

Load path method in the interpretation of masonry vault behaviour

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Abstract

Born as a method to design Strut and Tie Models in reinforced concrete structures, the Load Path Method (*LPM*, Schlaich and Schafer, Vitone A. and Vitone V., Palmisano et al.) has also become a simple and effective instrument to understand the behaviour of masonry structures (De Tommasi et al.). This paper deals with the effectiveness of the application of the Load Path Method to the diagnosis of the pathologies of the masonry vaults. While proposing the principles and the general criterions of *LPM* for the case of the arch, it will try to gather the correspondences with the classical approaches to these themes. The method offers an interpretation of masonry vault behaviour that immediately exhibits the correlation among form (geometry) and structure (distribution of loads and of thrusts): a particularly narrow correlation in the case of the masonry vaults.

Keywords: masonry structures, arch, vault, strut and tie, load path.

1 Load Path Method: basic principles

The most suitable orthonormal Cartesian system of architectural forces to physical environment in which they flow, is the one capable to bring back them only to vertical loads and horizontal thrusts. According to this, structure can be read as the trace of loads path (De Tommasi et al. [1]). The form of the structure is the result of their mutual integration and mainly of the influence of profiles traced by thrusts path, forced to deviate their natural horizontal flows to the soil,



in order to go in search of equilibrium. Forces represent loads that, in the way from their application points (S) to the restraints (E), in every deviation node have to apply thrusts (H) to the rest of the structure and to receive deviation forces equal in value and opposite in direction to thrusts in order to respect equilibrium (fig.1).

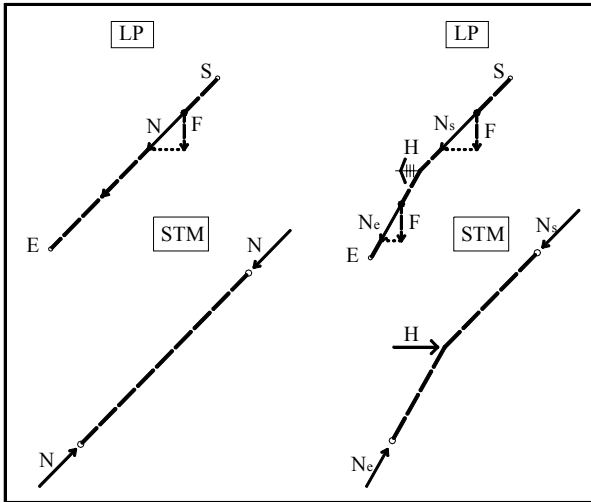


Figure 1: LP and STM.

The design of this load flowing through the structure can be approximated by polygonal lines in which there are thrusts in every deviation node. Structure will be crossed by fluxes in compression (dashed lines), when loads travel in the same direction of their path, and by fluxes in tension (continuous lines) along which loads go in the opposite direction respect to their path. According to classical theory, the basic principles that lead Load Path Method are the respect of equilibrium and of compatibility. Thrusts in deviation nodes are necessary in order to respect equilibrium and every path is possible if it is equilibrated.

Among infinite paths in equilibrium, loads have to choose the one in which their vectors invest the minimum quantity of strain energy, that is the only one equilibrated and congruent. At this purpose loads get energy from their own potential energy that decreases. Along a generic path (polygonal in this model), the calculus of the invested strain energy (D) is simplified in the summation of the terms relative to every side of the traverse:

$$D = \frac{1}{2} \sum N_i l_i \epsilon_i$$

where “i” is the generic side of the load path, N_i is the intensity of the *vector* that *brings* load on that side of the load path, l_i is the length of the generic side and ϵ_i is the relative strain that is medium constant on l_i .

2 The arch and the vault: the assumptions

It has been assumed that a masonry barrel vault can be thought as a series of parallel arch rings in the hypothesis that the generic ring is sufficiently far from the “discontinuity” regions such as the extremity tympana (fig. 2), the areas with concentrated loads, etc.

