Transactions on the Built Environment vol 4, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

Geometry of structures. Studies on historical buildings

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ABSTRACT

Although greatest masterpieces of Architecture in the history seem to be based on sophisticated knowledge of engineering, many of them have been conceived with a spirit independent of structure and even contrary to the optimization.

INTRODUCTION.

There is a mechanistic tendency to explain the design of structures as a universal relationship between analysis and geometry and also a tendency to conceive forms as a result of tensional needs. This relationship is certainly less obvious that it seems and not clear although in engineering it is accepted as a guarantee of goodness of design. In architecture nevertheless they participate items very complex that can justify the lack of such relationship and make design contradictory.

Objectives of Architecture are only remotely to solve human shelter needs with a maximum of economy. It is immediate the appeal for beauty, invention, fashion and other cultural factors not able to be submitted to scientific norms. Even it is frequent to consider as a characteristic of freedom and geniality to attempt against basic principles of statics. Obviously, this choice should be inevitably catastrofic if architects would not develop clever tricks to hide a correct work below an arbitrary appearance.

Actually it is easy to build absurd structures because reinforced concrete and steel work in a wide range of stresses. But even in the past it was also frequent to take advantages of materials till the limit of its possible configurations. Stone or ceramic in compression, wood in flexion or vegetal fibers in tension were all that architects and engineers could use two centuries ago.

Nevertheless, with such restrictions some historical buildings seem to violate the laws of nature by following only geometrical designs and by taking account only of nice appearance or the symbolism of the tracery.

The fight of architecture to be free of servitude of nature's law is an aspect very little explored and difficult to be identified because our habitude to admit as logical only what we accept and know. But from the moment in which form does not follow the funicular of loads, structures are contradictory in some way. Anybody can believe that the Gaudi's Colonia Güell church design came from a net of strings and bags with sand. Geometry is the final argument in this case.

The thesis of this paper is to show how in the most relevant historical buildings well known because the success of their structure, design is justified by geometry and not by stresses. It is a paradox that I consider interesting to detail in some typical examples.

Pyramids in Egyptian architecture, Pantheon an Thermes in Rome, Mosques in islamic architecture and Gothic Cathedrals, as the most sophisticated justification of the thesis, are conceived from a geometrical point of view. Structures try to solve better or worse geometrical arbitrarity.

In the other side Greek temples, Byzantine churches, Romanesque churches and Renaissance domes are examples of pure structural conception. No concession to geometry is done in them. It seems contradictory, Gothic tracery is less structural than heavy Romanesque. Weren't they a simple evolution one from the other ? Not at all. With such considerations our attitude in front of the fact of repair and rehabilitation of historical buildings can not be passive. It must be clever and respectful with intentions of architect designs. Even when they were not as logical as we would expect with our actual knowledge and the state of the art.

EXAMPLES

Egyptian Pyramids are the first great example of structural contradiction where an enormous structure renounce to optimization in favour of geometry. Among an extensive amount of interpretations which can justify this gigantic human effort no one is negligible but none gives answers to all questions. As Mesopotamic ziggurats it seems that the main objetive of the pyramids was to rise as much as possible over the ground. To achieve this goal with a primitive technology it was used a natural scheme of piling up materials forming a hill. Ziggurats rose complex levels of inclined walls and stepped platforms

with buttresses and reinforcements that stabilize the brick fabric. The broken plant had a structural justification. The height to achieve was not too much important because walls were not able to support intensive horizontal loads. The Ziggurat of Ur 62.5 x 43 sqm. was 16.7 m. high on the upper platform. This will of ascension united with the stepped conception of ziggurats were used in the egyptian mastaba with more modest results. But applied to more ambitious objectives got greatest monuments of the history (Figure 1)[1].

Pyramid of Zoser 121 x 109 sqm. was 60 m. high, not surpassed till some romanesque towers after 1000 A.C. except by other higher pyramids. Rectangular plant and stepped walls were justified because the construction by phases. This geometric model was able to be enlarged if economic possibilities or long life of emperors would permit it. The main objective of arriving as high as possible is confirmed by Biblical references about Babel tower. Pyramids of Gizeh and particularly Pyramid of Cheops are the masterpieces in the scale of competition. The use of resistant stone and walls with adecuated slopes guaranteed to arrive till a height never again achieved before 19th. century. 147 m. height with a basis of 227 x 227 sqm. 2.5 million cubic m. of stone and 10.000 workers during 20 years. Nevertheless the structural objective was determined by a geometric requirement. Herodotos defined pyramids as structures which devoured their shadow. With a circular plant it would be posible to save material and even would not be necessary to build pyramids by succesive shells. It would be possible to build only one shell cover filled with sand (Figure 2)[2].

Greek Architecture nevertheless is so clearly structural and reproduced with such precision wooden original motifs that can not be considered a geometric style. All attempts of interpretation of traces have failed against optical corrections, symbolic references and self imposed architraved construction with columns and lintels or trusses which admits only a few different posibilities (Fig. 2)[3].

