Structural behaviour of Gothic vaults

J. P. Valcárcel[1], J. Domínguez[1], E Martín[1] & F. Escrig[2]
[1] *Escuela Técnica Superior de Arquitectura La Coruña, Spain*
[2] *Escuela Técnica Superior de Arquitectura Sevilla, Spain*

Abstract

For a long period of time Gothic vaults have been one of the most controversial elements of Gothic constructions. The rib vault system that supports the infill, and its indisputable efficacy has brought about numerous studies and hypotheses. Some, following Viollet-le Duc, consider the ribs as unique in effectiveness; others, sharing Pol Abraham's opinion, consider them to be totally useless. For several years our team has been working on systems of modelling and calculus of historic buildings, which are generally of stone in Spain. In this research we have paid much attention to the structures of great Gothic buildings of the northeast of the peninsula and especially to the modelling of their vaults. Due to the research carried out we have been able to produce applications and programmes that could be used in the general study of Gothic vaults. This paper gives analyses of diverse Gothic vaults with different modellings and most relevant conclusions of their structural behaviour.

1. Introduction

The history of Gothic vaults still presents numerous doubts and provokes discussions among the historians. But the subject of our research is a different one: it treats neither the constructive nor the historical analysis of the evolution but the analysis of the structural efficacy of the different types of vaults used during the Gothic period, relatively independent of their historical context.

In a first fundamental classification we can distinguish two types of very different Gothic vaults: cross vaults and fan vaults. In their turn the cross vaults can be:

-six-part vaults (divided into six parts by three hooped diagonal ribs)
-four-part vaults (divided into four parts by two hooped diagonal ribs)
-ribbed vaults

These will be the basic types of our research, to which a special one coming from of Romanesque period will be added and which will be interesting as a preceding type: the vaults of La Trinite of Caen which we shall call *diaphragm vaults* as there is no a more exact definition.

2. Difficulties in the analysis of the gothic vault

The structural analysis of the Gothic vault presents some serious inconveniences that should be taken into consideration.

- Generally the vault thickness is unknown. Sometimes, due to partial destruction or drillings made in the vaults, it is possible to carry out precise measurements, but in general it is necessary to consider the indirect measurements. In some cases it was possible to find drillings carried out during the construction itself or later, but in the Middle Ages, permitting the movement of the stage machinery. In the Middle Ages theatrical performances were frequent in the cathedral itself, like the “Los Misterios” that sometimes required complicated staging, as it happens in the “El Misterio de Elche” which is still being performed. When it is not possible to determine the vault thickness and we turn to the indirect measurements, the error in the estimation of vault strains is of little importance but the error in the transmitted thrusts and loads is of the same magnitude as the error of the thickness estimation.
- Moreover, there is not any pattern and the measured thickness can be of 12 cm like that of Leon or of 60 cm. like that of Reims and they usually vary in the same vault. In Leon Cathedral, for instance, thickness varying between 12 and 20 cm. have been measured.
- The used construction system, according to the evidence of the medieval architects themselves, was based upon a criterion of proportion and not of measurement[1]. Because of that, even conceptually and proportionally similar cathedrals are quite different in size. For example Leon Cathedral has a design, which is rather similar to that of Reims, but its size is smaller by one third. For that reason any comparative analysis among different vault typologies should take these factors into consideration. The thickness and the size must be homogenized to be able to compare the results.
- An essential aspect for analysing the behaviour of a vault is the counteraction system. It is evident that vault supporting flying buttresses reduce the horizontal movements of the vaults but their efficacy is limited. Our studies have revealed the importance of such factors as the height of the pillars, the height and the weight of the side naves, the foundation effectiveness and the support point of vault supporting flying buttresses. Hence, if we want to obtain a homogeneous analysis of vaults, it is necessary to eliminate these factors.

