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Dynamic analysis and earthquake response of Hagia Sophia

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ABSTRACT

Recent results of field measurements and finite element studies of the Hagia Sophia, a sixth century masonry edifice, in Istanbul, Turkey, provide insight to the structure's response to dynamic loads. The church contains four great brick arches springing from stone piers that offer primary support for a 31-meter diameter central dome and two semidomes. Eigenvalue analyses using finite element models of the primary dome support structure provide estimates of mode shapes and frequencies for the structure. The material properties for the model are calibrated to match ambient vibration measurements. Motions induced by a recent 4.8 magnitude earthquake are discussed. Frequency and time domain analyses are used to identify mode shapes and frequencies. Analysis of distinct subintervals of the recorded motion demonstrates a subtle variation of the frequency with increasing intensity. Animation of measured responses relate the mode shapes and frequencies of the eigenvalue model to the actual response of the structure.

INTRODUCTION

This paper addresses the present day dynamic behavior of the primary dome support structure. A finite element (FE) model has been constructed for the purposes of eigenvalue and forced vibration analysis. Frequencies, mode shapes, and predicted responses are developed using standard structural dynamics techniques.

A low-level event of magnitude 4.8 was recorded on March 22, 1992, with epicenter at Karabacey, Turkey, about 120 km south of Hagia Sophia. Time histories recorded for this event have been analyzed in both the time and frequency domains. System identification is performed using standard spectral analysis procedures with the aid of a notch-filtered animation process.

EIGENVALUE ANALYSIS

A linear elastic FE model for eigenvalue analysis of the primary dome support structure called *dyn26a* was created whose natural frequencies would match the observed values given in Table 1 which were obtained from ambient vibration measurements on the actual structure [1].

TABLE 1. Ambient Calibration[1]

Mode	Observed	FE Model[2] Primary	FE Model[2] Complete	Dominant Motion
1	1.84	1.97	1.78	E-W (X-axis) translation
2	2.09	2.08	2.16	N-S (Y-axis) translation
3	2.41	2.37	2.43	Torsional (Z-axis) rotation

[1] All frequencies are in Hz
[2] Primary= *dyn26a*, Complete = *whole4*

The primary structure FE model is a more complex version of the initial large model employed in a static analysis [2]. The tympanum walls and columns, east apse semidome and piers, and exedrae semidomes, arches, and columns were added. The main piers and the bottom portion of the buttress piers are comprised of stone masonry, and the remainder of the primary structure is comprised of brick masonry. By experimenting with the moduli of elasticity of the two masonries, frequencies agreeing with the ambient measurements were obtained. The frequencies for the primary structure FE model listed in Table 1 were obtained using the elastic moduli given in Table 2. The same density, ρ, was used in all regions, except that a 25 percent reduction was applied in the pendentives.

TABLE 2. Elastic Moduli[1] for Primary Model

Region	Youngs modulus, E
Stone masonry	10.0
Brick masonry	5.0
Surcharge	2.5
Tension Areas	1.0

[1] Units of moduli are 10^9 Pa

The first three mode shapes for the primary structure model are shown in plan in Figure 1. These shapes correspond to horizontal translation (modes 1 and 2) and torsional rotation (mode 3) of the entire primary structure system.

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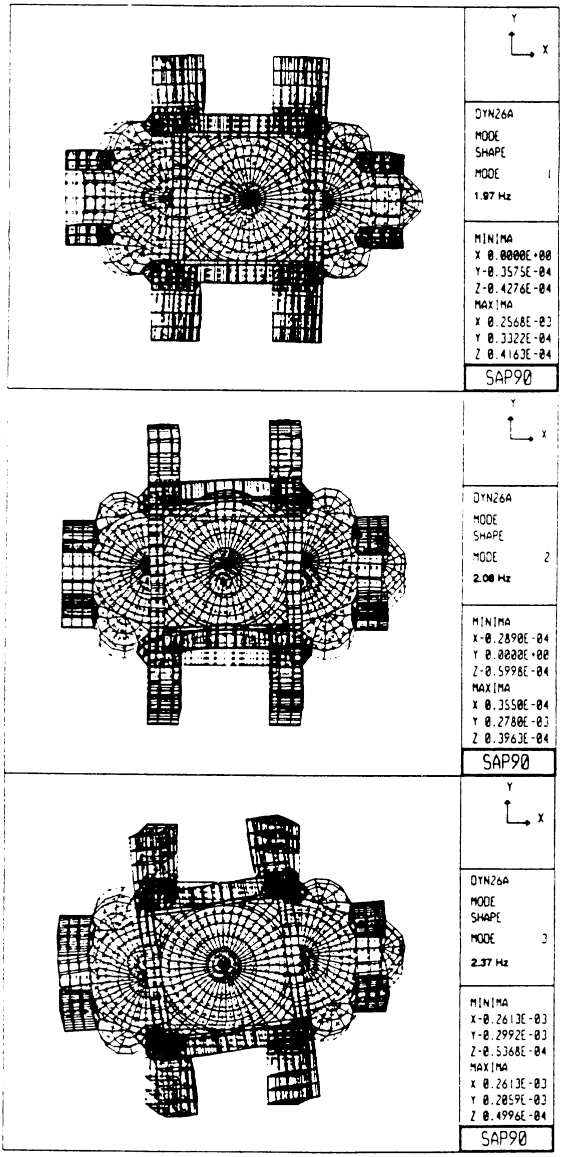

igure 1. Natural frequencies and mode shapes of primary structure model.

While the above frequencies agree with those of the ambient measurements, the elastic moduli necessary to obtain them are orders of magnitude larger than those obtained in a static analysis [2]. It may be that the structures surrounding the main building- the walls, floors, ceilings, narthex, and flying buttresses added much later- have influenced the frequencies measured in the ambient vibration study. To investigate this hypothesis, the surrounding structures were added to the primary model. This alternative model, called *whole4*, is apparently stiffer in modes 2 and 3, resulting in frequencies that are slightly higher than those obtained for the primary structure model. The frequencies for this complete structure model given in Table 1 are based on the same moduli and density as in the primary structure model except no softening was imposed in surcharge and tension areas.