Roman Architecture is the opposite. It seems pure engineering and structural justification but it is based in the half circle arch that is geometry but not optimization. Romans knew perfectly well parabolic and pointed arches. They knew of their advantages for strength and geometry because they permit any height of the arch independently of the span. But Rome preferred geometrical limitations. Pantheon is a clear example of clever inventions to defend the geometry against structural errors:

a) Liberation of weight of the crown suppressing the dome at this point by means of an opening.

b) To charge with heavy loads the base of the dome to stabilize its profile.
c) To lighten the dome by means of coffers that participate intensively in the decoration.
d) Deep walls to compensate horizontal loads.
e) A cohesive material like roman morter with some capacity to support tension (Mark)[4].
f) A complex system of fabric made with arches of brick leaning each one on other.

Nevertheless, in spite of such inventions the Pantheon is not a structural machine except by its great size. All in it was designed by geometric conditions (Figure 3) (Goshina)[4]. Roman Architecture recovered the trace that was primordial in Egypt and almost all results can be justified with a complex norm called "Orders".

Byzantine Architecture is a natural continuation of Roman Architecture and therefore it participates in some of the structural contradictions. But it renounce to the norm and even to the "orders" or traces.

Hagia Sophia Dome is not hemispheric. This is a heresy in form but a success in structure. Forces are supported by buttresses and quarters of sphere. They produce large tympans to fill the space between the dome and buttresses that contain oppenings to lighten the inner space. Walls seem be empty and unable to support loads. The main structure is leaning upon other smaller structures: arches or domes. The thickness of the dome is minimum, 1.1 m. in the first version. The result is a system of elements leaning one on the other whenever they are claimed by structural behaviour. There is not a form which conditions the result. Rather the building is born as a consequence of piling structural elements till they achieve the equilibrium. The exterior appearance is resented because this. But the interior was at this moment the most grandiose space never created; greater than Constantino Basilica in Rome; more spacious than Pantheon and built in only five years by 10.000 workers (Figure 4)[5].

It was a pity that poor materials, bad construction and earthquakes ruined the bright initial idea. The Hagia Sophia that we know today is different of primitive design. Now tympanums are heavier, the dome is deeper, and the façades have lost equilibrated composition. But it conserved the main lines and till 19 th century we do not will find other space so coherent like this one.

Islamic architecture heirs the geometric trace and forget structural conception. It is true that this architecture never conceived great structures and that

they were reduced to solve with efficacy necessities of building and civil engineering. But solutions arrived from craftmen of geometry for whom caligraphy was above structure.

In Cordoba Mosque we can find a wide synthesis of geometric proposals. From the solution of an immense room in which narrow columns of marble sustain thick arches of brick and stone, till domes carried on interlocked squash arches with a fine beauty (Figure 5). Rows of archeries have precedents in Roman aqueducts but so lightened transversally that they have not a logical structure. They have not stiffness in perpendicular directions and heavy archeries seem to fly upon filaments unable to carry loads. It is clearly a good example of geometric predetermination. The same happens with domes. They are so small that their great complexity is almost a degeneration. But we are in front of an ornamental style and it is necesary to embroider a splendid silver work. Arches crossing in space with complicated encountering points are a prophecy of gothic style. It is true that these arches work correctly. They have not structural objections except that their conception is not structural. They have the same trace that we can find in tiles or carpentry.

Romanesque Architecture try to recover roman splendour but it is very different. Like byzantine architecture with which it has a close relationship, it believes that the use of the same forms: half round arches and barred vaults, groined vaults and domes, buttresses,... goes to the same solution (Figure 6). Nevertheless it is conceived too much rationally to admit norms of orders. Romanesque style is more serious and austere because all its parts have a structural justification. Geometry is not a primary consideration although it is forced to use barrel vaults, groined vaults or half sphere domes stiffened by round arches supported on piles or thick columns. They produce great horizontal loads resisted by lateral naves with one or two levels and by buttresses[5]. They are not capricious inventions and only its impressive sincerity avoid to convert the style in something cold and boring. Nevertheless its rationality did not allow to solve great problems and only was capable of reaching records with towers, higher than any other never erected before except pyramids[6].

Gothic Architecture is once more a rebellion against engineering. In this case, the last liberation of structural servitudes. In the end the structure will not be something fixed that can be combined but not altered. Structure will be a single component only to be managed, changed, deformed or redesigned till unlikely and even to be disposed in absurd forms.

Rose windows are a typical example. A vertical plan, with radial trace, with

elements of the same size but differently loaded, subjected even to tensions, are improbable structures (Figure 7). Gigantic gothic rose windows have no more justification than geometry. They are conceived with compass and need for some supporting to make them rigids. A postcompression with the wall loads.