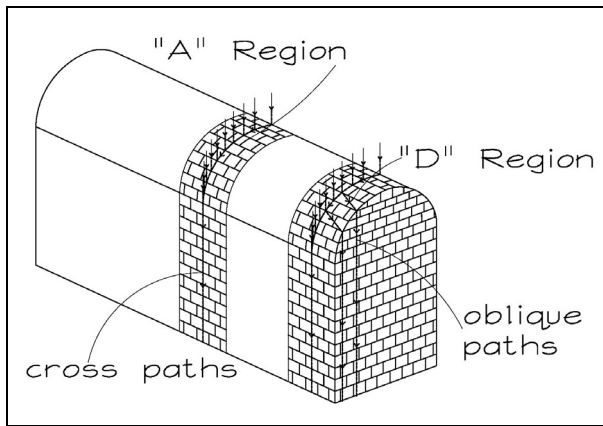


Figure 2: Masonry barrel vault.

In this paper the case of arches geometrically symmetrical and symmetrically loaded only with vertical loads will be considered. Despite this assumption, that benefits the simplicity of the analysis, the method has general validity.

As regards the masonry arches behaviour, the following three assumptions (Heyman [5]) have been made.

2.1 Assumption 1: masonry has no tensile strength

According to LPM, this assumption means that inside arches loads and thrusts can follow only paths in compression (fig. 3a,b). In other words loads are able only "to go down" but never "to go up again" and thrusts can only "advance" but never "move back". This is a behaviour totally different from the one of the beams (fig. 3c).

2.2 Assumption 2: stresses are so low that masonry has effectively an unlimited compressive strength

According to LPM this assumption is equivalent to admit that inside the arch the loads and thrusts path can approach the edges of the structure as far as it touches them, without implying the arch rupture. In fact the available resistant cross section when the LP touches the arch edge is equal to zero and the consequent compressive axial stress is equal to infinite.

According to Heyman [5] the assumption of unlimited compressive strength - although not real - is approximately correct if average stresses are low; in fact in this case high stress concentrations will cause only local ruptures that will not normally lead to overall failure of the arch.

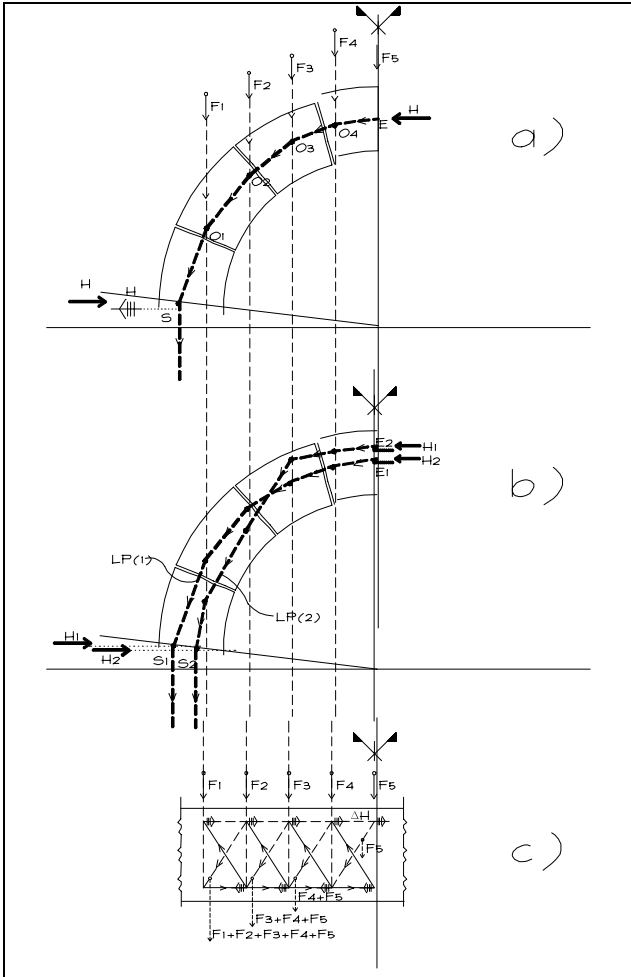


Figure 3: Arches and beams.