The above discussion summarizes the dynamic response characteristics of free or ambient vibration. The remainder of the discussion will focus on the forced vibration characteristics. To understand results of the system identification, principal aspects of the structural system, material, and measured time histories will first be highlighted.

MAIN DOME SUPPORT STRUCTURE AND INSTRUMENT ARRAY

The primary structure supporting the main dome of the Hagia Sophia and its orientation are illustrated in a cutaway view in Figure 2. Structure north (N) is taken along the positive Y-axis of the plan. The longitudinal axis of the basilica complex is along the east- west (E-W) or X-axis of the plan. The main dome is spherically shaped and rests on a square dome base. Major elements include the four main piers supporting the corners of the dome base and the four main arches that spring from these piers and support the edges of the dome base.

The two arches on the N and south (S) sides are stiffened by additional arches with infill provided between the primary and secondary arches. The N and S arches have approximately the same dimensions and are symmetrically located. A secondary colonnade system also provides some resistence to lateral motion of the main piers in the E-W direction. The E and W arches span an open area with buttress piers providing lateral support to the main piers in the N-S direction. The E and W arches are not symmetric in dimension and are slightly asymmetric in location. This asymmetry is a due to a reconstruction and enlargement of the W arch after a partial collapse [3].

The instrumentation array [4] has been designed to capture the motion of the major elements comprising the main dome support structure during earthquake events. Figure 2 indicates the approximate locations of the nine accelerometers relative to the primary system elements.

The ground motion at the base of the main piers is measured by the instrument labeled 1 identified by a solid square in Figure 2. Response locations labeled 2 through 9 are identified by solid circles in Figure 2. The motion at the tops of the main piers is measured by the array labeled 2-5. That at the tops of the arches is measured by the array labeled 6-9.

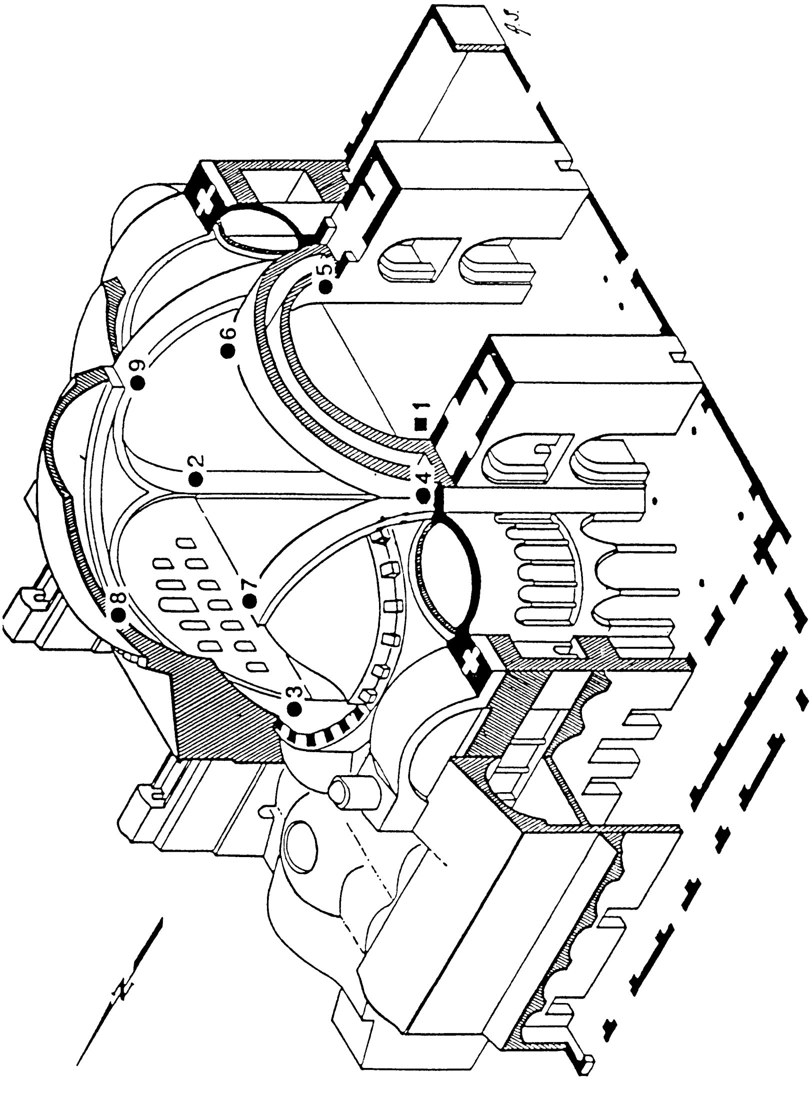

Figure 2. Main dome support structure and strong motion instrument array.

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MASONRY PROPERTIES INFLUENCING DYNAMIC RESPONSE

The masonries of Hagia Sophia which are composites of either brick or stone and mortar play an important role in determining the stiffness and damping of the primary structural elements. This section highlights important aspects of the masonry to be considered when identifying apparent system properties and calibrating analysis models using observed responses.

The main piers are comprised of stone masonry. The stone is either a limestone or a local type of granite. The blocks are .45 m deep on average and up to about 1 m long. The mortar layer is thin in this instance, being nonexistent or porous in some locations [2]. The stone blocks are almost rigid, whereas the mortar is relatively compliant.

The main arches and dome are comprised of brick masonry. The mortar thickness is on the order of that of the bricks (about 50 mm) [5]. Brick lengths in the main arches exceed .6 m. Preliminary analysis [6] of specimens taken from the basilica indicates that the mortar is pozzolanic rather than pure lime, containing brick dust and fragments in the matrix. The matrix also contains aluminum, calcium, and carbon. The fragments act as an aggregate and the other constituents indicate a cementitious material.

In effect, the pozzolanic mortar used in the brick masonry may be considered to be a form of concrete with tensile strength exceeding $3.5(10^6)$ Pa. By this description the presence of the bricks takes on a different meaning than the conventional one of a primary load bearing constituent. The bricks in Hagia Sophia may be thought of as providing stiffness rather than strength to the composite. The composite is also lighter than present day concrete in which denser stone is used for aggregate.