Very much has been said about structural boast of great cathedrals and nevertheless nor towers are higher than pyramids, nor vaults are larger than Roman basilicas, nor spaces are so wide as Hagia Sophia. But gothic architecture won all them in some aspects. It is a case so clear of independence respect static laws and a radical self affirmation in front of gravity dictatorship that they have arrived to be a paradigm of all structural, technologic and engineering styles. The opposite that it is: geometric and traced design. As Panowsky said "pure ph*i*losophy of light"[9].

Gothic vaults are spatial grids of bars that can be either support or supporter. The structure arrives even to escape out of the vault and to renounce to their mission with flying ribs (Figure 8)[10].

With the **Renaissance Architecture** architects think that they are emulating the roman architecture. But in fact they only consider anecdotic aspects: composition and ornament. When Brunelleschi decided to accept a poisoned command that nobody knew how to solve, took as a model the Florence Baptistery structure changing its scale and almost duplicating the size. Brunelleschi conceived a machine with such precision to transmit loads that it was repeated in several other domes. The pointed arch to "quinto acuto" transmits an horizontal load half than a hemisphere with the same span. Prestressing with chains and great thickness of tambour guaranteed perfect stability not affected till now except little cracking. With several structural innovations Brunelleschi gained the higher point on a dome never before built, 93 m. And something more important, to build it without scaffoldings (Figure 11)[11].

In the meantime, Bramante and Alberti tried the pure recuperation of geometric concepts of roman architecture without achieving the same success in structures. St. Pierre's Dome, designed at first by Bramante could not be built by him nor by any other of his followers, Michel Angelo included. Della Porta was who by changing the profile, raising it to a line similar to Florence's Dome, could finish it in 1.590 (Robinson)[4]. Even so, historically, this structure has been problematic and only surpassed 11 m. Brunelleschi's dome (Figure 11).

When we enter into **Baroque style** we do not find impressive structures.

Architecture becomes not only geometric but also capricious and fantastic. Only islamic renaissance allows to consider other important buildings.

Sinan's works in Otoman Empire like Suleymaniye Mosque in 1.556 with Hagia Sophia scheme and a dome with 26 m. span or Selimiye Mosque in 1.575 with a dome of 31 m. span stand out (Figure 9)[12]. These structures are contemporaries of italian works and have the same structural character.

A hundred years later, in India, Islamic Architecture retook brightly the spirit purely geometric in Gol-Gumbaz of Bijapur (1.625 - 60) with the greatest covered area ever built before with a single dome of 1.683 sqm. upon a square plant (Figure 10)[3].

What comes afterwads is an other history. But it is surprising that from 3th till 19th centuries main domes have 43 m. more or less of span and news records have been reached only in height:

Panteon	Ø 43.2 m.	H= 43.2 m
Hagia Sophia	Ø 31x$\sqrt{2}$ = 43.4 m.	H = 48.4 m.
Ste. M^a Fiore	Ø 43 m.	H = 87.2 m.
St. Pierre	0 42 m.	H = 106 m.
Gol.Gumbaz-Bijapur	# 44x44	H = 53 m.

Roman authority with a single geometric decision conditioned the human ambition along sixteen centuries (Figure 11).

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FIGURES

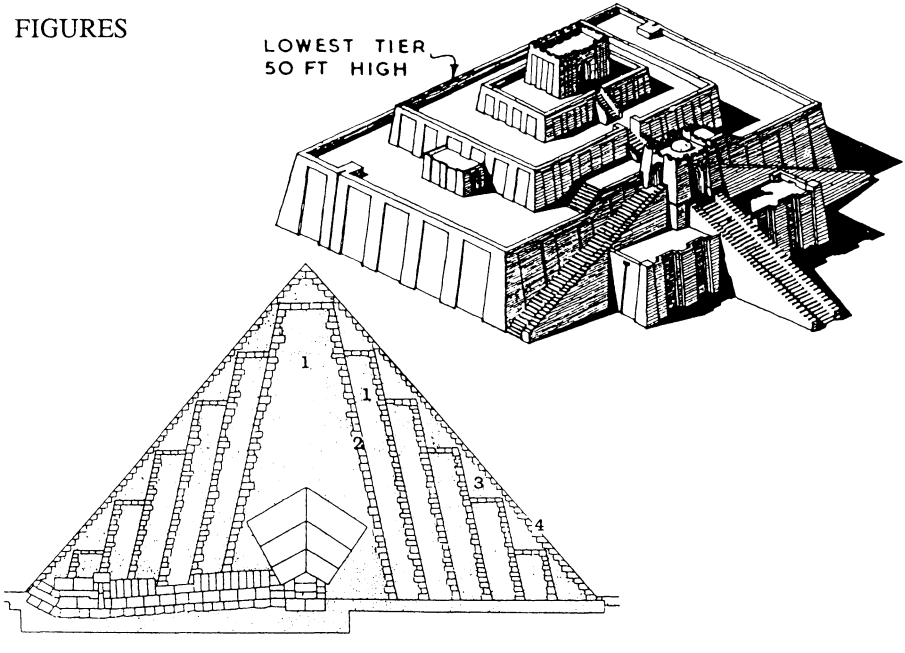


Figure 1. The Ziggurat of Urnamnu. Ur (Fletcher)[1]
Figure 2. Section of the Pyramid of King Sahure (Clarke)[2]