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[1] See for example the discussion on Milan Cathedral

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3. Comparative model of gothic vaults

To work out a homogeneous model of Gothic vaults we have chosen the Leon Cathedral as a reference. There are a lot of reasons for this election. From the structural point of view it is a middle-size cathedral, due to which the obtained results can be easily extrapolated. Throughout the history numerous very well-documented pathology problems have been presented, because of which the structural models can be verified by some real states of the cathedral. Besides, thanks to the collaboration of the architect D. Mariano Saez de Miera, at our disposal we have very exact data about vault dimensions, heights, and thickness. Finally, our team has carried out a complete measurement of the only reliable structural reference, i.e. the inclinations by which it is possible to adjust the calculation mode with certain precision. So taking the Leon Cathedral as a reference, we shall define a standard model with the following characteristics

-Span of the vault 12m

-Thickness of the vault 20cm

-Fixed supports

This model is always formed by ribs and infill sections. The purpose of this article is not to engage in the classical controversy between Viollet-le-Duc and Pol Abraham schools about the effectiveness or non-effectiveness of the ribs.

That is why, in any case, we have realized three calculations:

-Model with resistant ribs and infill sections as dead-weight (Viollet-le-Duc).

-Model with inefficient ribs and resistant infill sections (Pol Abraham)

-Model with resistant ribs and resistant infill sections.

The following vaults have been calculated:

-Trinité de Caen Abbey – cross vault with interior diaphragm (false six-part vault)

-Vaults of Bourges Cathedral (six-part vault)

-Vaults of Leon Cathedral (four-part vault)

-Vaults of St. Mary church of North Leigh (simple fan vault)

-Vaults of King's College Chapel of Cambridge (crossed fan vault)

From these ones, our team has obtained precise data about León Cathedral whereas other researchers' data have been used for the rest of the cathedrals.

The first result of interest refers to the geometry of the vault. When we started our research on Leon Cathedral and obtained the real data by a tachymetric survey, we have verified with exactitude that the vaults do not respond to simple geometric figures. It may be thought that cross vaults are obtained by the intersection of cylindrical surfaces, but it is not so for two reasons:

Firstly, because the directrix is curved and not straight and moreover the surfaces are of different radii, because of which their intersection would be a warped curve in the space For a Gothic builder such a possibility was inconceivable, above all for conceptual reasons, as when he tried to realize the ribs of the curved plan, warped in the space, he did it without problems. That is why constructively the most sensible solution was adopted. In a classical Gothic vault the diagonal ribs are circular and expand over a vertical diagonal plane (see figure). The solution to

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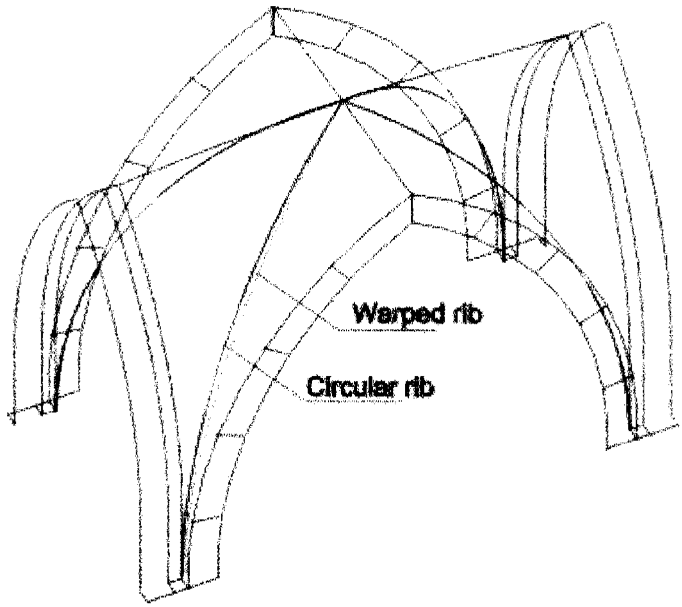
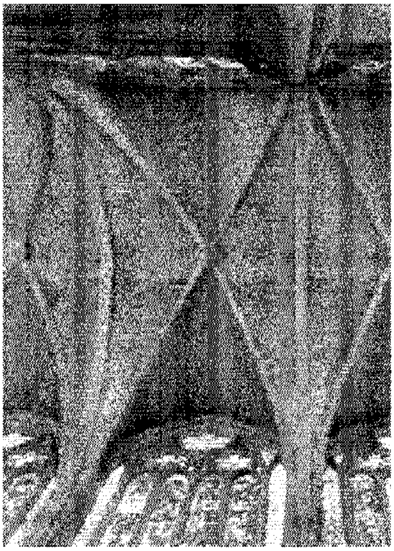
the construction of vousoirs and the design of the scaffolding is much simpler, but the infill sections need to be warped to be supported by the ribs. This is exactly what can be observed without difficulty in Leon and what we have been able to measure with precision (below figures).