MOTION MEASUREMENTS

All nine accelerometers in the instrument array were triggered during a magnitude 4.8 earthquake on March 22, 1992. The epicenter was at Karacabey, Turkey, 120 km (75 mi) south of Hagia Sophia. Figure 3 shows the earthquake induced N-S (Y), E-W (X), and vertical (V or Z) acceleration component time histories at the nine locations. The three components for each response location are displayed in a plan arrangement corresponding to that of the locations themselves (see Fig. 2). The records in the center of the plan correspond to the base motion at location 1 which is directly beneath location 2 in plan. The range of the plot ordinates is identical for all locations at the same elevation.

In all cases the records indicate a nonstationary process acting with three approximately stationary periods or time intervals. The first interval extends from 2 to 12 s and corresponds to the action of compressional waves; the second from 15 to 25 s corresponds to the arrival of shear waves; and, the third from 30 to 40 s corresponds to a period of decay in the energy of the base motion. The most intense shaking occurred during the second interval.

N-S motion dominates the ground motion throughout all three intervals. E-W motion is significant in the second interval only. V motion is moderate

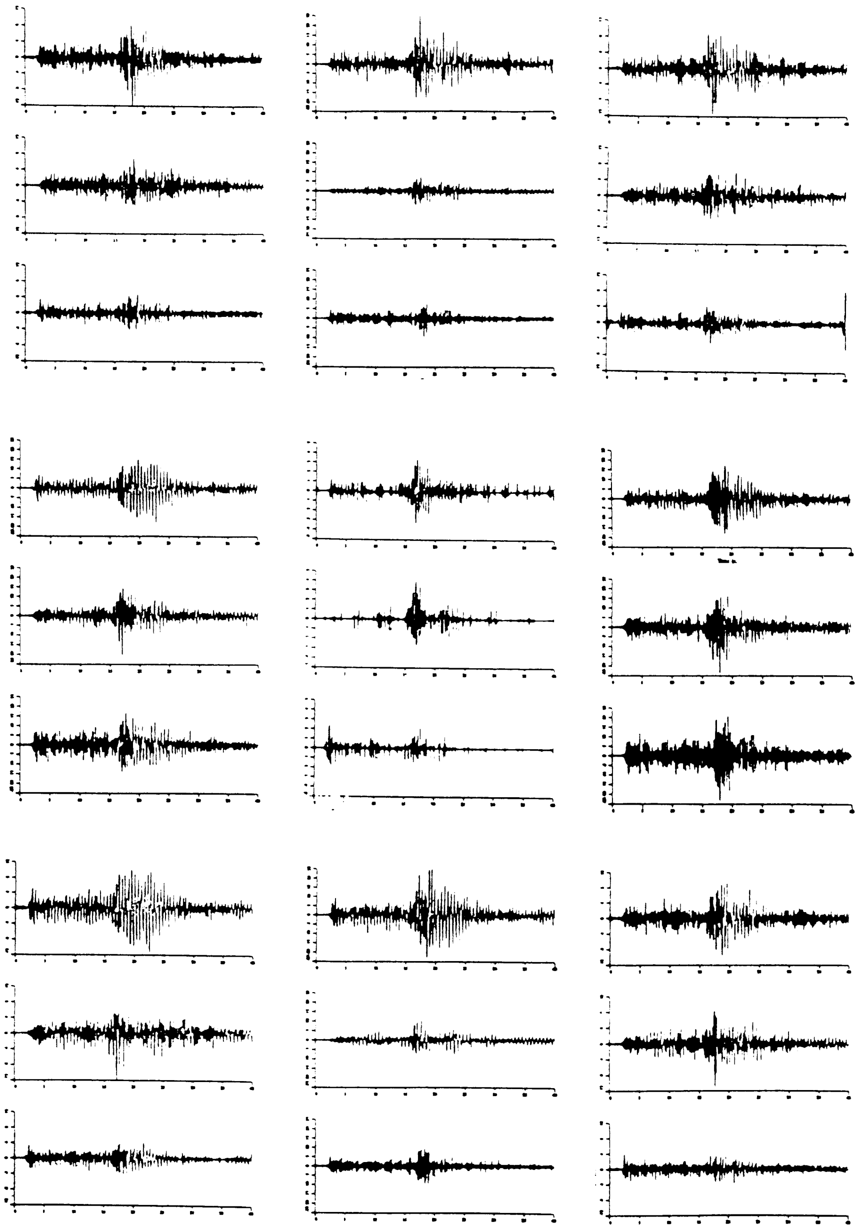

Figure 3. Measured accelerations for March 22, 1992, Karacabey earthquake.

in the first two intervals but negligible in the third. Response motion is dominated by horizontal (N-S and E-W) motion except at the tops of the E and W arches where V motion is significant. In general the N-S motion dominates the horizontal motion. This is particularly evident at the tops of the N and S arches where N-S motion corresponds to out-of-plane motion of the arches. The peak acceleration of about 25 cm/s^2 occurs at these locations between times of 16 s and 20 s. Significant E-W response motion occurs at the tops of all four main piers and at the tops of E and W arches. V response motion is significant only at the tops of the E and W arches.

A clear tendency toward higher accelerations is apparent as higher elevations are reached. Motions of locations at the same elevation exhibited comparable intensities with the exception that more intensity (as measured by the standard deviation) is apparent in the N-S motions at locations 4 and 6. The peak displacement at location 4 is estimated as .071 cm which is double that of the locations 2, 3, and 5. Similarly, the peak displacement at location 6 is estimated to be .13 cm which is almost double that at location 8. This behavior is probably associated with the foundation of the southwest main pier.

SYSTEM IDENTIFICATION

Spectral Analysis

Non-normalized one-sided power spectral density (PSD) estimates have been computed for each horizontal motion record using standard software [7] for Fast Fourier Transform (FFT) analysis. The Tukey-Hanning spectral window [7] has been applied to obtain smooth representations that illustrate the dominant frequencies. Figure 4 presents the spectra obtained for three levels of the primary structure treating each entire record as the realization of a single non-stationary process. The input energy is found to be concentrated in N-S motion near a frequency of about 2.4 Hz. Noticeable energy is also observed in E-W motion having frequency peaks between 2.7 and 4.2 Hz. The response energy is concentrated in N-S motion at frequencies of about 1.8 Hz, 2.4 Hz, and 3.3 Hz. The 3.3 Hz response energy diminishes at the level of the top of the arches, however.