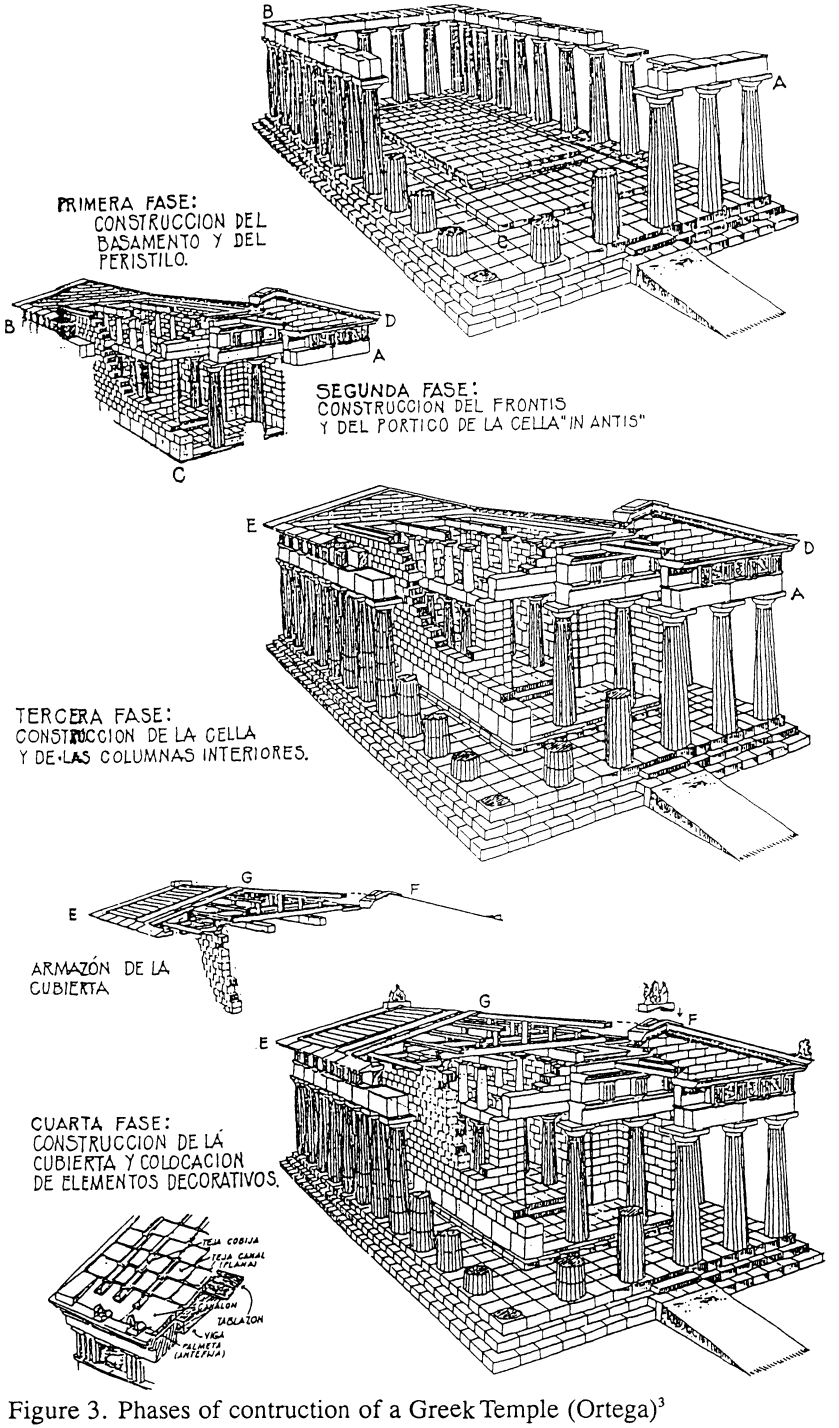


Figure 3. Phases of contruction of a Greek Temple (Ortega)[3]

