Limit analysis of masonry walls with rectangular openings by equivalent shear panel model

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Abstract

The present study deals with the limit analysis of masonry walls with rectangular openings by an equivalent shear panel model. Many experimental results on masonry walls with rectangular openings subject to both vertical and horizontal loads show importance of shear resistance of wall panels. Based upon these experimental results, in-plane failure patterns of single masonry walls are categorized into a) the shear diagonal crack opening, b) local panel rocking, c) bed joint sliding, and d) full or partial overturning of an entire wall. The equivalent shear panel model can evaluate strength for typical in-plane failure patterns due to earthquake damages. By means of the present model the limit analysis of masonry walls can be formulated as a searching problem for a collapse mode with minimum failure load, to which the genetic algorithm (GA) can be applied. The application of both the present shear panel model and the GA is illustrated in a numerical example of a masonry wall with rectangular openings subjected to constant vertical and incremental horizontal loads.

1 Introduction

Unreinforced masonry is constitutive material for all the historical architectural heritage of a large number of regions in the world. For some countries, as Italy, the architectural heritage is so large and important to put the problem of the analysis of safety of existing masonry structures under focus of structural engineering research development. Earthquake is by sure among
the main causes of loss of historical buildings. Therefore significant efforts have been made to build good analysis models and strategies to assess the seismic vulnerability of masonry structures. For analysis of masonry structures collapse mechanisms should be modeled, and reliable constitutive laws have to be defined.

Finite Element Analysis (FEA) model for masonry, which is most widely used, is based on the assumption of isotropy and homogeneity for material, Drucker-Prager plastic failure criterion with low level cutoff on tensile stresses [1]. Other FEA non-linear models are based on the damage mechanics. Lourenço [2], Gambarotta [3], and Aoki [4] analyse the non-linear behavior of masonry confining the elastic-plastic failure to mortar bed-joints. Lourenço [2] applies also an anisotropic continuum model to masonry structures.

Based upon the experimental results, Kohama et al. [5] introduce an equivalent truss model in order to express failure patterns of wall panels, and the genetic algorithm (GA) is applied to the limit analysis of masonry walls with openings. Various types of GAs have been proposed and they are prominently applied to structural optimization problems because near optimal solutions are frequently required instead of the optimal solution from the engineering point of view. Jenkins [6] applies the GA to the optimization of structural design of a trussed beam, where stochastic processes generate an initial population of designs and then apply principles of natural selection/survival of the fittest to improve the designs. Ghasemi and Hinton [7] apply the GA to the size optimization of several 2D planar trusses based on the concept of rebirthing with involving continuous and discrete design variables. Kocer and Arora [8] apply the GA to the optimal design of 3D latticed towers subject to earthquake loadings, in which the truss elements are optimally selected from the sections in a manufacture’s catalog.

By means of our previous truss model [5], the limit analysis becomes very difficult to solve for sizable masonry walls because of increment in the number of truss elements. Therefore a new shear panel model is proposed, and the GA is applied to the limit analysis of masonry walls with openings.

2 Equivalent shear panel model

Observations after strong earthquakes show that unreinforced masonry walls with openings are separated into sound and damaged parts. As the sound part can be expected to behave like rigid body, unreinforced masonry walls can be modeled as equivalent shear panel model, which is composed of rigid panel elements and shear panel ones. According to the loading conditions, the behaviors of panel elements are characterized by both pier and beam. Figure 1 shows an example of a masonry wall with openings and the corresponding equivalent shear panel model. Rigid panels exist between pier panel elements and/or beam panel ones. As shown in this figure, rigid
Figure 1: Masonry wall with openings and an equivalent shear panel model.

A panel element has three degrees of freedom, that is, displacements \( u \), \( v \) and rotation \( \theta \). Therefore, the nodal displacements of 4 nodes in each rigid panel can be expressed by \( u \), \( v \) and \( \theta \). As the possible movement of each component for the rigid panel element are three, that is, positive and negative directions and nil. Taking into account of symmetry, the number of possible deformation modes of each pier and each beam panel element are \( 3^3 - 3^2 - 3^1 = 15 \).