As described in the preceding section, the time histories indicate that the full record is more realistically described by three shorter approximately stationary processes having different levels of intensity. Each of the three intervals has been defined with a duration of 10.24 s in order to obtain the best resolution in the PSD estimate. The power spectra for the separate intervals indicates that the power in the second interval dominates that in the other two. Figure 5 shows the spectra for the second interval. The frequency range of 1.6 to 2.8 Hz has been isolated, because it is in this range that the first three modes are expected to lie based on the ambient vibration results. The patterns observed above for the spectra obtained using the entire records are seen to be controlled by the middle interval spectra.

The system identification is obtained through the estimation of the transfer or frequency response function. The system frequencies and damping are defined point-wise and component-wise throughout the structure

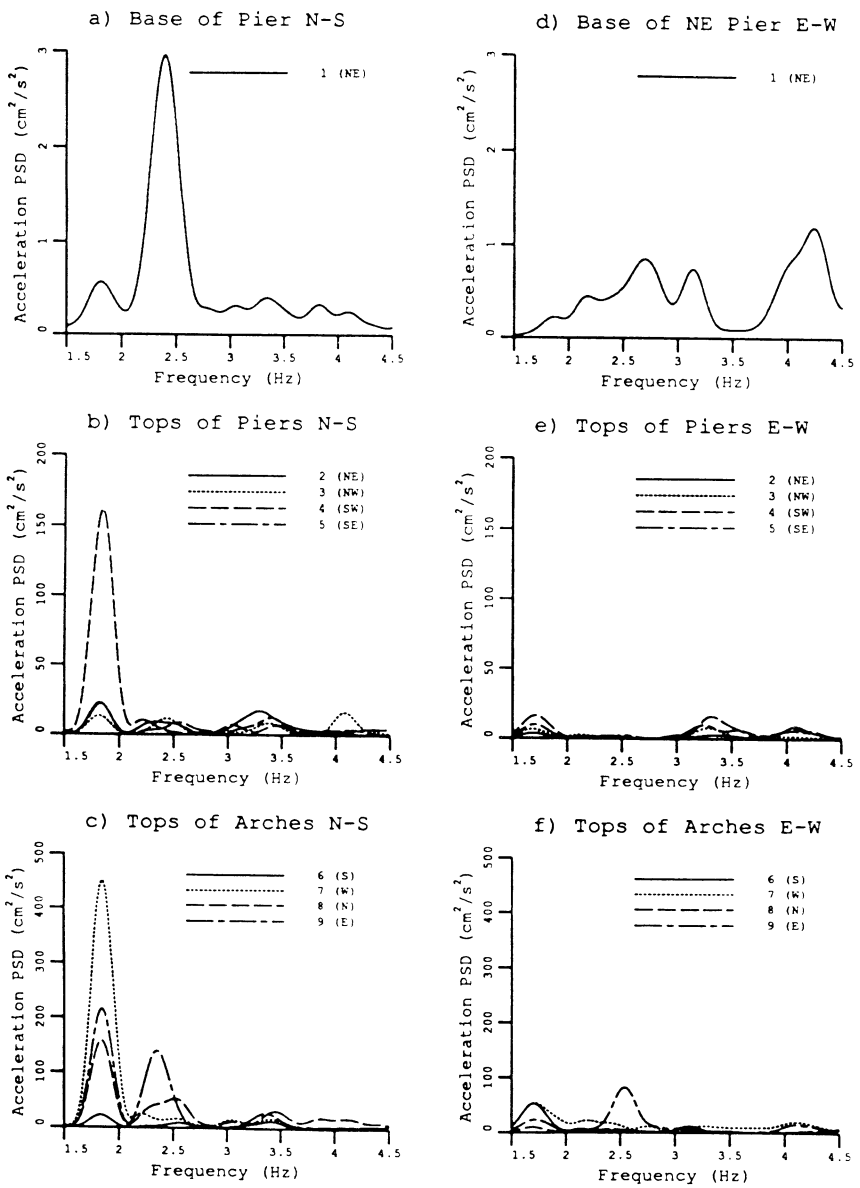

Figure 4. Acceleration power spectra using entire records.

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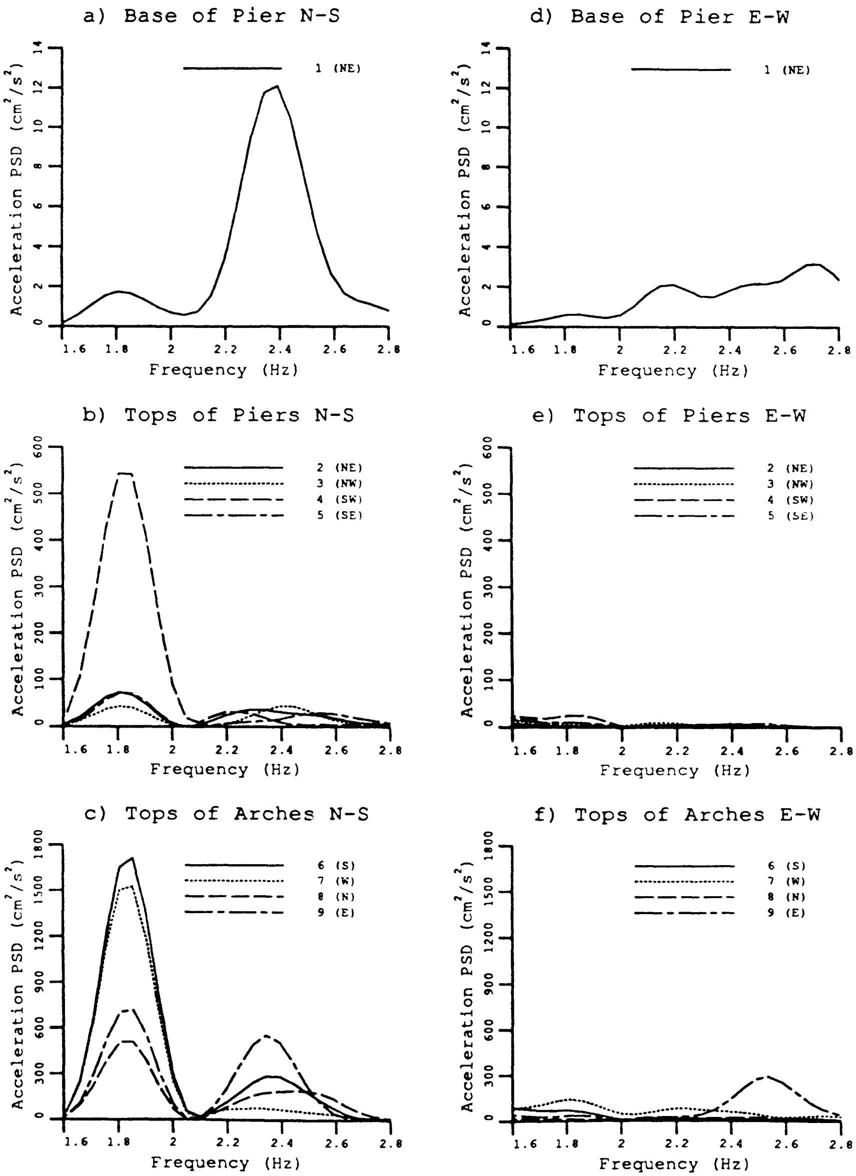

