Abstract

Unilateral contact analysis with friction can be performed by general purpose finite element programs, provided that the user has basic theoretical background knowledge. These mechanisms are of great importance for the safety evaluation and structural rehabilitation of monuments. This is demonstrated by the investigation of some damage scenarios for a stone bridge.

1 Introduction

Mechanics of stone structures built without reinforcement is a challenging field of applications. The study of the masonry arch has a long tradition in the engineering literature [1]. Although some of these monuments would not fulfill the requirements of modern building codes, they have demonstrated their strength by surviving a number of earthquakes during their long history. Energy dissipation mechanism arising along contact interfaces are certainly responsible for the beneficial aseismic behavior of these structures. Since similar problems arise in the design of granular materials, one would think that discrete element methods could be used. Nevertheless, a complete analysis of a stone masonry with thousands of particles and interfaces would not be efficient for the structural engineer. Therefore the use of some phenomenological interfaces, or potential interfaces along the lines where cracks and other damages are possible to appear, is a promising alternative. On the other hand, several years before, large-scale finite element computations including unilateral contact and friction effects were beyond the ability of everyday engineering practice. The situation
dramatically changed. Theoretical developments in the area of nonsmooth and contact mechanics [5,6] and the availability of powerful computers triggered this development.

In this paper the authors demonstrate the use of unilateral and frictional structural analysis models for damage prediction and strength evaluation of stone bridges, by using a concrete analysis of Plakas’s bridge in the region of Epirus in Greece.

2 Unilateral contact effects with friction effects.

The possibility that some separation appears between two parts of a structure coming into contact is known as a unilateral contact phenomenon. This is a typical variable-structure nonlinearity, which involves either-or decisions in the mechanical model. The frictional stick-slip nonlinearity is an analogous phenomenon. Both problems belong to the area known nonsmooth mechanics (after P.D. Panagiotopoulos) [6], [5]. The reason is that the arising models (functions) are nondifferentiable in the classical sense. The short discussion of this section uses a minimum of knowledge from the above-mentioned broader field. With this knowledge one can use effectively currently available general purpose finite element software for the static and dynamic analysis of stone structures with unilateral frictional joints (interfaces).

Unilateral contact along interfaces is a suitable model for nonlinear analysis of masonry structures. From this practical point of view one defines a number of potential interfaces. Along these interfaces opening (separation) and frictional (stick-slips) effects are considered. The actual state at each point of the interface will be found after the solution of the problem. It should be noted that the numerical solution of problems with nonsmooth nonlinearities is quite demanding. In general, elements of nonsmooth and nonconvex optimization can be used for the construction of general-purpose algorithms, see, among others, [5] and the references given there. In the particular case of unilateral contact and friction, being one of the oldest studied problems in this field, several empirical or semi-empirical algorithms (including, trial-and-error techniques) have been proposed. A more general approach is to use algorithms based on numerical optimization, mathematical programming and variational inequalities [6].

Modern general-purpose finite element software can be used for the solution of several real-life problems with unilateral-contact elements. For this study we used MARC [4]. Effective use of the available models and the limits of their applicability require the use of some theoretical knowledge. In this case, a mixture of augmented Lagrangian type procedures, for the treatment of the unilateral contact mechanism, and of smoothing techniques, for the frictional part, is used. Technical details can be found, among others, in [1, 3, 8, 9, 10]. Previous applications on contact between rubber-coated cylinders and on dynamic contacts between concrete reinforcement and masonry walls have been presented in [7], [8].
For practical applications of contact analysis it is necessary to define the contact bodies, the contact tolerance, the area in which the contact possibly occur, the contact procedure, the separation procedure and the type of friction. A Lagrange multiplier approach was used in this work for the solution of the unilateral contact problem. The developed contact forces are calculated directly from the external load and the nodal point forces, equivalent to the current element stresses. The resultant force transmitted from one surface to the other through a point of contact is resolved into a normal force, due to unilateral contact, and a tangential traction, which is due to friction. The region of contact is often unknown prior to the analysis and large changes in the contact area are possible including relative sliding with Coulomb friction or possible separation after contact. In the modelling of friction, usually it is assumed that Coulomb's law of friction is applicable with a constant coefficient of friction. The coefficient of friction was considered equal to 0.1 in the here presented applications.

