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The concept of proportion in heritage architecture: a study of form, order and harmony

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Abstract

This is an ongoing investigation conducted by the author to examine how different concepts of proportion were employed by Western civilization to create "form, order, and harmony" in architectural heritage structures. The study covers the period from Antiquity to the Middle Ages and up to the Renaissance. This is indeed an enormous undertaking in view of the fact that there were so many different systems of proportion employed by such diverse civilizations that makes it impossible, if not futile to attempt to discuss the "history" of proportions in the present article. In view of these practical restrictions, this paper discusses how a particular system of proportion was applied to extant monuments constructed by the ancient civilizations of Egypt and Greece. Future articles will consider Rome, the Middle Ages and the Renaissance and their views towards the concept of proportion in its varied forms.

1. Introduction

Since ancient times, the practice of architecture and building construction was based primarily on craft tradition and starting from this tradition architectural styles and forms were created by employing systems of proportion. These architectural styles and forms, developed over the centuries with hardly any or no application of analytical structural mechanics. Higher civilizations believed in an order based on numbers and relations of numbers, and they sought and established a harmony, often a mystical one, between universal and cosmic concepts and the life of man. As long as monumental art and architecture were devoted to religious, ritual, cosmological and magical purposes they had to be expressive of this order and harmony by means of proportions. Thus,

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architectural forms were created by applying different systems of proportion to the design of historic constructions. A design system was created that established the structural stability, aesthetics of order and harmony, and topology of the monument. This development brought about design concepts and methods of construction employed in architectural heritage. The ratio or proportion commonly known as "The Golden Section" appears to be the principal invariant among the various systems of proportion employed by the builders of heritage architecture.

The present work discusses in a condensed form, how the Golden Section was employed by the Egyptians in the Great Pyramid of Khufu (Cheops) and used by the Greeks in the Parthenon.

2. Concept of proportion

The general concept of proportion defines a combination or relation between two or several ratios. The notion of proportion then, follows immediately that of the ratio. A ratio is the quantitative comparison between two aggregates belonging to the same kind or species. If we are dealing with segments 'A' and 'B' of a straight line, the ratio between these two segments will be symbolized by A/B measured with the same unit. This ratio A/B has all the properties of a fraction. As declared by Euclid in his Elements: "Proportion is the equality of two ratios".

Systems of proportion [1] can be classified according to a *practical method* based on the repetition of geometric shapes or by the type of *mathematical* relationships employed to obtain the dimensions of the shapes. A brief discussion follows.

2.1 Systems of proportion

The systems of proportion which evolved over time can be classified in two general categories, either by the *practical method* which is used to put them into effect, or by the type of *mathematical* relationships which they embody.

According to the *practical method* these systems consists of geometrical systems (based on "shapes") which aim directly at the repetition of similar shapes; patterns of proportional relationships developed among the dimensions. The *mathematical method* divides proportions into systems using dimensions (based on "mathematical" relationships) which are commensurable and which are related by geometric progressions based on whole numbers (rational numbers); and systems using dimensions which are often incommensurable, and which are related by geometric progressions based on other numbers (irrational numbers).

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As indicated earlier, it is not our purpose here to study the history of proportion but to discuss how a particular system of proportion was employed in heritage architecture.

In general, the most important systems of proportions were based on geometric, arithmetic and harmonic relationships.

2.1.1 Geometrical proportion

The name of the geometrical proportion (A/B = C/D) was in Greek and in Vitruvius, *analogia*; harmoniously ordered or rhythmically repeated proportions or analogies introduced "Symmetry" and analogical recurrences in all consciously composed plans. As defined by Vitruvius: "Symmetry" resides in the correlation by measurement between the various elements of the plan, and between each of these elements and the whole...as in the human body...it proceeds from proportion - the proportion which the Greeks called *analogia* – it achieves consonance between every part and the whole. This symmetry is regulated by the modulus, the standard of common measure of the work considered, which the Greeks called the Number. When every important part of the building is thus conveniently set in proportion by the right correlation between height and width, between width and depth, and when all these parts have also their place in the total symmetry of the building, we obtain eurhythmy."

It is the modulus (the common measure) or the Number to the Greeks, that regulates the symmetry of design.