In actual fact, according to LPM, we should say that the loads and thrusts path cannot touch the edges of the arch, but it has to keep a distance from them sufficient to assure that stresses don't locally exceed the resistance of the material. On the other side, assuming a LP that "touches" the edges of the arch we make a mistake in the assumption of the geometry of the LP that is negligible if the gap between the real position of the LP joint and the assumed one is small compared to the whole height "h" of the arch cross section.

The assumption of masonry unlimited compressive strength excludes the possibility that the arch collapses if there are possible equilibrium conditions - independently from the intensity of the stresses - on condition that the equilibrium is possible retaining the original shape of the arch (see the third assumption). According to LPM this happens when there is no "possible" (that means "in equilibrium") loads and thrusts path that remains completely inside the arch (in the assumption that the arch has retained its original shape). If the load (or thrust) path goes out of structural body the arch collapses because of the assumptions made. In fact, because of the lack of tensile strength (assumption 1), it would be impossible to equilibrate an axial force applied outside the arch cross section (fig. 4).

2.3 Assumption 3: sliding failure does not occur

This assumption can be considered correct in most of the cases because, in general, local masonry ruptures don't involve consequent appreciable shape modifications (Heyman [5]). According to LPM this assumption is equivalent to admit that local ruptures involve neither an appreciable change of the shape of the loads and thrusts paths, nor local incongruities such as the slippage of two faces of the same cross section (fig. 5).

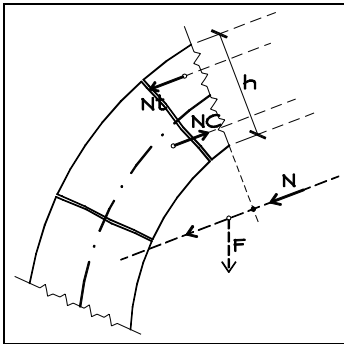


Figure 4: Axial force outside the arch cross section.

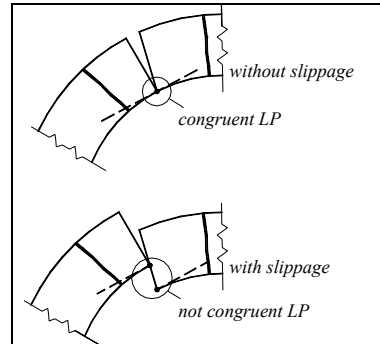


Figure 5: Slippage of two faces of the same cross section.

3 Loads path in arches and vaults

3.1 Possible and limit paths

An arch, of thickness "h" and perfectly fixed to the imposts, gives loads (and thrusts) infinite "possible" (\equiv in equilibrium) paths, depending on the positions of the start "S" and of the end "E" of the thrust.

In figure 6 only three “possible” paths are shown. According to par. 2 they are “possible” because they lie inside the structure.

If, in a first limit case, the bearings (abutments) have infinite stiffness compared to the arch, among infinite possible paths the only one congruent is the one that invest the minimum quantity of strain energy inside the arch.

If, on the contrary, in the other limit case, the arch has infinite stiffness compared to the bearings, among infinite possible paths the only one “preferred” is the one that invest the minimum quantity of strain energy inside the abutments. In this case the path will be the one having the minimum value of the thrust at the impostes.

