Creep modelling of masonry historic towers

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Abstract

This work illustrates a theoretical model developed to reproduce the behaviour of ancient masonry subjected to sustained stresses. Starting from a model recently proposed by the authors, two damage tensors have been introduced into a rheological model: the components of these tensors change both according to the intensity of the applied stress and, in case of sustained stress, to the duration of the load history. Evolution laws found in the literature for brittle materials have been employed. The principal directions of damage are meant to represent the directions of the experimental cracks; accordingly, when any damage direction is activated, it remains unchanged throughout the subsequent load history. The presence of second-order damage tensors makes it possible to describe the damage-induced anisotropy of the microcracked material. Also, since the possible increase of damage in time is accounted for, the model is able to describe creep failure and to predict the creep time to failure of the material under given stresses. The model parameters can be obtained through uniaxial creep tests on masonry samples at increasing stress levels and up to failure. The model was implemented into a finite element code, and structural analyses were carried out to assess the safety of middle-age masonry towers. The results obtained for one of these towers are briefly described and discussed.

1 Introduction

The action of sustained stresses in structures made of brittle materials can be particularly dangerous for their life-time. This statement can be true not only if the stresses are of very high intensity, but also if they are of a level relatively lower than the short-term strength of the material. Indeed, random changes in the stress intensity, even if exerted upon the structure for a short time, can act as "accelerators" for the evolution of damage throughout the structure: this was
pointed out, e.g., in an experimental program carried out on samples taken from the ruins of the Civic Tower of Pavia (Italy), a masonry building dating back to XII century, which collapsed in March 1989 [1]. The collapse of this Tower can be attributed to the creep strains, induced by the heavy persistent compressive stresses associated with the self-weight of the tower. Other middle-age buildings that show serious evidence of damage are the belltowers of the Cathedrals of Monza and Cremona, near Milan. For these reasons, an extensive experimental investigation on the behaviour of brittle materials subjected to sustained stresses of high intensity has been carried out at the Department of Structural Engineering of Milan Technical University (Politecnico). Other sources of damage, e.g., physico-chemical effects, were not accounted for in the experimental program, that aimed at investigating only damage of mechanical origin. This program included monotonic and creep tests on masonry samples taken from the ruins of the Tower of Pavia, as well as a number of experiments on masonry specimen taken from the crypt of the Monza Cathedral [2].

The description of the experimental results requires a mathematical model to be available, capable at least of accounting for the material damage, the damage-induced creep acceleration and, possibly, the change in the elastic and failure domains associated with creep damage. A rheological model with these characteristics was proposed by the authors in [3]; this model, however, is not suitable to the description of tests including non-proportional load histories. Accordingly, here the authors propose to describe the creep behaviour of brittle materials by modifying the previous model, assuming the principal damage directions to remain fixed along any strain and stress evolution. The features of the model are described in Section 2, whereas Section 3 is devoted to the implementation of the model into a finite element code. In Section 4 a procedure to identify the parameters that define the damage model is outlined, with particular emphasis to creep tests on masonry specimens taken from Monza Cathedral. Finally, in Section 5, the capability of the model to capture the salient features of the creep behaviour of a masonry tower to failure is illustrated.

2 The viscoelastic model with damage

To reproduce the structural response of masonry subjected to persistent loads, the authors introduced into a classical Burger’s rheological model two damage variables. In this way it is possible to describe the decay in the mechanical properties induced by increasing stresses, as well as creep failure.

The essential features of the model are described in Fig. 1 in its uniaxial version. It consists of a Kelvin element in series with a Maxwell element. Both elements are composed by a spring and a dashpot: in the first one, the two components are in parallel, whereas the Maxwell element has the two components in series. The stiffness constants and the relaxation times of the springs and the dashpots are denoted by $E^K, E^M$ and $\tau^K, \tau^M$, where $K$ and $M$ are for Kelvin and Maxwell, respectively. The Kelvin element permits to describe the primary creep of the material; the spring of the Maxwell element accounts for the instantaneous (elastic) response of the material, whereas the Maxwell
A dashpot simulates the steady-state (secondary) creep. A "frictional" (or Bingham) element is inserted between the spring and the dashpot of the Maxwell element in order to avoid the activation of the secondary creep at low stress levels (say, lower than $\sigma_0$ in uniaxial conditions).