If we have established two ratios A/B and C/D between comparable quantities, then the equality A/B=C/D (A is to B as C is to D) is said to be a proportion. The four magnitudes A, B, C, D are connected by the proportion. This is the geometrical proportion, called discontinuous in the general case when A, B, C and D are different. If two of these numbers are identical it is then called a continuous geometrical proportion, as for example when A/B = B/C; or $B^2 = AC$. Then B = AC, is called the proportional or geometrical mean between A and C.

2.1.2 Arithmetic proportion and harmonic proportion

These systems of proportion are based on arithmetic and harmonic relationships. In the arithmetic proportion the middle term overlaps the first term by a quantity equal to that by which it is itself overlapped by the last term, as in 1, 2, 3. In the harmonic proportion the middle term overlaps the first one by a fraction of the latter equal to the fraction of the last term by which the last term overlaps it, or 2, 3, 6 (here the fraction is ½) or 6, 8, 12 (here the fraction is 1/3).

All of these systems of proportions are a recurring theme throughout the history of architecture, but it is the geometrical proportion, discontinuous or continuous, which is generally used in architecture. In particular, the application of the

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Golden Section, to architectural heritage structures in order to create "form, order, and harmony" in the design of these monuments.

3. The Golden Section Φ

Starting as early as Egyptian times the design of the most important monuments of every great period or style of architecture were executed according to a very subtle and rational "dynamic symmetry" in which the special properties of the Golden Section were used to obtain the most flexible and varied "eurhythmy" [2].

The Golden Section is a proportion found in nature. Apparently, it was developed by the Ancient Greeks who applied it to designing their temples. It is described as *"the whole is to its larger section, what the larger section is to the smaller"*. It is also known as the Golden Mean, the Golden Ratio, or the Divine Proportion. As shown in Figure 1, it is a way to divide a line in such a way as to create an ideal relationship between the parts.

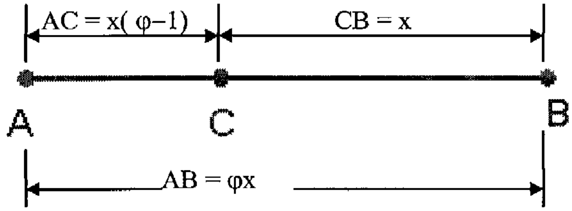

Figure 1. Golden Section of a line

From the figure, AB/CB = CB/AC or,

$$\frac{\varphi x}{x} = \frac{x}{x(\varphi - 1)}$$, that is:

$$\varphi^2 - \varphi - 1 = 0 \text{ (solving for } \varphi \text{ we obtain } \varphi = \frac{1+\sqrt{5}}{2} = 1.61803..) \qquad (1)$$

The qualities of the Golden Section φ, (given the name Phi probably after the Greek sculptor, Phidias) were central to the numerological philosophy of Plato and Pythagoras. The mathematician Filius Bonacci (Fibonacci) wrote a treatise on the number series related to the Golden Section.

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0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,.......

If we take the ratio of two successive numbers in Fibonacci's series, we find:
1/1=1; 2/1=2; 3/2=1·5; 5/3=1.666..; 8/5=1.6; 13/8=1.625;
21/13=1.61638..; 34/21=1.61904..;
144/89=1.61797..;....610/377=1.61803...;...1597/987=1.61803...
The ratio converges to a value very close to the Golden Section (φ) just as for the" line" above!

Throughout history Phi has been observed to evoke emotion or aesthetic feelings within us. The ancient Egyptians used it in the construction of the great pyramids and in the design of hieroglyphs found on tomb walls. The Greeks studied Phi closely through their mathematics and used it in their architecture.

The Parthenon at Athens is a classic example of the use of the Golden Rectangle. The Golden Rectangle, shown in Figure 2, is produced mathematically when the side DC of a square ABCD is bisected at X and an arc with radius XB is swung on to DC at F. The resultant rectangles AEFD and BEFC are both golden rectangles whose sides conform to the golden section, having the ratio 1: 1.618.

That is: DF/AD = BC/CF = 1.618.