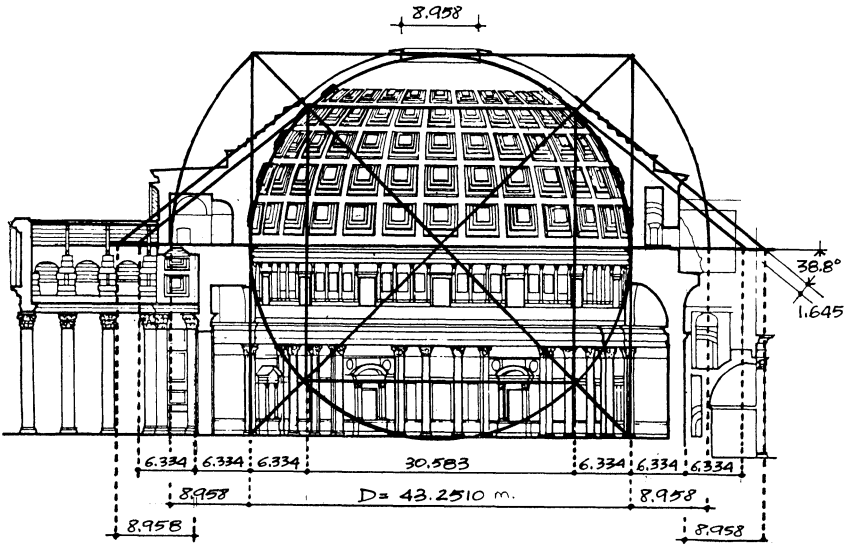


Figure 3. Geometry of the Pantheon. (Goshina)[4]

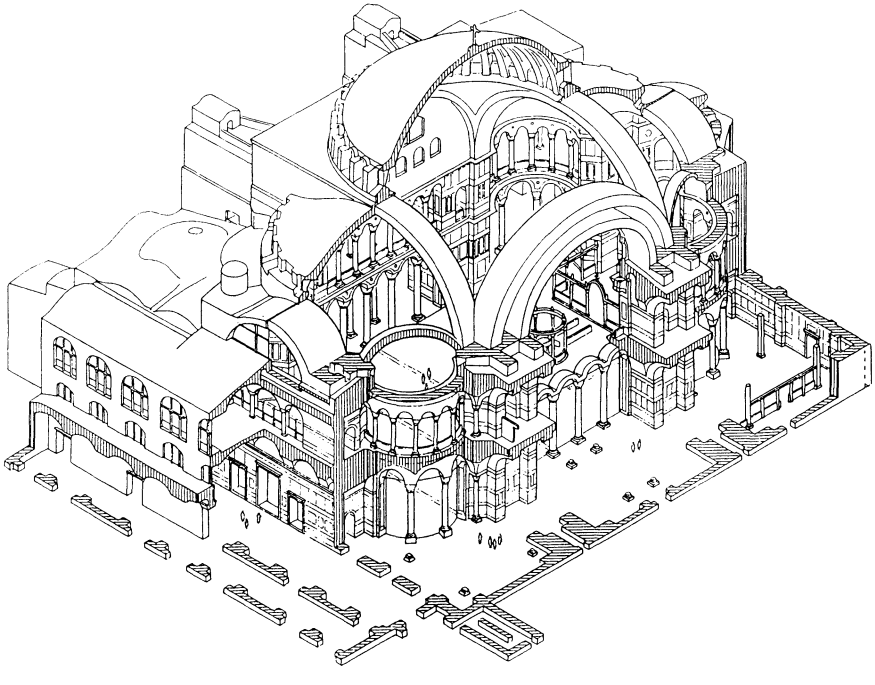


Figure 4. Structural Scheme of Hagia Sophia (Mainstone)[5]

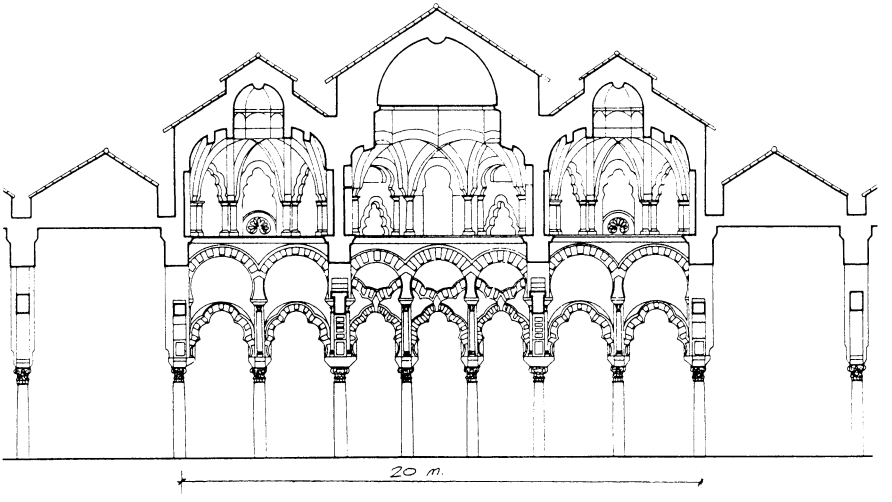


Figure 5. Section of the Mihrab area of Cordoba Mosque.