The purpose of this research is the analysis of the structural behaviour of the Gothic vaults. That is why the realized analysis is elastic and lineal, since the

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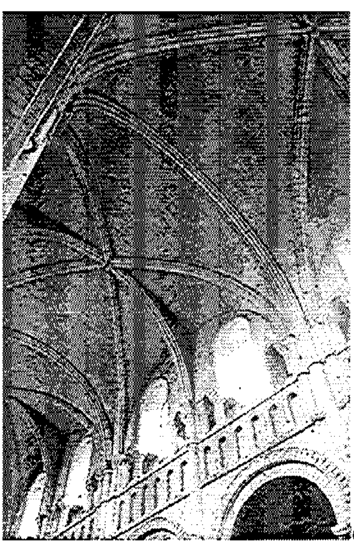
parameters that are looked for are the pressures and the deformations on the vault and especially the actions that are transmitted to the rest of the structure, such as the vertical loads and the horizontal thrusts. The vault pressures give us data about the zones presenting major interest and so that is where we must look for pathologies. And the data of loads and thrusts show us the damages that the vault can produce in the rest of the structure. As our purpose is to compare the results and as the geometry of each vault reproduces the real vault, although the vault span is homogenized, the data of loads and thrusts is presented for each support and for each metre of the vault width.

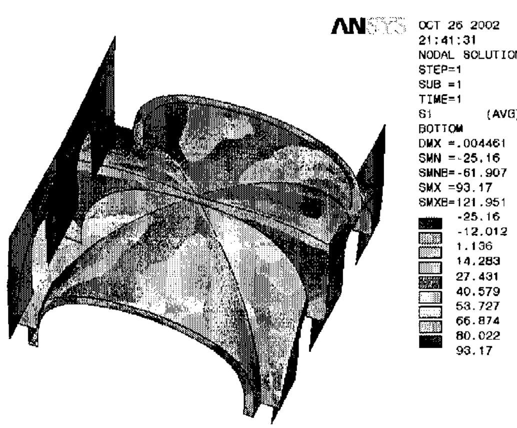
4. Particularized study of vaults

4.1. Trinité de Caen Abbey (France)

This vault is unique since it is a edge vault intersected by a diaphragm arch. In the below figures he real vault and the used calculus model can be observed.

The results show the little efficacy of the system. The diaphragm arch hardly modifies the vault stresses and thrustes that are very elevated. Besides the central support makes that the vault present strong stresses of traction on the top part. Surely cracks were produced in the vault and as a matter of fact the system does not seem to have been used later on.





Boveda de la Trinite de Caen: Bovedas falsa sexpartita.

4.2. Bourges Cathedral (France)

This vault is a typical example of a six-part vault, which is relatively common in the early Gothic buildings. In the below figures the real vault and the used calculus model can be observed.

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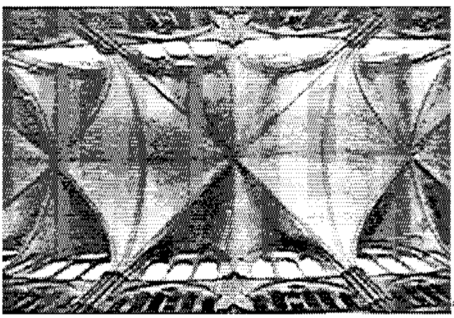
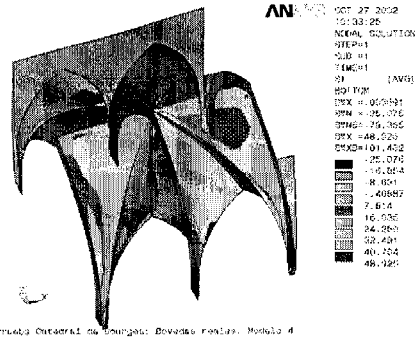
The results show that the vault has a highly resistant behaviour. However the stresses in the central supports are much inferior to the other ones. Many of the early six-part vaults leaned upon pillars of different cross sections what makes us think that the medieval architects intuited it. Despite its good behaviour, the six-part vault is difficult to build and this system was abandoned with the time.