Figure 5. Acceleration power spectra using second interval.

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by the complex-valued frequency-dependent input-output relationship expressed by the transfer function. A simple algebraic relation exists between this function and the ratio of the power spectra of input and output quantities for a linear system undergoing stationary motion [8]. Figures 6 and 7 show the experimental transfer functions, H_{YY}, and H_{XX}, obtained using this relation. H_{YY} represents the transfer function for the Y (N-S) response motion caused by Y (N-S) input motion at the base, and H_{XX}, the X (E-W) response caused by X (E-W) input. Using the second interval, the observed frequencies given in Table 3 are obtained.

TABLE 3. Earthquake Calibration[1]

Mode	Observed	Simulation[2]	Dominant Motion
1	1.53	1.74	E-W (X-axis) translation
2	1.85	1.88	N-S (Y-axis) translation
3	2.15	2.10	Torsional (Z-axis) rotation

[1] All frequencies are in Hz
[2] Simulation using FE model *Hsdyntc4*

The system responds primarily at 1.85 Hz in the second interval. This represents a 10 percent reduction in the Mode 2 frequency relative to the ambient vibration frequency of 2.09 Hz. The first interval indicates 2.00 Hz is dominant, corresponding to a 5 percent reduction. Such lowering of frequency with increasing intensity of the forced vibration is consistent with the behavior of a nonlinear or damaged structure. Damage is unlikely here because the peak acceleration at the base is less than 1 percent of gravity. If such a low level event could cause damage, significant accumulation would have occurred over the long history of the structure, and some evidence of this accumulation would be noticeable. Nonlinearity associated with some of the masonry material aspects described earlier is also surprising at such low levels.

The transfer functions also show the tendency of the SW main pier and the S and W main arches that spring from it to respond at higher amplitudes than the other piers and arches in mode 2 motion. The behavior in mode 3 which involves rotation is somewhat different. Evidence of this mode is weak relative to the mode 2 response. Secondary peaks appear in the range of 2.2 to 2.6 Hz which is consistent with the ambient vibration value of 2.4 Hz. Little indication at the level of the tops of the main piers is found except in the third interval which shows a small peak at 2.3 Hz. At the tops of the piers, the E arch appears to respond the most in mode 2 motion at frequencies between 2.2 and 2.4 Hz.

Animated Motion

In order to examine mode shapes corresponding to the measured response, the relative motion of the locations at the different levels were examined