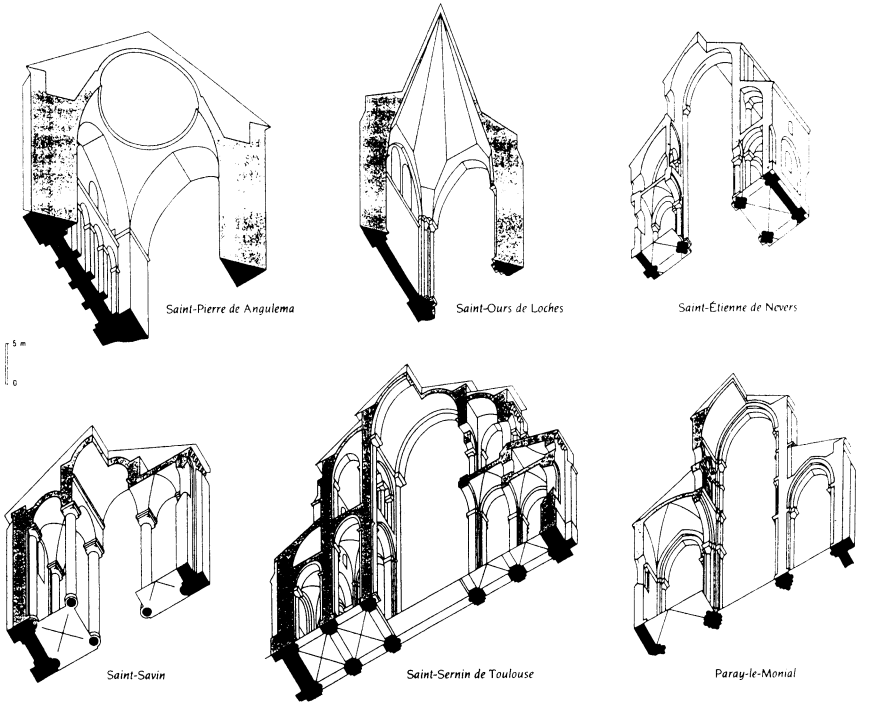


Figure 6. Romanesque Structures (Choisy)[6]