![Rheological Burger's model diagram](image)

**Figure 1:** Modified rheological Burger's model.

The classical Burger's model cannot describe neither the loss in stiffness of the material under increasing stress, nor the tertiary stage that precedes creep failure. Accordingly, a static damage variable $D^s$ and a viscous damage variable $D^v$, which reduce the elastic modulus of the Maxwell spring and the relaxation time of the Maxwell dashpot, respectively, are introduced in the model. It is because of the presence of the latter damage variable that tertiary creep and creep failure can be captured. The damage variables grow only after that a suitable threshold has been reached and their value ranges between 0 (virgin state) and 1 (complete failure). The two damage variables evolve differently, according to the conjugate static and viscous "damage forces" $Y^M$ and $Y^S$ (see [3] for details). The first one is associated with the elastic strain in the Maxwell's spring, whereas the second one is supposed to depend on the irreversible strain accumulated in the dashpot of the Maxwell element.

The model was generalized to the three-dimensional case, by introducing two second-order symmetric damage tensors, $D^s$ and $D^v$. Each principal direction of these two tensors is assumed to be associated with the normal to a plane microcrack that forms and grows at any point in the solid. Once the microcrack has formed, its normal is supposed to be fixed along any subsequent load history. This is physically reasonable, considering that the orientation of any microcrack in a brittle solid is likely to remain unchanged, without rotating. In the model previously developed by the authors the principal directions of the damage tensors were supposed to coincide with the principal strain directions. With this assumption, the previous model could not correctly reproduce the behaviour of brittle materials subjected to non-proportional loading, and, in general, to load histories involving an important rotation of the local stresses.
The evolution laws of the components of the two damage tensors were obtained by modifying laws proposed by other authors for different brittle materials. The principal values of the static damage tensor can increase according to an evolution law similar to that proposed by La Borderie et al. for concrete [4]:

$$\dot{D}_h^S = \frac{a_H b_H}{1 + a_H (Y_{hh}^M - Y_{O0H}) / E^M} \left( Y_{hh}^M / E^M \right)^{b_H - 1} \left( Y_{hh}^M / E^M \right)^{b_H} (h = 1,2,3)$$ (1)

where $Y_{hh}^M$ are the direct components of the damage force tensor

$$Y_{hh}^M = \frac{1}{2} E^M \varepsilon^{el} \cdot \varepsilon^{el}$$ (2)

conjugate to $D^S$, along any principal direction of damage; $\varepsilon^{el}$ is the elastic strain tensor. Macauley brackets are used to make static damage vanish if any damage force does not exceed the threshold value $Y_{O0H}$. $a_H$ and $b_H$ are model parameters, that assume different values according to the sign of the relevant principal strain ($H = C$ for compressive strains; $H = T$ for tensile strains).

The viscous damage variables evolve according to a law originally formulated by Chan et al. for rocksalt [5]:

$$D_h^V = \left( \frac{Y_{hh}^B}{E^M} \right)^{A_{2H}} A_{1H} D_h^V \left( \log \frac{1}{D_h^V} \right)^{A_{3H}} (h = 1,2,3)$$ (3)

where $Y_{hh}^B$ are the direct components of the damage force tensor

$$Y_{hh}^B = \frac{1}{2} E^M \varepsilon^{B} \cdot \varepsilon^{B}$$ (4)

conjugate to $D^V$, along any principal direction of damage; $\varepsilon^{B}$ is the irreversible creep strain tensor. $A_{2H}$ and $A_{3H}$ are nondimensional material constants, whereas $A_{1H}$ is dimensionally the inverse of time. Also these parameters take different values in presence of compressive ($H=C$) or tensile ($H=T$) strains. Both static and viscous damage component depend only on the relative damage force which is function of the corresponding strain component.

Note that the undamaged (virgin) material is assumed to behave isotropically, but the model could be quite easily generalized to materials which are anisotropic in the initial state (such as brick masonry).
This new version of the model is based on two assumptions:
- when damage activates at any Gauss point, one of the principal direction of damage is defined and remains unchanged, whatever the subsequent strain and stress evolution be;
- the local orthotropy axes of the damaged material are the principal directions of the damage tensors.

Before damage activates, the principal directions of strain and stress are coincident because of the assumed isotropy of the virgin material, and so are the principal directions of damage at the time of first appearance of damage at any point. Nevertheless, the situation changes after that one or more directions of damage are formed. If damage is represented by 2nd order symmetric tensors, the principal directions of damage are mutually perpendicular and the damage-induced local material anisotropy is, in the more general case, orthotropy. If only one damage direction \( \mathbf{n}_1 \) is activated, the other two directions will have to activate in the plane perpendicular to \( \mathbf{n}_1 \). The authors’ proposal is that, at any point where a microcrack has formed (normally to \( \mathbf{n}_1 \)), an additional orthogonal microcrack can form normally to the direction \( (\mathbf{n}_1 I) \) in which the component \( Y_{11} = \mathbf{n}_1 (Y \mathbf{n}_1) \) (with \( \mathbf{n}_1 \) perpendicular to \( \mathbf{n}_1 I \)) of the damage force tensor takes its greatest value. \( Y \) is either the static \( (Y^S) \) or the viscous \( (Y^B) \) damage force tensor. This direction can be identified e.g. by means of the Mohr’s circle applied to the projection of \( Y \) in the plane orthogonal to \( \mathbf{n}_1 \). When a second damage direction has been determined the third one \( \mathbf{n}_{III} \) can be deduced through a simple vectorial product.