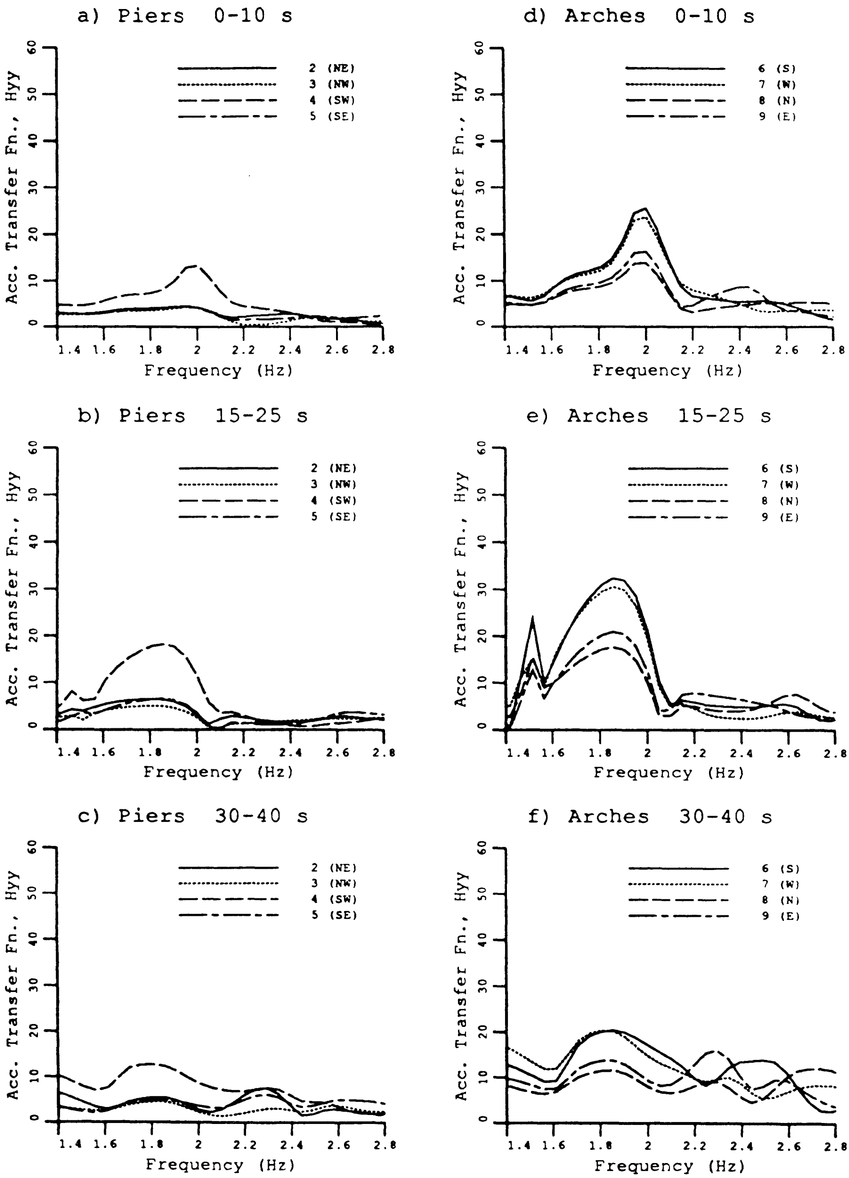

Figure 6. Transfer functions for N-S response to N-S base input.

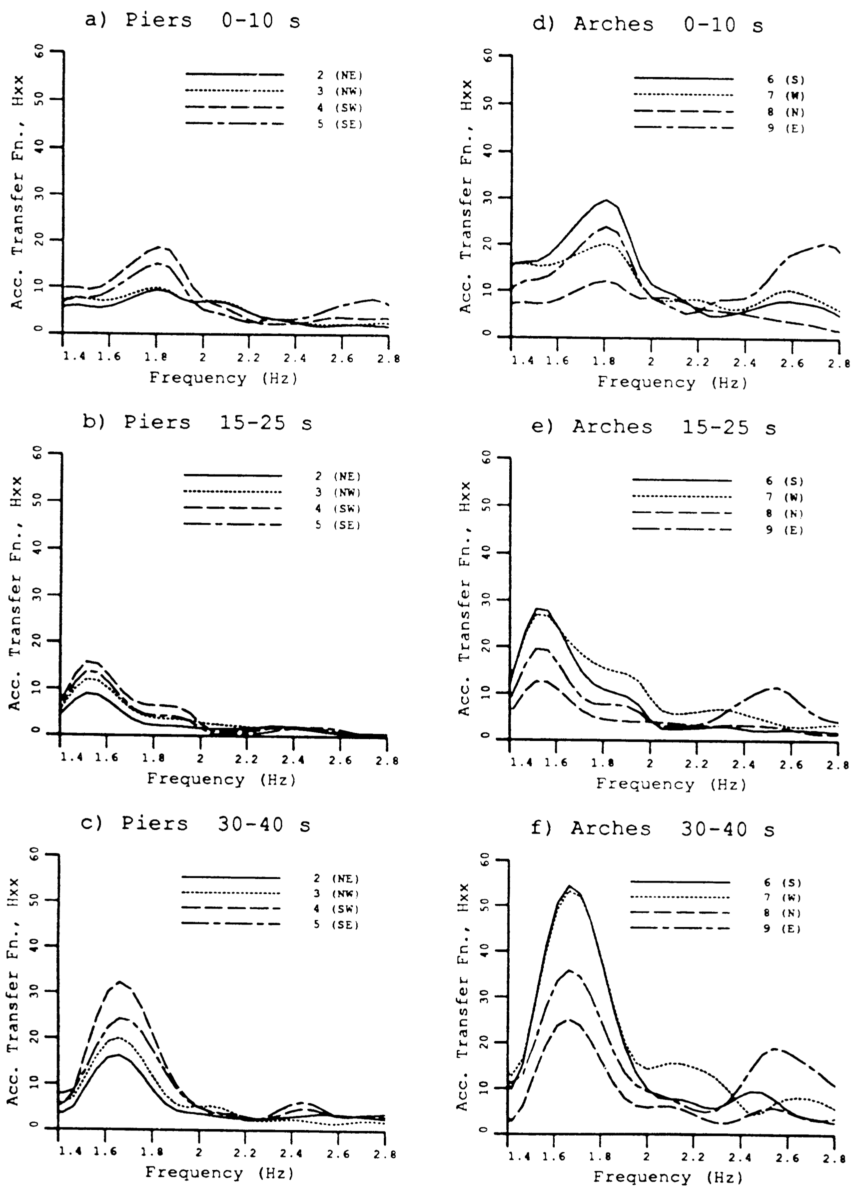

Figure 7. Transfer functions for E-W response to E-W base input.

visually with the help of animation. The motions were first filtered to isolate the anticipated frequency for the mode of interest. An animation graphics routine called *Animhs* was developed to accomplish the visualization part, while the filtering was performed using standard data analysis techniques [9]. The graphics routine idealizes the plan view of the dome base as a circle enscribed by a square defined by the axes of the arches. Actual motions filtered from the original time histories in a narrow band around the selected frequency are plotted in uniform time increments. The deformation of the square and enscribed circle are then interpolated in bilinear fashion.

The unfiltered and filtered displacements for the top of the NE main pier are shown in Figure 8. Figure 9 shows snapshots of the animated response which combines simultaneous filtered displacements at the tops of all four main piers and all four main arches. Cross hairs define the relative translation or rotation of the dome base by establishing the initial and displaced positions. Mode shapes were observed in real time and found to be more complex than those of the finite element models. As Figure 9 shows, mode 2 response contains a small E-W component resulting in an elliptical orbit of the center of mass, and mode 3 is not a pure rotation. In the animated mode 3, the W and E arches move in an almost shear type motion in the N-S direction.

Simulation

An attempt at matching the recorded response using a linear FE model has been made to test it's validity under forced vibration. The model named *Hsdyntc4* has been constructed which is similar to the primary structure model, *dyn26a*, but has more refinement of the mesh in the region of the arches. The elastic properties have been adjusted to account for the observed change in frequency during the second interval of the earthquake response motion. The elastic moduli and density used in this model are the same as those for the *dyn26a* model (see Table 2), except that the E values in areas other than stone masonry and tension areas were reduced 20 percent to $4.0(10^9)$ Pa. Also, no distinction was made for surcharge areas. The frequencies obtained by eigenvalue analysis of the *Hsdyntc4* model are given in Table 3.