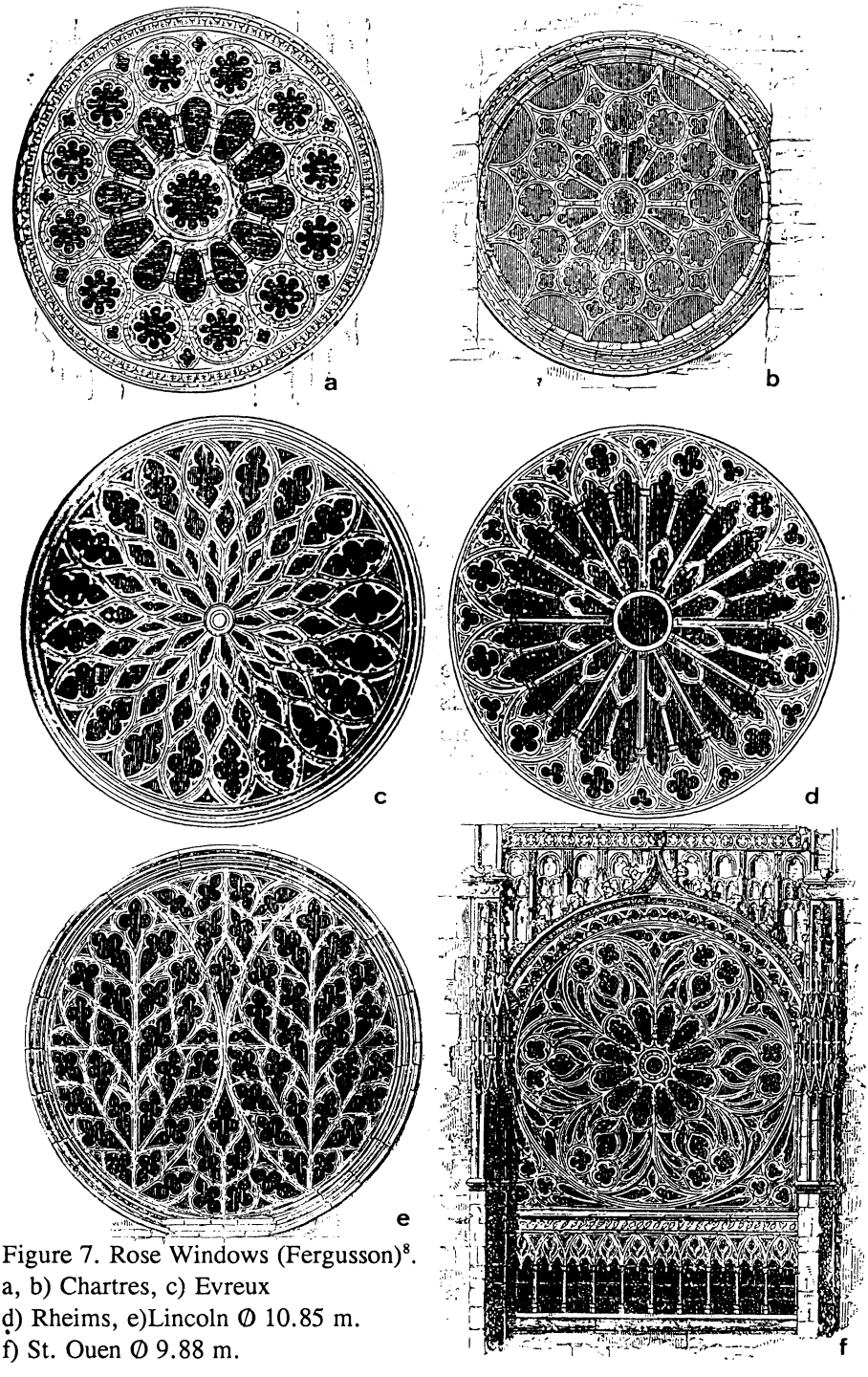


Figure 7. Rose Windows (Fergusson)[8].
a, b) Chartres, c) Evreux
d) Rheims, e)Lincoln Ø 10.85 m.
f) St. Ouen Ø 9.88 m.

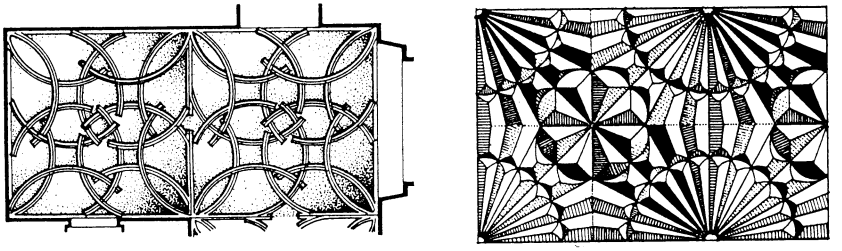


Figure 8. Gothic Vaults (Acland)[10]

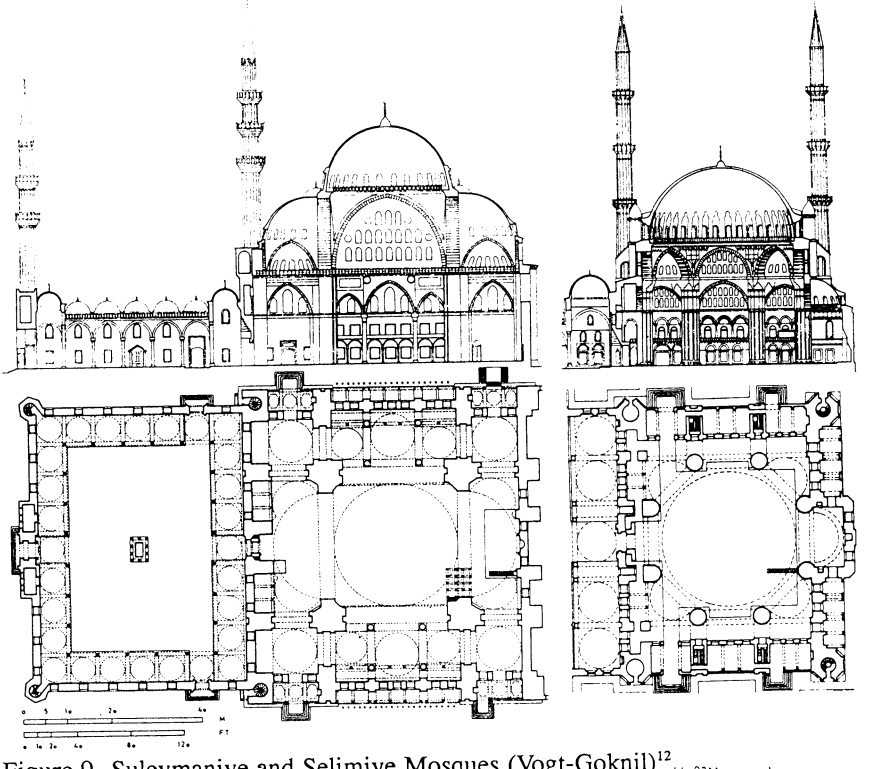


Figure 9. Suleymaniye and Selimiye Mosques (Vogt-Goknil)[12]