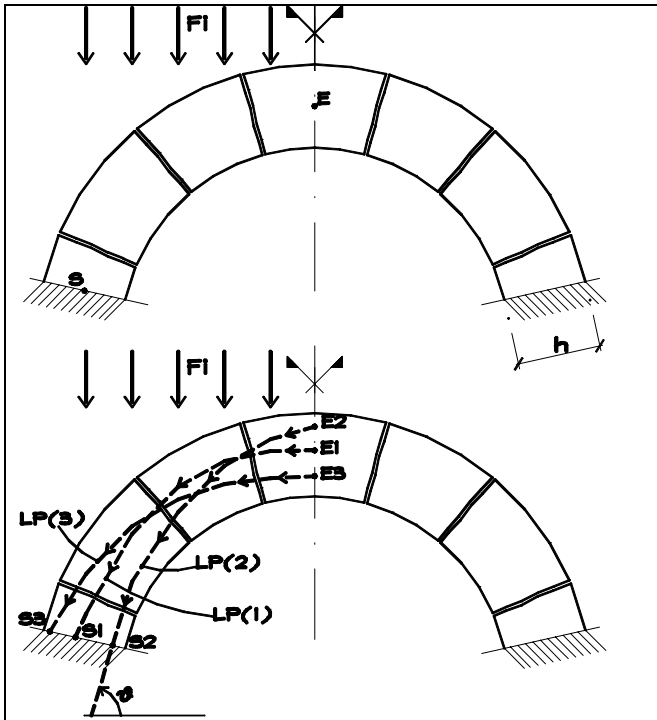


Figure 6: Possible loads paths in an arch.

3.2 Slenderness and saddle backing of the arch

First of all, the case of only one point vertical load “R” acting on the arch axis of symmetry will be examined (fig. 7). The value of the thrust “H” will be proportional to the load:

$$H = (R/2) \cot\theta$$

As figure 7 shows, the minimum value of H is reached when $\cot\theta$ reaches its minimum values, respecting the condition that the LP remains completely inside the arch. Fixed, for instance, the end “E” (fig. 7a), the minimum value of H is reached when the start “S” is in the closest position to the R axis; this position is the one with the minimum span ($\ell_2 < \ell_1$).