Simulated response of the model to the earthquake was calculated using the mode superposition method with input acceleration at all base points in the model identical to that measured at the base of the NE main pier. Figure 10 shows the measured and simulated N-S acceleration time histories at the tops of the NW pier and E arch which represent a continuous load path to the ground. Ten modes were superimposed assuming damping of 2 percent critical in each mode. At the tops of the piers, response envelopes are in fair agreement. The measured response envelopes at the tops of the arches are significantly higher than those computed, however.

CONCLUSION

Dynamic modeling of the Hagia Sophia has been performed using calibrated finite element analyses and animated representations of measured earthquake response motions. Using data from a recent low-level earthquake, the dominant frequencies identified in an ambient vibration study have been

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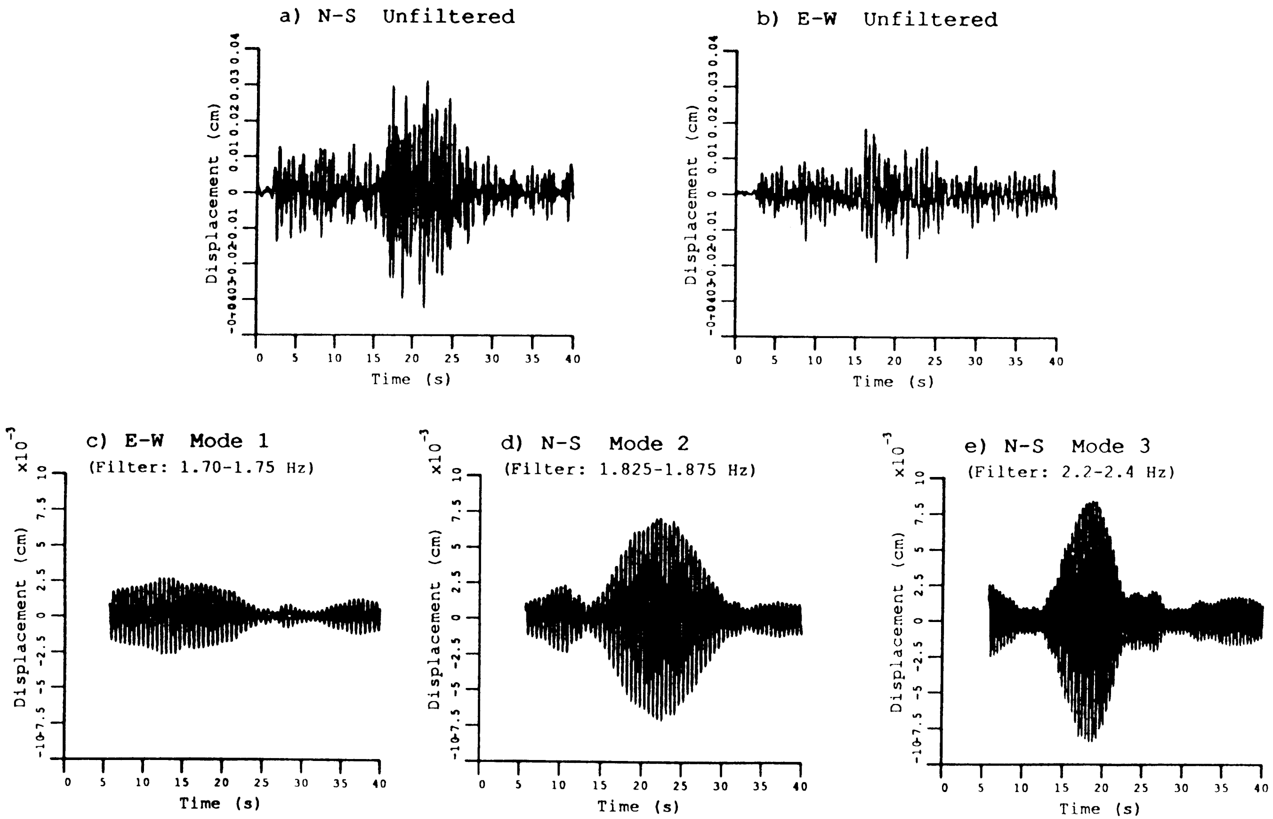

Figure 8. Unfiltered and filtered time histories for top of NE pier.

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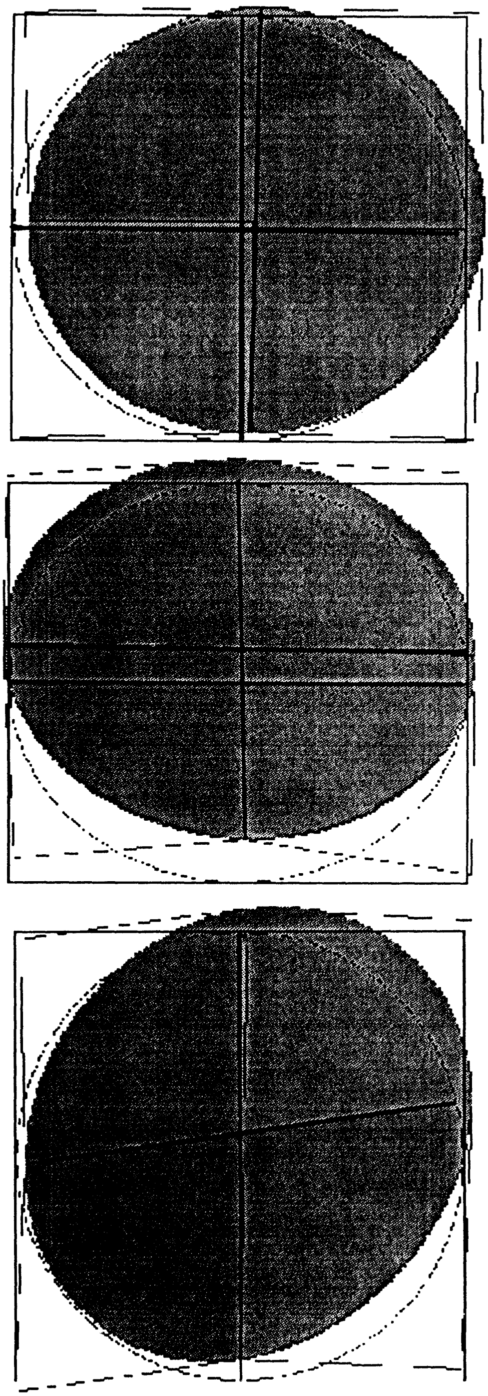

Figure 9. Animated mode shapes of primary structure.