3 Implementation of the model

In the subroutine developed to implement the model into a finite element code, two different reference frames have been used: the Cartesian reference system, which is the same for all the Gauss points, and a local reference system, which is associated to the three principal damage direction at any Gauss point.

The Cartesian reference system is used as fixed basis throughout the entire analysis. The component of the inelastic Kelvin strains, the elastic strains and the stresses are calculated in this system. On the contrary, the component of the inelastic Maxwell strains, the stiffness matrix and the principal values of the damage tensors are calculated in the local reference system related to damage. The variables are eventually reprojected from one reference system to the other one using suitable rotation matrices which contain the direction cosines of the principal strain and damage directions with respect to the Cartesian reference system.

Suppose that the loading history is divided into several time steps; suppose that at time \( t_0 \) all variables at each Gauss point are known from the preceding computations, i.e. \( \mathbf{\sigma}_{ij}, \mathbf{e}_{ij}^{el}, \mathbf{e}_{ij}^{in}, \mathbf{D}_{ij}^{S}, \mathbf{D}_{ij}^{V}, \mathbf{u}_{ij} \). Vector \( \mathbf{u}_{ij} \) collects all the free (unconstrained) nodal displacements of the element aggregate. The finite increments \( \Delta\mathbf{\sigma}_{ij}, \Delta\mathbf{e}_{ij}^{el}, \Delta\mathbf{e}_{ij}^{in}, \Delta\mathbf{D}_{ij}^{S}, \Delta\mathbf{D}_{ij}^{V}, \Delta\mathbf{u}_{ij} \) are to be determined as the response to given increments \( \Delta F_{ij} \) of the equivalent nodal loads conjugate to \( \Delta\mathbf{u}_{ij} \) during any time step \( \Delta t_{ij} \). The solution of this finite-step problem, leading
to the new state at instant $t_{i+1} = t_i + \Delta t_i$ can be achieved by the sequence of steps outlined below:

a) The nodal displacement $u$ and the relevant total strains $\varepsilon$ at any Gauss point are computed as if the response to the current load vector were fully elastic ("prediction").

b) The principal strains and their directions are calculated;

c) The stresses are computed and projected onto the damage reference system;

d) The viscoelastic strain increments corresponding to the Kelvin element in the model of Fig. 1 are calculated in the global Cartesian system;

e) The irreversible creep strain increments corresponding to the dashpot of the Maxwell-Bingham element are computed in the damage reference system and then projected onto the Cartesian one;

f) The elastic strain increments (corresponding to the spring of the Maxwell-Bingham element in the model of Fig. 1) are computed in the global Cartesian basis;

g) The total elastic strains and the inelastic strains accumulated in the Maxwell-Bingham element are calculated in the global Cartesian basis;

h) The damage forces are computed in the global Cartesian basis and then rotated (along with the elastic strains) in the local damage reference system;

i) The values of the damage variables activated in the previous time steps are updated and the activation of new damage direction is checked;

j) The elastic and inelastic strains are recalculated using the new damage values;

k) Starting from the compliance matrix computed in the damage reference system, the stiffness matrix is obtained;

l) This matrix is projected onto the global Cartesian basis and the stress components in this basis are computed.

4 Parameter identification

A correct application of the model described in the previous Section requires that the parameters are correctly determined. Considering that the damage laws involve different parameters in tension and compression, the model is completely defined by 17 parameters.

The elastic constant $E^M$ and $\nu$ and the threshold values $Y_{0T}$ and $Y_{0C}$ for the static damage forces are determined from the first linear part of the stress-strain plots (both in tension and compression) relative to a monotonic test. By best fitting the non-linear part of the stress-strain plot in uniaxial tension or compression, the model parameters $a_C$, $a_T$, $b_C$ and $b_T$ that define the static damage evolution laws can be derived. In particular, as reported in [4], $a_C$ and $a_T$ affect the strength value in compression and tension, respectively, whereas $b_C$ and $b_T$ are related to the value of the strain at the peak stress and to the shape of the softening branch.

On account of the goal for which the model has been conceived, particular attention must be paid to the identification of the model parameters that affect the creep behaviour of the material and the damage evolution laws. The
parameters $E^K$ and $\tau^K$ can be determined according to the duration of the primary creep phase, whereas $\tau^M$ is function of the secondary creep rate. To correctly identify these parameters, creep tests at low stress levels should be used, to avoid the presence of damage.