Within MARC, the unilateral contact mechanism is considered in an exact way and solved by an iterative solution method. On the contrary, the stick-slip frictional mechanism is treated by a smoothing approach. For theoretical and analytical results related to these methods the reader may consult [1, 6, 10].

3 Case study: the Plaka bridge in Epirus.

The arch of the stone bridge of Plaka, in the area of Epirus in the north-west part of Greece, constructed in 1866, is the largest span in Northern Greece, which has many old bridges, and maybe in the whole of Greece with an impossing arch of 40.00 m wide and 18-20 m height and two small relief arches in either side 6.00 m wide (see, Figure 1, for more details see [9]). There is a crack of 15 cm wide inside the main arch, while in a large area the stones that form the arch are dislocated and misaligned. Minor damage exists at the lower part of the crown of one span of the bridge near each end to the side of the mountain.

The bridge from the monolith region is supported on earthy ground and there is the possibility of displacement of the bridge due to the movement of the ground and the thrust of the nearby mountainous terrain. Previous investigation has indicated that from the two possible scenarios, settlement and earthquake, basement settlement seems to be the cause of this damage [9]. Further investigation in this direction is reported in this paper.
3.1 Finite element modeling.

The details of the geometry of the bridge are given in Figure 1. The following material data were used for the linear elastic part of the analysis: modulus of elasticity $8.825 \times 10^6$ Pa, Poisson's ratio 0.15 and density $19.44$ N/m$^3$.

Quadrilateral finite elements have been used for the analysis of the arch ring, and triangular elements for the remaining of the structure. Only two translational degrees of freedom per node were considered. In addition, because of the considerable thickness, the analysis was carried out with plain strain conditions. The model consists of 1831 nodes and of 2622 elements. The finite element mesh is shown in Figure 2.

3.2 Structural analysis assumptions.

The model was analysed first for a vertical settlement of the foundation equal to 0.05 m and second for a vertical settlement of 0.2 m together with a horizontal movement of 0.2 m. Due to the large width and height of the arch, 40 m and 17.61 m respectively, the 1.56 m thickness at the apex of the bridge is considered small and we expected that some problems would appear in that region of the arch. The results showed instead that a concentration of stresses appeared in the region of the arch where the actual crack exists. A preliminary study and the dynamic characteristics of the structure have been reported in [9]. In this paper, a
more detailed finite element model is considered. The eigenfrequencies and
eigen-shapes do not differ substantially between the two models, therefore only
static results are presented here.

3.3 Results of the linear structural analysis.

The results of two loading cases are schematically shown here. First loading case
has both vertical settlement and horizontal displacement for one of the piers
equal to 0.1 m. The magnitude of these settlements is set equal to 0.2 m in the
second loading case. Some results of the second loading case are shown in
Figures 3-6.

![Deformed configuration](image1)

Figure 3: Deformed configuration.

![Horizontal stress component](image2)

Figure 4: Horizontal (s_11) stress component
3.4 Structural analysis including unilateral contact effects.

Let us now consider the case with unilateral contact effects. The whole model is divided into parts connected with unilateral frictional interfaces (see different parts of Figure 7).
Again, several loading cases have been considered: a vertical settlement of 0.04 m, a vertical settlement together with a horizontal one, both equal to 0.2 m, etc. The latter loading case is shown in Figures 8-9.

From a parametric investigation we observed that, for static loadings, opening of the unilateral contact mechanisms does not appear in the equilibrium configuration. On the contrary, slip mechanisms of friction are activated. Near the existing cracks contact stresses are considerably reduced. Therefore the possibility of crack initiation from this point during an additional loading (e.g., an earthquake) is high. Moreover at the crown of the lower side of the right-hand-side opening some tensile stresses arise from linear elastic analysis, while
these stresses disappear, under the same loading, in the unilateral frictional contact model. Since at that point minor damage exists, we estimate that some initial tensile loading caused this damage and stress relaxation followed after entering the nonlinear behavior.

![Figure 9: Shear stress components](image)

4 References