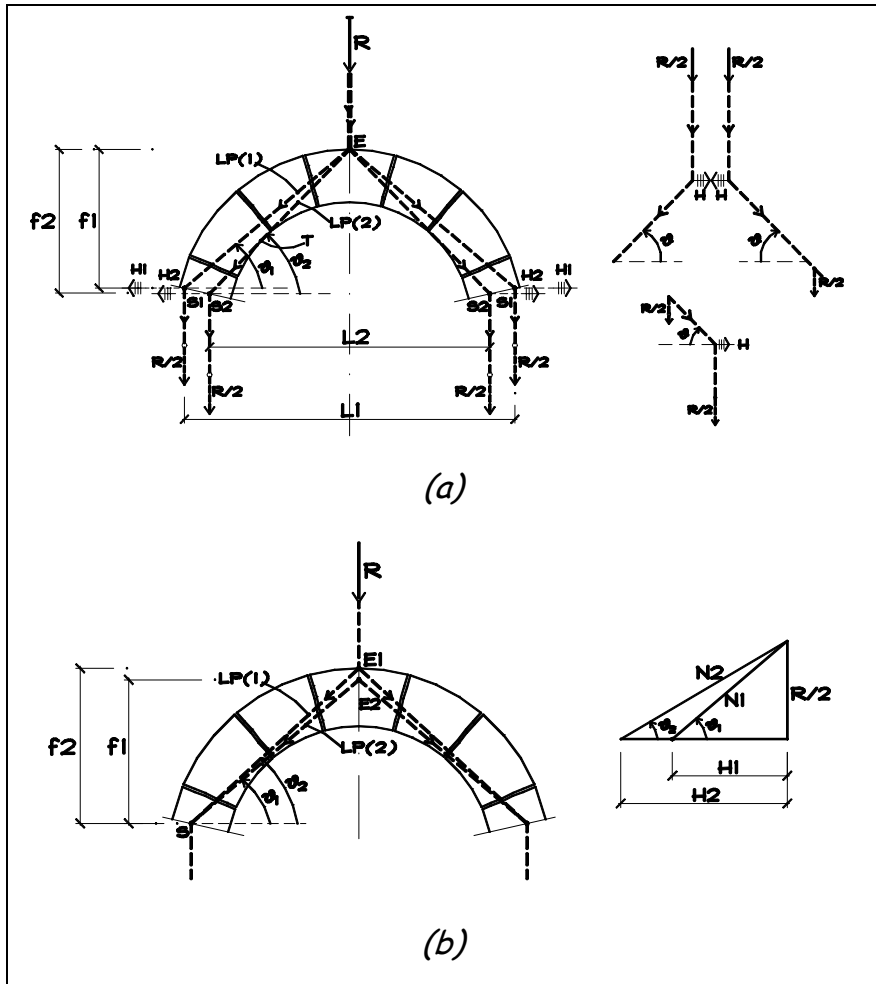


Figure 7: Limit load paths.

In a similar way, fixed the start “S” (fig. 7b), the minimum value of H is reached when the end “E” of the thrust is in the highest position of the crown section of the arch; this position is the one with the maximum rise ($f_2 > f_1$).

The optimal situation (absolute minimum value of H , respecting the condition that the LP remains completely inside the arch) is the one represented

by LP(2) in fig. 7a. In this situation “E” is at the top edge of the crown section and the path is tangent to the intrados line of the arch (in T).

Figure 8 shows that equilibrium is not always possible. In fact, it can happen that the most “internal” path crosses the intrados line (in U1 and U2): this case corresponds to a collapse mechanism (for every value of R) because the arch thickness is too thin as regards the “saddle backing” of the extrados line. The “saddle backing” is therefore a specific (geometric) characteristic of the structure, very important also for the static behaviour. It is measured (fig. 8) by:

$$\rho_0 = \tan\theta_0 = f_0 / (l_0/2).$$

3.3 The hinges formation

According to LPM, in the arch cross sections where the loads (or thrusts) path “touches” the edge of the section (either the extrados or the intrados) hinges will form.

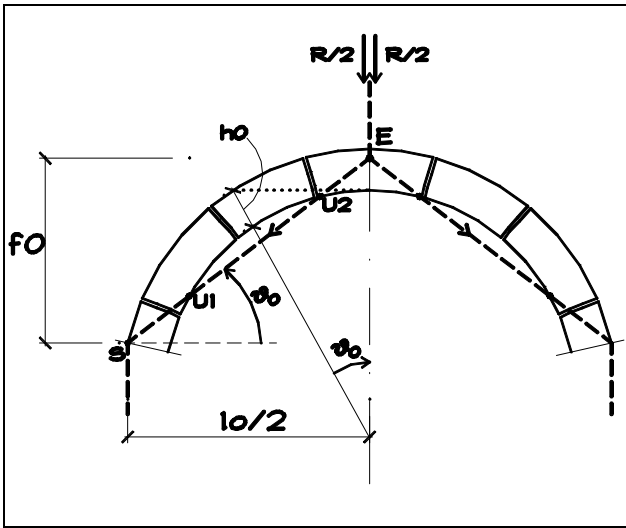


Figure 8: A load path not in equilibrium.

As figures 9 and 10 show, it’s possible to have different situations depending on the ratio of slenderness ($\lambda_0 = h_0/l_0$) to saddle backing ($\rho_0 = 2f_0/l_0$) of the arch.

In the figure 9a case a LP from the crown to the internal edge of the imposts is possible: this LP has the absolute minimum value of the thrust and forms three hinges.

In the figure 9b case the previous situation is not possible because the arch is too slender as regards its saddle backing. In this case the path with the minimum value of the thrust is the one that reaches the intrados line in O1 where the hinge will form (and not at the imposts).

In the figure 10 case the load path reaches the intrados and the extrados line (respectively in O2 and in O1). The arch is on the point of collapse by the formation of five hinges.