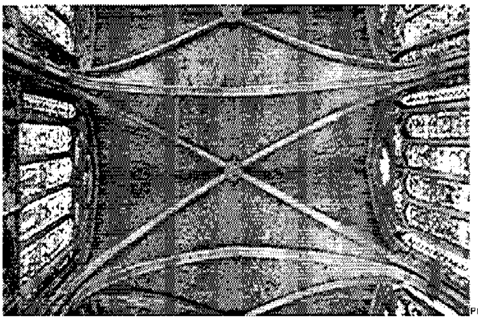


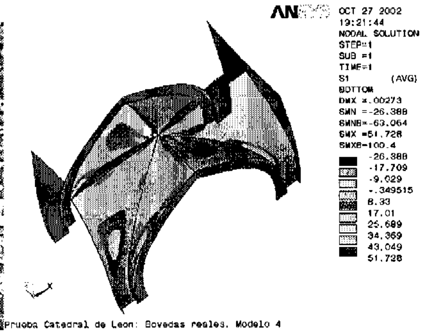


4.3. León Cathedral (Spain)

This vault is a typical example of a four-part vault, that is the much used type during the Gothic period. It corresponds to the classic period, later the vaults become complicated by the introduction of tiercerons and curved ribs, but their resistant desighn is practically the same. In the below figures the real vault and the used calculus model can be observed.

The results show that the vault has a highly resistant behaviour. The thrusts are relatively low and very concentrated and the stresses are reduced.





4.4. St Mary of North Leigh (England)

This vault is a typical example of the fan vault, that is very characteristic of the English Gothic architecture. In this case the fans are complete with incorporated ribs and allow wide plane soffits on the keystone. In the below figures the real vault and the used calculus model can be observed.

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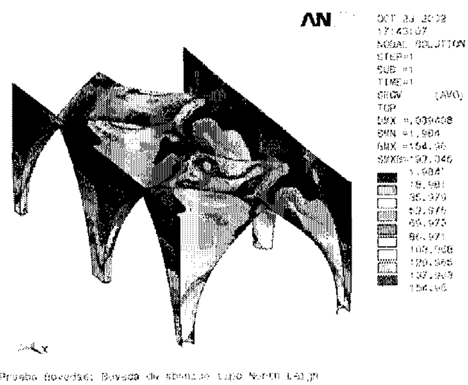
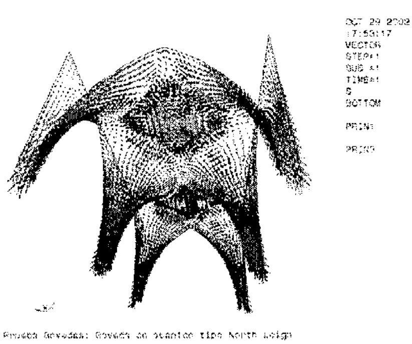
The results show that the behaviour of the vault is highly resistant in the fans and worse in the soffits. The thrusts are relatively low and very concentrated and the stresses are reduced, except for the soffits. This fact should be observed, because the early vaults like those of the cloister of Gloucester use this scheme, while later vaults unite the fans in order to reduce the size of the soffits.

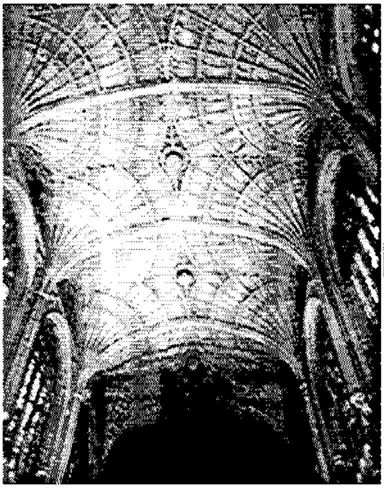


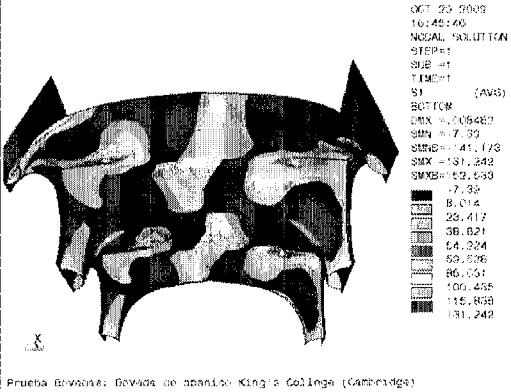


4.5. King's College Chapel in Cambridge (England)

This vault is a typical example of a later period fan vault with united fans, reduced soffits and jut out ribs. In this case the profile of the vault is very low and the arch is of four centers. In the below figures the real vault and the used calculus model can be observed.