The collapse mechanism is described by a failure pattern connecting to several local failure mechanisms. The choice of the correct collapse pattern is the result of a process of combinatorial optimization [5]. The local numerical model adopted here is a wall-panel "functional" macro-element. A wall panel can fail in different ways. The actual realization of one or another among all possible failure patterns is substantially connected to the geometrical shape of the panel itself and the amount of vertical loads acting on the top edge.

The constitutive law of the panel elements is rigid-perfectly plastic. Their strength is assessed in such way to agree with the ultimate shear forces due to rocking and/or overturning, which are expressed in the following formulas, depending on the collapse mechanism actually expected.

As mentioned above, masonry structures have a large scatteredness of their material properties, therefore, the accurate formulation becomes less important [9]. In this paper, elastic equilibrium equations are adapted. The principal failure mechanisms of masonry panel subjected to seismic loadings can be modeled as follows:

1. Compressive (tensile) failures: as horizontal load or displacement increase, external pier panel in masonry structure is subjected to large compressive (tensile) stress. On the other hand, if the movements of two pier panels are different, beam panel in masonry structure is subjected to large compressive (tensile) stress. In both cases, final failure is obtained by crushing in compression (cracking in tension) of the panels.
2. Rocking failure: as horizontal load or displacement increase, bed joints crack in tension and shear is carried by the compressed masonry. Final failure is obtained by overturning of the wall and simultaneous crushing of the compressed corner.

3. Shear failure: peak resistance is governed by the deformation and development of inclined diagonal cracks, which may follow the path of bed and head joints or may go through the bricks, depending on the relative strength of mortar joints, brick-mortar interface, and bricks.

4. Sliding failure: during reversed seismic action, potential sliding planes due to the deformation of tensile horizontal crack in the bed joints can form along the cracked bed joints. This failure mode is possible for low levels of vertical load and/or low friction coefficient.

For a slender panel the local rocking mechanism can prevail. This kind of behavior is ductile, as it allows a floor drift quite large under constant lateral force, but the energy dissipation is nearly null. The ultimate shear force, $V_r$, due to rocking is calculated by the following equation.

$$V_r = D \cdot t \frac{f_u}{2(1+r)} \frac{D}{H} \left\{ r + (1-r) \left( \frac{p}{f_u} \right) + \left( \frac{p}{f_u} \right)^2 \right\}$$  \hspace{1cm} (1)

where $D, t$ represent length and thickness of the wall panel, $p = P/(D \cdot t)$, the mean stress on the wall panel due to the vertical load $P$, $f_u$, compressive strength of masonry, $r = f_t/f_u$, the ratio of the tensile strength $f_t$ to $f_u$, respectively.

Other possible failure patterns are connected with sliding and/or diagonal shear failure. In this case the lateral shear force, $V_d$, is given by the following equation.

$$V_d = D \cdot t \cdot \frac{2}{3} (c - \mu p)$$  \hspace{1cm} (2)

where $c$ represents an unbiased value of cohesion coefficient, $\mu$, an unbiased value of friction, respectively.

Still another failure mode is possible. It is connected with the global overturning of an entire wall panel for the whole height of the building.

Masonry is an anisotropic material. Therefore, the biaxial strength envelope of masonry must be either described in terms of the full stress vector in a fixed set of material axes, or in terms of principal stresses and the rotation angle between the principal stresses and the material axes [10, 11]. The tests have been carried out with half scale solid clay units. Both the orientation of the principal stresses with regard to the material axes and the principal stress ratio considerably influence the failure mode and strength.

From the experimental results by Page, the following relationships of strengths between the pier panel element and the beam panel element are
obtained.

\[ f_{u,\text{pier}} > f_{u,\text{beam}}, \quad f_{t,\text{pier}} < f_{t,\text{beam}}, \quad f_{s,\text{pier}} < f_{s,\text{beam}} \]  \tag{3}

where \( f_u, f_t, f_s \) mean the compressive, tensile and shear strengths, respectively. The factor \( f_o \) between the orientation of the principal stresses and that of the material axes is adopted here, in another way, the compressive, tensile and shear strength of the beam panel element can be obtained by \( f_o \times f_u, 1/f_o \times f_t, \) and \( 1/f_o \times f_s \), respectively. In case of the beam panel element the unit load \( p \) is negligible. Beyond the capacity of the materials, compressive, tensile and shear strengths become less. The equivalent rigid-plastic envelope adopted here is \( V_u = 0.7V_{\text{max}} \).