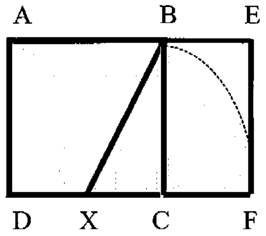

Figure 2. Golden Rectangle

The golden section has been utilized by numerous artists and scientists since (and probably before) the construction of the Great Pyramid of Khufu (c.2600 BC). As scholars and artists of past eras discovered (i.e.: Pythagoras, Plato, Leonardo Da Vinci), the intentional use of the golden section in art of various forms expands our sense of beauty, balance, and harmony to optimal effect.

3.1 The Golden Section Phi Φ in the ancient world

3.1.1 Egypt

Although pyramids may be found in other parts of the world, notably in Central America, it is this remarkable collection of pyramids, particularly the Great Pyramid of Khufu at Giza, Figure 3, which has most captured the world's

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imagination. According to historical sources [3], the ancient Greek historian Herodotus (c. 484 – 420 BC) was told by Egyptian temple priests that the Great Pyramid was constructed in such a way that the area of each of its four faces is equal to the square of its height. Following Herodotus argument and referring to Figure 4, let the height of the pyramid be $h=\sqrt{x}$ units, and the length of the base edges be 2 units. Then the square of the height is $h^2 = x$, and the slant height (or apothem) of the pyramid is x units, as this gives the area of the face to be x.

Thus: $$x^2 = 1^2 + [\sqrt{x}]^2 \qquad (2)$$

or, $$x^2 = 1 + x \qquad (3)$$

Therefore,

$$x^2 - x - 1 = 0 \text{ (compare with equation 1).} \qquad (4)$$

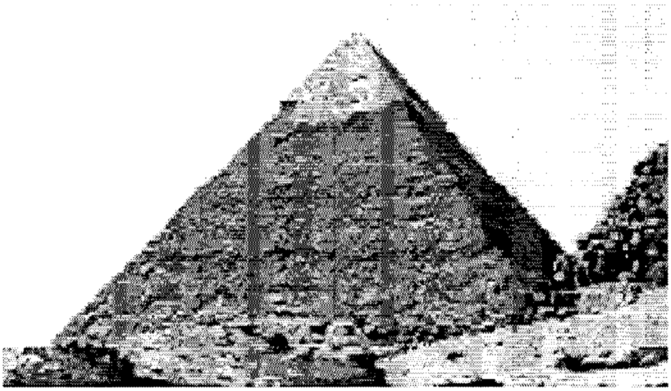

Figure 3. The Great Pyramid of Khufu, Egypt

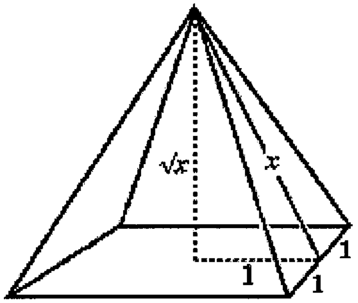

Figure 4. Egyptian Triangle

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This equation is the defining relation for 1.618033... the golden section. In other words, for a pyramid of a square base of length 2, the height of the pyramid is $\sqrt{\phi}$ and the slant height (apothem) is ϕ.

The original dimensions of the pyramid in royal cubits were: the base length 440 (231m), height 280 (147 m), and apothem 356 (186.9m), where 1 royal cubit = 0.525 meters, the 'standard' length of a forearm from elbow to finger. These dimensions are a consequence of the proportions 1, $\sqrt{\phi}$,ϕ (respectively: half the side of a square base, the height, and the apothem).

3.1.2 Greece

The Parthenon was built during the 5th century B.C. in the 'Golden Age of Pericles' when Athens was at the height of its glory. It was a temple dedicated to the honor of Athena Parthenes, the patroness of Athens, whose highly ornate 12-meter high statue was erected in an inner chamber. The building was a proportioned sculpture constructed to house the goddess, rather than a place of worship as one normally thinks a temple to be. The architects Iktinos and Kallikrates worked under the chief designer, the sculptor Phidias.

The Parthenon is an interesting example of a mathematical approach to art. Once its ruined triangular pediment is restored, the ancient temple fits almost precisely into a golden rectangle, Figure 5. Further classic subdivisions of the rectangle align perfectly with major architectural features of the structure.

