey in figure 3) is equal to 122.

From this solution, a stress state appears which does not markedly differ from the linear elastic one (shown in figure 2); in particular, in the points where the linear analysis gives the higher stresses, the stress increase, due to the smeared cracking approach, results negligible. Such an issue sets up a characteristic of the class of structures under consideration, that is due to their massive shape and the low span of the vaults.

The volumes concerned with cracking are: the crowns of both the central and the lateral vaults, the connections between the top of the columns and the arch and, finally, the vaults in the attic. In the central barrel vault there is a significant volume of cracked material; however, given the low level of the tensile stresses, it follows that the variations of the stress states with respect to the linear elastic ones are negligible. On the other hand, a cracked zone of extremely limited size appears in the lateral vaults. The cracking of the whole joint at the top of the columns determines a decompression in the columns themselves, which nevertheless appears of modest importance. Finally the cracking of elements in the vaults of the attic is relatively unimportant when linked with the low level of stress there obtained.
Dynamic response to seismic actions

The site where Constantine's Arch was built is denoted by slight seismic activity; however, it seems a high intensity earthquake occurred in Rome several centuries before Christ, and hence before the construction of the Arch. In any case, it is well known that there are two seismic areas which ought to concern the city of Rome [6]: the Albani hills, located at a limited distance (about 30 km) and characterized by earthquakes of modest magnitude, limited duration and with a high frequency content; the Appennine mountains, more distant but characterized by earthquakes of greater intensity, which produce long time histories with significant contents in low frequencies. With reference to stiff structures, the most relevant seismic actions are induced by the Albani Hills earthquake spectrum.

The modal analysis of the Arch has been developed by considering the above mentioned model, which has been properly restrained in order to obtain both the translational (longitudinal and transversal) and the torsional modal frequencies and shapes. The obtained results constitute a first approximation in the process of dynamic identification of the Arch; the completion of such a process will be possible once the measurements of the dynamic response of the Arch to environmental actions, such as the vibrations induced by the underground, are available.

The fundamental modal shape corresponds to a transversal vibration with a frequency \( n_1 = 4.1 \) Hz (figure 4a); the second modal shape occurs in the longitudinal direction with a frequency \( n_2 = 6.1 \) Hz (figure 4b).
and involves, because of the high longitudinal stiffness of the attic, only the deformation of the pillars; the third modal shape is related to the torsional behavior and corresponds to the frequency $\nu_3 = 6.6$ Hz (figure 4c).

Figure 4: Modal shapes: a) transversal; b) longitudinal; c) torsional; d) 4$^{th}$ mode.
It is worth noting that the considerable height of the central vault, together with the presence of the attic room, determine a transverse modal shape with counterbalanced movements of the central and lateral modal shapes.
pillars ($n_t=11.8$ Hz). Finally, in the higher modal shapes a marked deformation of the columns is observed.

The analyses to seismic actions have been performed through the response spectrum technique by considering in the transversal and longitudinal directions respectively the first and the second mode. The response spectrum adopted here is that proposed in [7] for compact soil deposits which is analogous to the Albani Hills one; the maximum acceleration assumed is 0.06 g [6].

The obtained stress states, superimposed to those ones due to gravity loads, are shown in figure 5 with reference to the symmetry sections of the Arch. The response to transversal action is characterized by a maximum horizontal displacement of 3.7 mm and by a relevant increase of the stress level (figure 5a); in particular, the compressive stress near to the base is about -1.5 MPa, whereas tractions are obtained in the plinths of the columns. Localized high stresses are attained in the neighborhood of the column bases (-2.1 MPa). The longitudinal action produces the pillars deformation (maximum horizontal displacement 1.5 mm) and a relevant increase of the compressive stresses at their base (maximum value -1.8 MPa).

REFERENCES