### 3 Formulation of limit analysis of masonry wall

By means of the present shear panel model, deformation of any pier or beam panel can be determined by displacements of all rigid panels \((u_i, v_i, \theta_i; i \in \{\text{rigid panel numbers}\})\). Moreover, the total internal work for any collapse mechanism of a masonry wall can be obtained by the deformations of pier and beam panels. The virtual work equation, therefore, for any collapse mechanism can be given as follows.

\[ \lambda p_v^T d + p_c^T d = W_I(u) \]  \tag{4}

where \( \lambda \) represents the failure load factor, \( p_v, p_c \), the variable nodal load vector and the constant nodal load vector, \( u \), the wall panel deformation vector, and \( W_I(u) \), the total internal work of a masonry wall, respectively. \( d \) means the nodal displacement vector, which can be calculated by the following equation in terms of \( u \).

\[ d = Au \]  \tag{5}

From eqns (4) and (5), the failure load factor, \( \lambda \), can be obtained as follows.

\[ \lambda = \lambda(u) = \frac{W_I(u) - p_c^T Au}{p_v^T Au} \]  \tag{6}

Consequently, the present limit analysis can be formulated as the following minimization problem of the failure load factor with the decision variable \( u \).

\[ \lambda(u) \rightarrow \text{min.} \quad \text{subject to} \quad \lambda(u) \geq 0 \]  \tag{7}
4 Application of genetic algorithm

4.1 Genetic algorithm

The genetic algorithm (GA) is one of stochastic optimization methods, which is based upon the process of natural evolution. By analogy to natural evolution, the solution candidates are called individuals and the set of individuals are called the population. Each individual is usually expressed as a binary bit string. The GA can search for an optimal solution by an iterative implementation of genetic operations. First, an initial population is created at random, which is the starting point of the evolution process. Then a loop consisting of the steps evaluation, selection, crossover and mutation is executed a certain number of generations. Each loop iteration is called a generation. At the end, the best individual(s) in the final population is the outcome of the GA. Parameters necessary at the beginning of the GA are the chromosome length, the population size, the maximum number of generations, the probability of crossover, and the probability of mutation.

4.2 Coding

For the GA the deformation vector, \( u \), should be expressed as a binary bit string. Although a decimal number can be transformed into a binary number (a binary code), which can be used as an individual for the GA, the gray code is adopted in the present GA. The relationship between the gray code \( (g_t, g_{t-1}, \ldots, g_1) \) and the binary code \( (b_t, b_{t-1}, \ldots, b_1) \) is defined as follows.

\[
g_k = \begin{cases} 
  b_l & \text{if } k = l \\
  b_{k+1} + b_k \pmod{2} & \text{if } k = 1, 2, \ldots, l - 1 
\end{cases}
\]

(8)

Inversely,

\[
b_k = \sum_{i=k}^{l} g_i \pmod{2} ; \quad k = 1, \ldots, l
\]

(9)

where \( l \) is a chromosome length of each variable.

It is said that the gray code is superior to the binary code, and this fact has been confirmed in our trial analyses.

4.3 Fitness assignment

The quality of an individual is represented by a scalar value, the so-called fitness, which is calculated by a fitness function. In the general GA, the fitness is maximized during the genetic iteration. The present GA inversely searches an optimal solution which minimizes the fitness by means of the
selection operation mentioned later. Consequently, the objective function is treated as the fitness function. Taking the constraint \((\lambda > 0)\) into account, the present fitness function, \(F\), can be defined as follows.

\[
F(u) = \begin{cases} 
\lambda(u); & \text{if } \lambda(u) \geq 0 \\
M; & \text{if } \lambda(u) < 0
\end{cases}
\] (10)

where \(M\) is a constant large number.