The parameters defining the viscous damage law ($A_{1C}$, $A_{2C}$, and $A_{3C}$) can be identified by best fitting the secondary and tertiary phases of creep tests performed at higher stress levels. $A_{2C}$ controls the viscous damage rate, whereas the other two affect the creep time failure of the material. Since it is very difficult to perform creep tests in tension, the parameters $A_{1T}$, $A_{2T}$, and $A_{3T}$ can be obtained by best fitting the stress-transversal strain plot relative to uniaxial creep test in compression.

For sake of illustration, the scheme used to identify the parameters $E^M$, $\tau^M$, $Y_{0C}$, $Y_{0T}$, $A_{1C}$ and $A_{2C}$ for the masonry of the bell-tower of Monza Cathedral is presented. Since true creep tests at constant stress are extremely time-demanding, since they keep testing machines busy for an unpredictable duration, the model parameters were identified using pseudo-creep tests. All the (prismatic) specimens were subjected to a uniaxial stress applied in subsequent increments of 0.25 MPa at a rate of $1.25 \times 10^{-3}$ MPa/sec. The stress is then kept constant for one hour and half, before the following stress increment is applied. Table 1 reports the specimen sizes and the main results of three of the tests performed.

Table 1. Characteristics of the experimental tests

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For each test, through a linear interpolation of the first phase of the stress-axial strain plots, the value of $E^M$ can be identified as the average slope of the plots. From the same plots, the threshold values of the damage forces can be also obtained as

$$Y_{0C} = \frac{1}{2} \cdot \varepsilon_0^2 \cdot E^M$$

where $\varepsilon_0$ is the average axial strain at the end of the (nearly) linear part of the curves. Since the ratio between the strain at the elasticity limit in tension and compression is about 1/10, the threshold value $Y_{0T}$ can be assumed equal to 1/100 of $Y_{0C}$.

Starting from the time evolution of the axial strain under a constant stress $\sigma_0$, it is possible to obtain the relaxation time $\tau^M$ of Maxwell dashpot. During the
steady-state creep phase, the inelastic strain rate $\dot{\varepsilon}^{in}$ is nearly constant and corresponds to the strain rate in the Maxwell dashpot of the rheological model in Fig. 1. Indeed, at intermediate stress levels, the experimental results show that $\dot{\varepsilon}^{in}$ increases with $\sigma_0$ with a nearly linear law. This fact seems to confirm that the use of a Maxwell element to describe the secondary creep is correct.

The parameters of the viscous damage law can be evaluated by computing the decrease in elastic modulus during the loading phases from one step at constant level to the subsequent one. The associate increase in damage, divided by the time spent between the two reloading phases, gives an estimate of the average damage rate which can be used to identify $A_{1G}$ and $A_{2C}$.

The comparison between the experimental curves and the model simulation (with some of the parameters identified according to the procedure outlined above and using for the remaining parameters the values obtained in previous studies [6]) is shown in Fig. 2a,b.

A similar work is actually in progress on other experimental tests carried out on specimens of Monza Cathedral, subjected to a stress constant for three hours.

Figure 2: Experimental (thin lines) and numerical (thick lines) results on three masonry specimens from Monza Cathedral: a) axial strain vs. time; b) transversal strain vs. time.
5 Numerical analyses on masonry tower

The theoretical model implemented into a finite element code was used to simulate the time behaviour of some massive masonry structures. The first analysis concerns the civic Tower in Pavia. The lowest part of the tower (22 m high) was discretized using three-dimensional finite element of different shapes. The applied loads are the dead weight of the discretized part and the weight of the upper part of the tower, including the belfry. They are supposed to grow "quickly" (in about ten year) up to their final value and then to remain constant. The model parameters were determined on the basis of the procedure outlined in the previous Section and according to the results of experimental tests on samples taken from the tower ruins (see [3]). In particular, it was found that $\tau^A=80$ years and $\tau^M=4000$ years.

Fig. 3 shows the evolution in time of the vertical displacement of a corner at the top of the finite element model. It is important to note that the failure is reached in about 400 years: this value matches the experimental collapse time reasonably well. The evolution of viscous damage is represented in the Fig. 4a,b,c,d: it is important to note that damage evolves initially slowly and then grows quickly after 250 years. Moreover, the static damage is not significant throughout the entire analysis.

Numerical analyses on the belltower of Monza Cathedral are in progress: the preliminary results seems to be encouraging, since viscous damage is mainly localized in the zones where significant cracks are present in the tower.

![Figure 3: F.E. analysis of Pavia tower: vertical displacement of a node at the top of the model vs. time](image)

References


Figure 4: F.E. analysis of Pavia Tower: contour plots of the trace of the viscous damage tensor at different times: a) 100 years; b) 210 years; c) 310 years; d) numerical collapse (418 years).