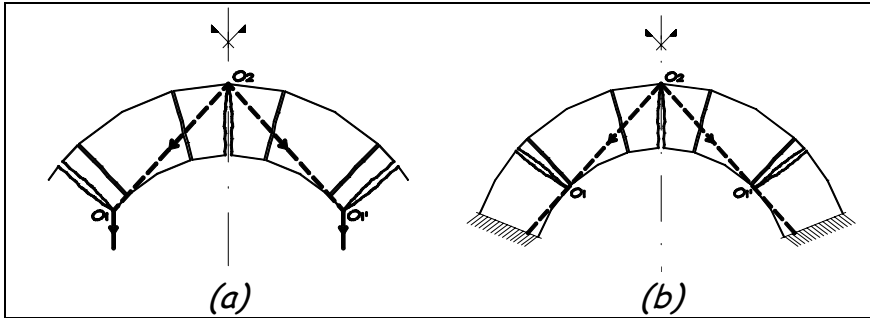


Figure 9: The formation of three hinges.

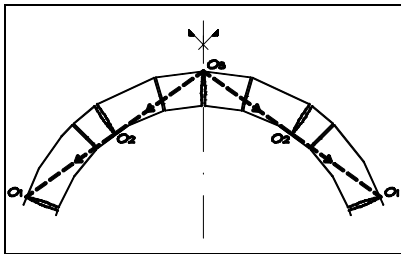


Figure 10: The formation of five hinges.

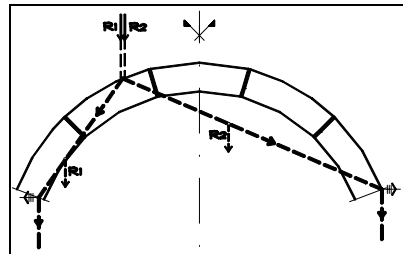


Figure 11: Not symmetrical load condition.

3.4 Lateral point load (not symmetrical load condition)

Figure 11 shows that if the point load is not applied to the crown of the arch, but it is lateral, that is if the load condition is not symmetrical, with the same value of the saddle backing it is necessary to have larger thicknesses to be safe that non-collapse mechanism will form.

3.5 Symmetrical loads. The arch as path of the vertical loads

It is always possible to transform a generic load condition in an equivalent point load condition (fig. 12a).

Figure 12 shows that the value of the thrust H of the F_i loads distribution is equivalent to the one of two point symmetrical loads $R/2$, each of them equal to

the resultant of the F_i on the half of the arch:

$$H = R/2 \cot\theta$$

If $R/2$ changes his position from $x_R=0$ to $x_R=l_0/2$ (f_0 fixed), the value of the thrust H will increase from 0 ($\theta=90^\circ$) to its maximum value H_0 :

$$H_0 = \frac{R}{2} \cot \vartheta_0 = \frac{R}{2} \frac{1}{\rho_0} = \frac{R}{2} \frac{l_0/2}{f_0} \tag{1}$$