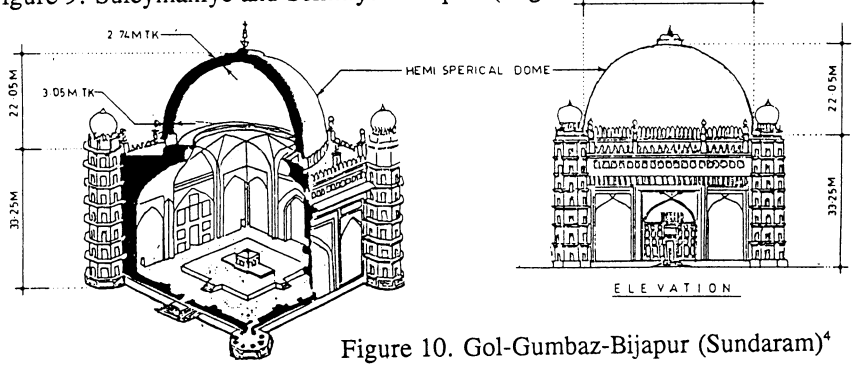


Figure 10. Gol-Gumbaz-Bijapur (Sundaram)[4]

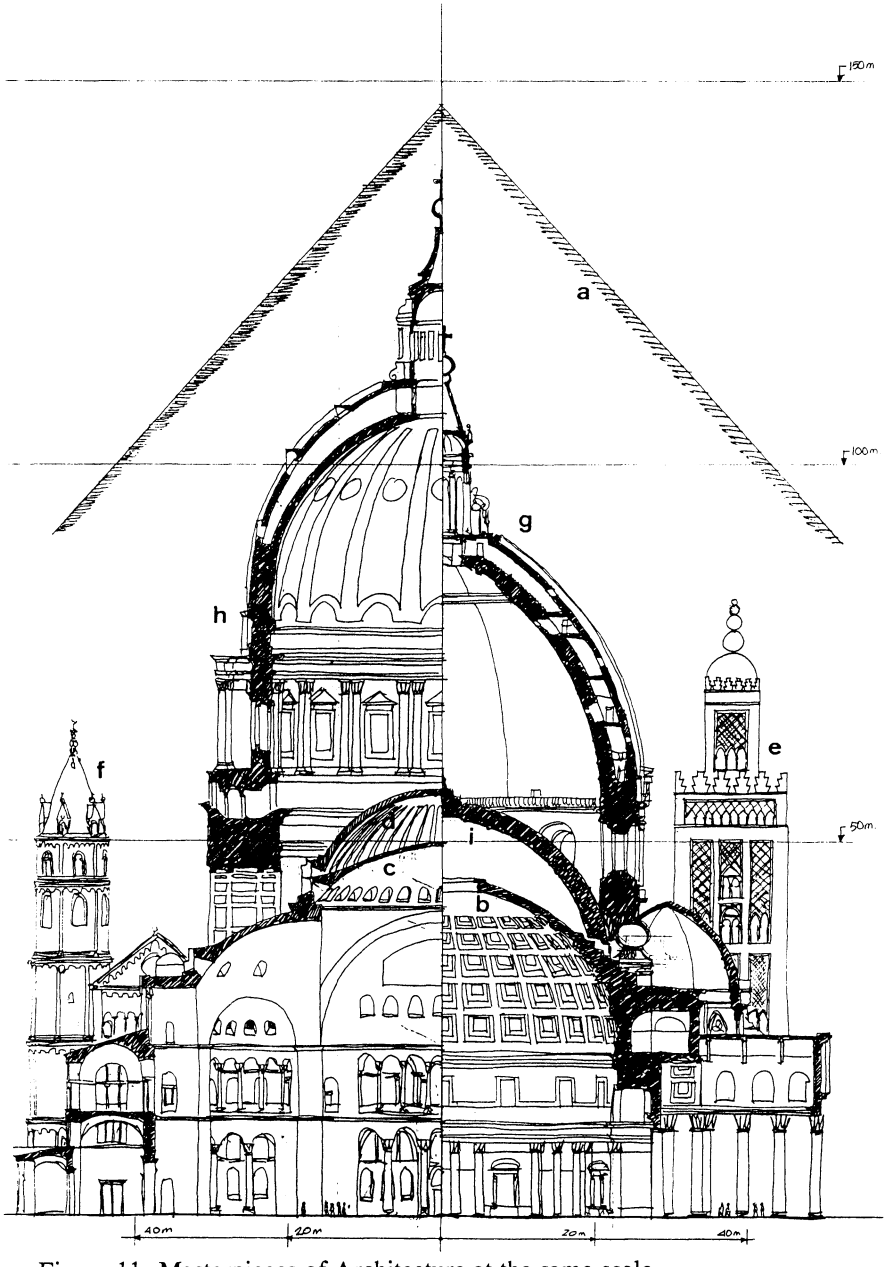


Figure 11. Masterpieces of Architecture at the same scale.

a) Pyramid of Cheops b) Pantheon c) First dome of Haiga Sophia.
d) Haiga Sophia now e) Giralda of Seville f) Spira Cathedral
g) Florence Duomo h) St. Pierre Rome i) Gol-Gumbaz-Bijapur. India.