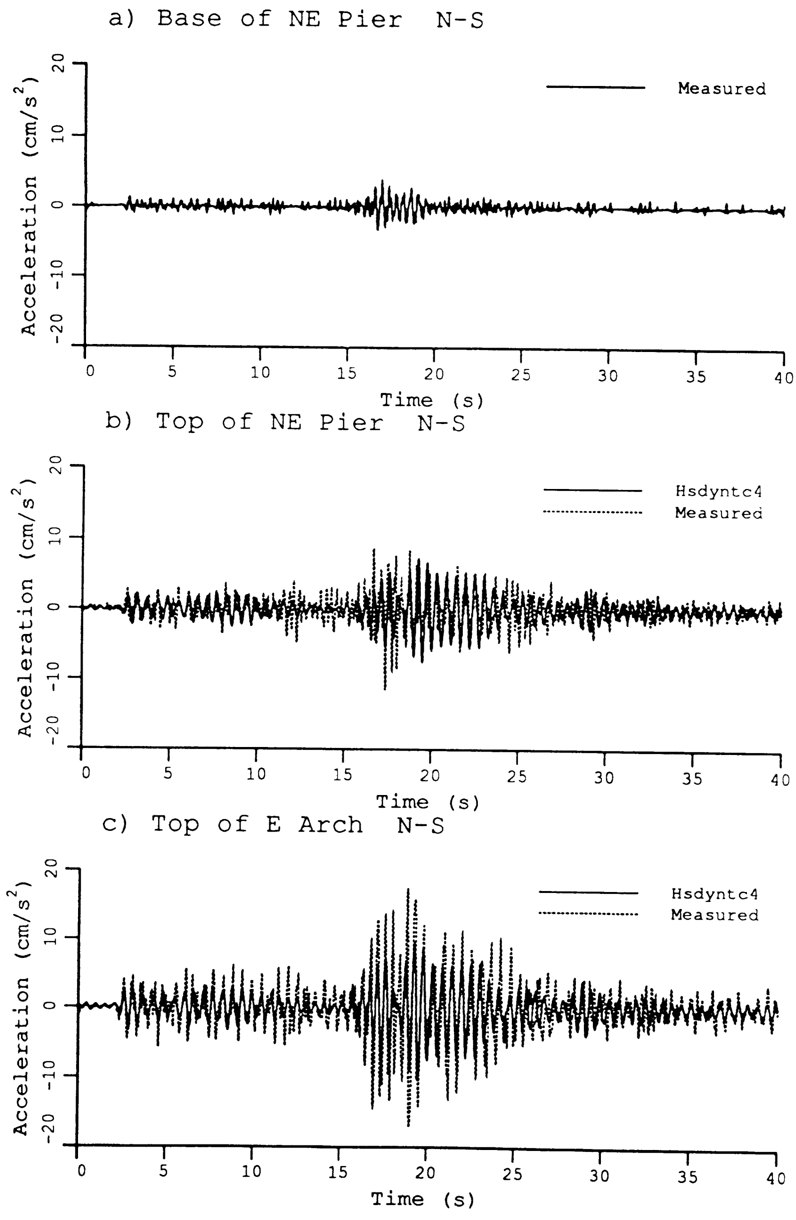

Figure 10. Simulated earthquake response using primary structure model.

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confirmed. The observed frequencies indicate a nonlinear behavior for the masonry structure even at very low response levels. The linear finite element models of the stone masonry portion of the main dome support structure are in fair agreement with measured responses, while the models underpredict the response of the brick masonry portion of the structure. Modeling of dynamic response provides an important means of monitoring the earthquake worthiness of Hagia Sophia. As larger intensities are recorded and appreciable damage becomes evident, it will become increasingly important to incorporate nonlinear action associated with the masonry construction in such models.

REFERENCES

1. M. Erdik and E. Cakti, 'Istanbul Ayasofya Muzesi Yapisal Sisteminin ve Deprem Guvenliginin Saglanmasina Yonelik Tedbirlerin Tespiti,' 2nd Research Report, Earthquake Research Institute, Bogazici University, November, 1990.

2. K. E. Hill, 'Hagia Sophia: Static Analysis of a Finite Element Model,' (unpublished senior thesis, Dept. of Civil Engineering and Operations Research, Princeton University, 1991).

3. R. Mark, A. S. Cakmak, and K. E. Hill, and R. Davidson, 'Structural Analysis of Hagia Sophia: A Historical Perspective,' *Proceedings of STREMA '93*, 3rd International Conference, Bath, UK, June 16-18, 1993.

4. M. Erdik and E. Cakti, 'Instrumentation of Hagia Sophia and the Analysis of the Response of the Structure to an Earthquake of 4.8 Magnitude,' *Proceedings of STREMA '93*, 3rd International Conference, Bath, UK, June 16-18, 1993.

5. R. J. Mainstone, *Hagia Sophia: Architecture,Structure, and Liturgy of Justinian's Great Church,* (New York; Thames and Hudson, 1988).

6. R. A. Livingston, P. E. Stutzman, R. Mark, and M. Erdik, 'Preliminary Analysis of the Masonry of the Hagia Sophia Basilica.' *Proceedings of STREMA '93*, 3rd International Conference, Bath, UK, June 16-18, 1993.

7. 'Users Manual: FORTRAN subroutines for statistical analysis,' *IMSL STAT/ LIBRARY*, Version 2.0, IMSL, Inc., Houston, TX, September, 1991.

8. D. E. Newland, *Introduction to Random Vibrations and Spectral Analysis,* (New York; John Wiley and Sons, 1984).

9. R. W. Hamming, *Digital Filters*, 3rd edition, (New Jersey; Prentice Hall, 1989).