The results show that the vault has a highly resistant behaviour in the fans and in the soffits. However the thrusts are rather strong and very concentrated, surely because of the little rise of the vault.





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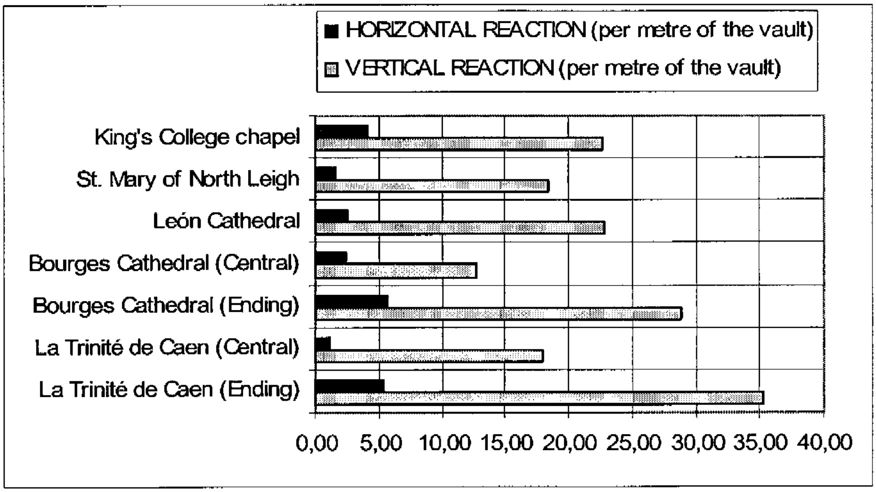
4.6. Comparative analyses of vaults

To be able to compare the structural behaviours of these vaults different finite element models have been realized. Homogenizing the obtained results it is possible to summarize them in the following table:

	σ_{eq} (max) N/mm^2	σ_1 (max) N/mm^2	τ_{xy} (max) N/mm^2	δ_{vert} (max) cm	δ_{horiz} (max) cm
Roman. vault	3.23	1.35	0.57	1.45	0.89
Trinité Caen	0.88	0.93	0.22	0.44	0.24
Bourges Cath.	0.84	0.49	0.19	0.37	0.21
León Cathedral	0.91	0.52	0.36	0.27	0.14
North Leigh	1.21	0.89	0.34	0.59	0.59
King's College	1.55	1.31	0.41	0.84	0.38

The reactions of the vaults have also been studied in detail, considering the fact that they are essential for determining the behaviour of the support structures. In this case the loads and thrusts have been determined per metre of the vault longitude, since having different separations of pillars, the results were not comparable. The loads and thrusts can be schematized in the following diagram.



5. Conclusions

The carried out study allows to prove that the solutions of the Gothic vaults correspond to architects of great wisdom and constructive sense, although having little technical knowledge. In all the cases the strains and stresses on the vaults are acceptable and in the points where the concentrations of stresses are predictible the resistant section is greater. For determining these results the numeric calculus turns out to be a simple verification corroborating the intuitively forseen data.

The excellent structural behaviour of the fan vaults is especially remarkable. The concentrations of stress are only produced in the soffits and in the supports. This fact should have been quickly intuited by the architects, since the supports were solidified by load plugs and the soffits were progressively reduced, thus seriously complicating the design of the ribs.

The thrust of the Gothic vaults is not less than that of the Romanesque ones. In fact the only advantage, although the fundamental one, is that the thrusts are concentrated in points that permits to balance them with flying buttresses. Moreover, our studies indicate the importance of the weight of the wall over the vaults for reducing the inclination of the thrust.

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