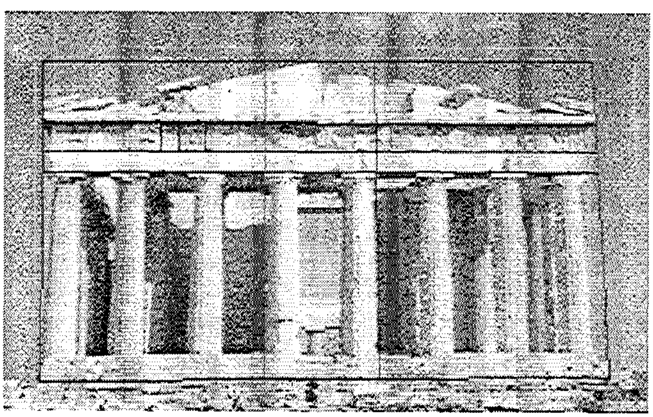

Figure 5. The Parthenon, Athens.

As shown in Figure 6, the facade of the Parthenon apparently was designed around the proportions of two large and four small Golden Section, or $\sqrt{5}$, rectangles (refer to Figures 6a, 6b), placed above four squares. These proportions are in agreement with the Golden Rectangle, shown in Figure 2.

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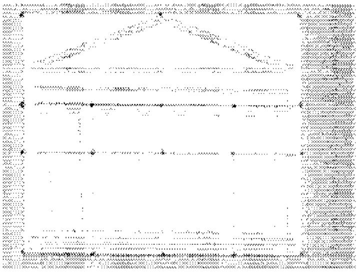
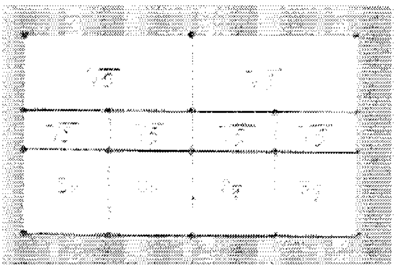
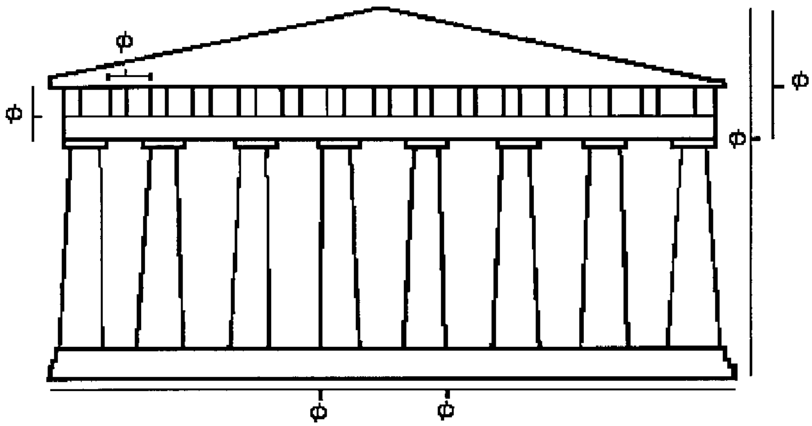


(a)



(b)



(c)

Figure 6. The Parthenon and the Golden section

4. Conclusion

Since the ancient world, different concepts of proportion were employed by Western civilization to create "form, order, and harmony" in architectural heritage structures. The ratio or proportion commonly known as "The Golden Section" appears to be the principal invariant among the various systems of proportion employed by the builders of heritage architecture.

As we have seen, important examples of the use of the Golden Section in ancient architecture are the Great Pyramid of Khufu at Giza, Egypt and the Greek Parthenon in Athens.

The Golden Section is clearly a pervasive topic in heritage architecture. Through its fundamental relationship with the Fibonacci sequence, found throughout nature and art, the Golden Section may very well be a definitive characteristic of architectural design and an element of aesthetic quality.

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It is possible that the aesthetic properties associated with the Golden Section were appreciated in ancient times, although its mathematical properties were not necessarily fully understood.

Nonetheless, and all the more beautifully, the mathematical relations in the form of the Golden Section proportion are there in the Great Pyramid of Khufu and in the Parthenon and they are unquestionably true.

Our fascination with the golden section and all it represents, as it have been in the past, will continue to be of interest for mathematical speculation and design considerations in heritage architecture.

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