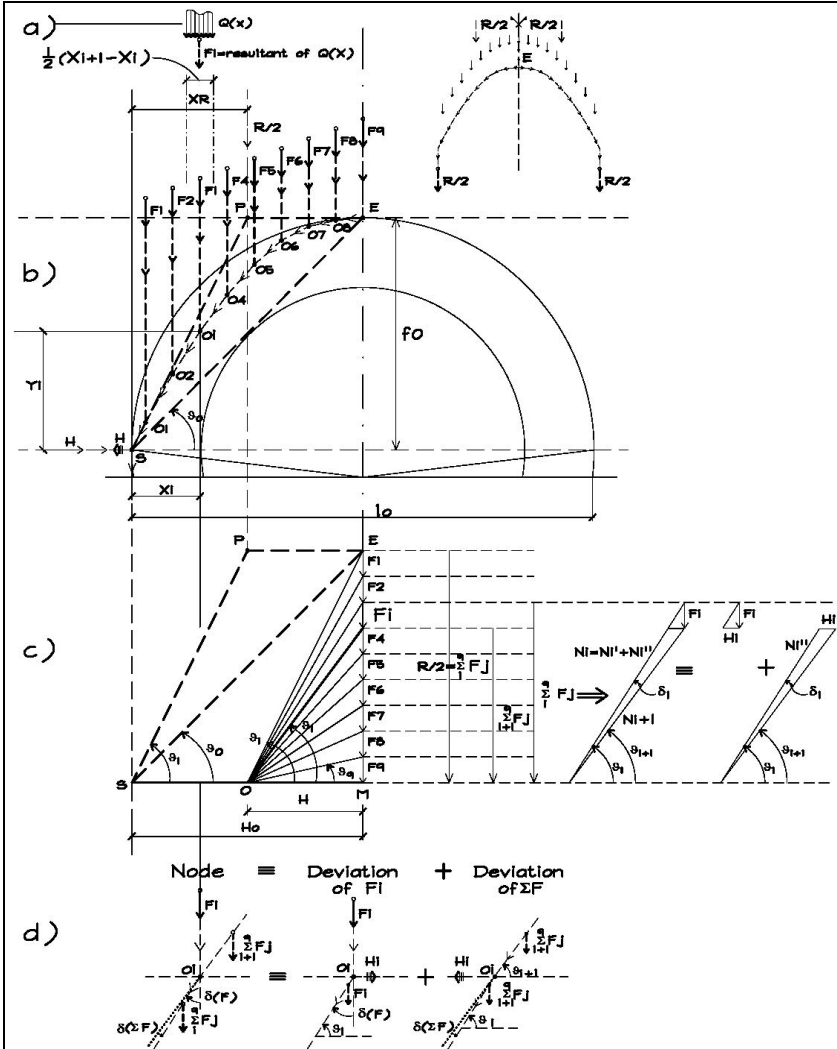


Figure 12: The arch as path of the vertical loads.



the travelling load, decreasing itself, has to reduce the inclination of its path to keep constant the value of the thrust H (fig. 12c). In the assumption that the direction of every load is from the top to the bottom, in every node O_i the inclination angle of the ΣF_j path must increase ($\theta_i > \theta_{i+1}$) because in every O_i , F_i deviate and apply to the structure a thrust:

$$H_i(F_i) = F_i \cot \theta_i \quad (2)$$

For equilibrium reasons (\equiv to make possible the F_i deviation) ΣF_j , coming from the top, must deviate in O_i of δ_i , increasing the inclination angle from θ_{i+1} to θ_i , and must apply a thrust equal in value (but opposite in direction) to the F_i one (fig. 12d):

$$H_i(\Sigma F_j) = \Sigma F_j (\cot \theta_{i+1} - \cot \theta_i) \quad (3)$$

Because of the equilibrium of these two thrusts it is possible that loads go from their application points to the imposts only by path in compression.

Equation (3) analytically shows what figure 12 exhibits graphically: the line of the “possible” (\equiv in equilibrium) LP is strictly related to the intensity and distribution of loads F_i .

3.6 The arch as path of the thrust

It is also possible to interpret the loads ΣF_j path as the one of the thrust H (fig. 13). In this case the travelling load doesn't change (if every F_i is vertical). In every node the thrust path (from the impost to the crown) deviates of δ_i , changing its inclination from θ_i to θ_{i+1} . This is possible (\equiv in equilibrium) because in every node O_i , the vertical thrust V_i (directed from the bottom to the top) of H is equilibrated by the load F_i which is equal in value but opposite in direction.

4 Conclusions

Load path method is an instrument to analyse structural continuum. It seems to have the peculiar capacity to make understand, using its form geometrical outlines, the physical behaviour of the structure, from the global behaviour to the most accurate details. As showed for the masonry vaults, it can be very useful to understand the link between form and structure and to analyse the pathologies of the masonry structures.

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