4.4 Selection, crossover, and mutation

Among many proposals of selection operation of parental individuals, herein, a combination of the elitist preserving selection and the binary tournament selection is applied. By the elitist selection the elitist individual with the minimum fitness in each population is survived in the next generation. The binary tournament selection is carried out by both random selection of two individuals in the population and subsequent choice of the individual with smaller fitness between the two. This operation is repeated \(N\) times, where \(N\) is the population size.

As the crossover operation the one point crossover with a certain crossover probability is applied. The present crossover is accomplished by selecting a pair of parents, selecting a cut-point at random, and simply swapping the strings between the chosen cut-point among the two parents. The crossover probability is defined as the ratio of the number of individuals generated in each generation to the population size.

The mutation is a changing operation of bits in strings, which consists of a switching of a 0 to a 1 or vice versa. This operation is carried out with low probability, the so-called probability of mutation.

5 Numerical example and discussion

An experimental project of special interest has been completed in the laboratory of the University of Pavia \([12]\). A two-storied full-scale prototype as shown in Figure 2 has been tested under a cyclic loading applied at each floor level. To make sure of the effectiveness of the present equivalent shear panel model, this experimental test of masonry wall in the laboratory of the University of Pavia is analyzed.

The floors are made of a set of isolated steel I-beams. Both vertical and horizontal loads are applied in the plane of the wall through these floor beams.

The analytical model of the present masonry wall consists of both 6 rigid panels and 10 equivalent shear panel elements. Rocking and shear strengths of the equivalent shear panels are given by eqns (1) and (2).

Using the present shear panel model, the number of variables are \(3 \times 6 = 18\). The minimum failure load factor, \(\lambda = 1.03\), can be obtained by
thickness: \( t = 25 \text{ cm} \)
weight per unit vol.: \( \gamma = 17 \text{ kN/m}^3 \)
vertical loads
  - 1st fl.: \( P_1 = 124.2 \text{ kN} \)
  - 2nd fl.: \( P_2 = 118.4 \text{ kN} \)
horizontal loads
  - \( H_1 = H_2 = 75 \text{ kN} \)
comp. strength: \( f_u = 7.9 \text{ MPa} \)
tensile strength: \( f_c = c/\mu \)
  - \( c = 0.14 \text{ MPa}, \quad \mu = 0.55 \)

Figure 2: Prototype of masonry wall with openings.

(a) Analytical result. (b) Experimental result.

Figure 3: Comparison of failure pattern.

the present GA with the chromosome length \( l = 180 \), the population size \( N = 20 \), the maximum number of generations \( N_g = 1000 \), the probability of crossover \( p_c = 1.0 \), and the probability of mutation \( p_m = 0.01 \). In the collapse mode with minimum load factor, internal and external walls at the first floor fail in shear as shown in Figure 3(a).

Figure 3(b) shows the crack pattern at the maximum drift level of 0.43% derived from the experimental results. The internal and external walls at the first floor fail in shear, while external walls at the second floor remain undamaged. Comparison of those results suggests that the collapse modes can well be simulated by the limit analysis with the present shear panel model.
6 Concluding remarks

The present study deals with the limit analysis of unreinforced masonry walls with rectangular openings subject to both vertical and horizontal loadings by means of the equivalent shear panel model approached by the GA. The subsequent concluding remarks can be obtained:

1. From some experimental results on failure of rectangular walls with openings subject to both vertical and horizontal loadings, typical failure modes can be described by the equivalent shear panel model whose structural parameters defined by the experimental results.

2. The limit analysis of the equivalent shear panel model is formulated as an optimality problem, that is, searching problem for the minimum load factor. Consequently, the GA is applied to the present limit analysis.

3. A numerical example of a single masonry wall with 4 rectangular openings is analyzed to illustrate the application of both the present shear panel model and the GA. Comparison of the analytical and experimental results suggests that the collapse modes can well be simulated by the limit analysis with the present shear panel model.

4. Unfortunately, no guarantee exists of converge to the rigorous optimal solution by the present GA. However, it seems powerful at identifying regions where the optimal solution exists, or selecting candidate solutions.

References


