Creep failure of ancient masonry: experimental investigation and numerical modeling

E. Papa, A. Taliercio & L. Binda
Dept. of Structural Engineering, Politecnico of Milan, Italy

Abstract

The recent collapse of some ancient massive buildings has raised attention on the creep behaviour of masonry. An extensive experimental and theoretical research program is being carried out on this topic at the Politecnico of Milan. The results are presented of accelerated creep tests to failure on brick masonry samples taken from a building (the Cathedral of Monza, Italy) that shows signs of serious damage. Then, a recently developed theoretical model is proposed to simulate the experimental results. It is based on the theory of viscoelasticity coupled with two anisotropic damage variables, allowing, respectively, for damage induced by monotonically increasing stresses and by sustained stresses. The numerical simulations performed so far are quite encouraging, so that the applicability of the proposed model to structural analyses can be envisaged.

1 Introduction

The work covers the study of the behaviour of ancient masonry under sustained stresses. Indeed, stresses of a level even much lower than the short-term strength of masonry have been found to be responsible of the failure of ancient massive masonry structures, as shown by some recently collapsed towers (Pavia, Italy; Goch, Germany) and churches (Noto Cathedral, Italy). In fact, towers as well as heavily loaded, slender structural elements (columns, pillars, etc.) turn out to be greatly influenced by creep deformations. Moreover, the inhomogeneity of these structural elements and buildings (often made of multiple leaf masonry) generally induces a non-uniform stress distribution within the load-bearing area. These factors, together with the fatigue effects due to cyclic actions (induced e.g. by temperature changes and wind actions), can be responsible for serious structural damages that, in some cases, can bring the structure to failure.
In the last decade a great effort was made in order to understand the long-term behaviour of masonry materials and structures, both with experimental tests and numerical studies. Several tests were performed on masonry samples taken from the ruins of the Pavia tower at increasing stress levels, at which the stress was kept constant for a certain time, and up to failure [1]. The results clearly show a creep behaviour of masonry, characterized by the three phases of primary, secondary and tertiary creep (at the last step). These tests also pointed out that changes in the stress intensity, even of short duration and small amplitude, can act as ‘accelerators’ for the evolution of creep strain and can anticipate the structural collapse. An experimental campaign has recently been made on another middle-age building that shows serious evidences of damage, that is, the belltower of the Cathedral of Monza, near Milan (Italy) [2].

To interpret the test data, the authors started from a rheological model proposed by Anzani et al. [3], able to capture only the first two parts of the creep. In order to extend the model to tertiary creep, two damage variables were introduced by the authors: in this way the proposed model can describe the creep failure and predict the creep time to failure of the material under given stresses. It was shown by Papa et al. [4] that the modified model is able to reproduce experimental results on samples taken from the ruins of the Pavia bell-tower. The model was also implemented in a finite element code to carry out structural analyses of the collapsed Tower. The results obtained were encouraging, since a realistic estimate of the time to failure could be obtained and the predicted collapse mechanism of the Tower was found to be consistent with the ruins [5].

The proposed model is here applied for a simulation of the tests performed on the material taken from the crypt of the Monza Cathedral. This is a necessary preliminary step toward the numerical analysis of the Monza bell-tower in order to assess its safety. The features of the theoretical model are reviewed in Section 2.1. Indeed, since it is very important to understand the influence that each parameter has on the model response and, in particular, on the prediction of the creep time to failure of the material at a given stress level, a parametric study of the model is performed in Sec. 2.2. Then the model is used to simulate experimental tests carried out on samples from the Monza Cathedral in order to identify the model parameter specific for that material (Section 3). The last Section is devoted to concluding remarks and future perspectives of the research.

2 Theoretical model

2.1 A rheological viscoelastic model with damage

In order to qualitatively interpret the complex response of masonry when subjected to persistent loads, the authors have proposed a simplified mathematical model based on the classical theory of linear viscoelasticity coupled with the mechanical theory of damage. Details on the model can be found in [4,5]. Due to the assumption of damage variables, the model can describe the decay of the mechanical properties induced by increasing stresses, as well as the creep failure.

Fig. 1 shows the essential features of the rheological model that corresponds
in its uniaxial version. Two classical Kelvin and Maxwell elements are assembled in series to form a so-called Burger’s model. The Kelvin element aims to describe primary creep: in this element, only reversible viscoelastic strains develop and damage effects are neglected. The Maxwell element consists of a spring, which accounts for the elastic response of the material to any stress increment, in series with a dashpot. A “static” damage variable, \( D^5 \), is introduced in the spring to account for damage induced by increasing stresses. In the dashpot strains evolve only when the stress exceeds a threshold, \( \sigma_0 \), corresponding to the slip strength of the frictional (or Bingham) element in the model. When \( \sigma_0 \) is exceeded, the secondary (or steady state) creep phase starts. Since “viscous” damage can develop in this element, according to the evolution of a damage variable \( D^V \), the relaxation time \( \tau^M \) of the dashpot is supposed to progressively decrease, so that the tertiary (or unstable) creep phase can start, according to the stress level, and creep failure can be described.

![Modified rheological Burger's model](image)

The evolution laws for the damage variables were obtained by modifying laws proposed by other authors for different brittle materials. Different laws are used in tension and compression, to match the so-called “unilateral” behaviour of most brittle materials, such as masonry.

Under constant loading only the viscous damage variable evolves, according to a law originally proposed for rocksalt by Chan et al. [6]:

\[
\dot{D}^V_j = \left( Y_j^B / E^M \right)^{a_{2H}} a_{1H} D^V_j \left( - \log D^V_j \right)^{(1+\alpha_H)/\alpha_H}. \tag{1}
\]

Here \( a_{1H}, a_{2H} \) are model parameters, where \( H = C \) or \( T \) applies for damage induced, respectively, by compressive or tensile strains. \( Y_j^B \) (\( j = 1 \) or \( 2 \), for axial and transversal strains, respectively) are ‘viscous damage forces’ [4] associated with the irreversible strain accumulated in the dashpot of the Maxwell element. \( \dot{D}^V_j \) is supposed to vanish if the stress does not exceed \( \sigma_0 \) (see Fig. 1).

Under monotonic loading, also the static damage variable can increase according to an evolution law similar to that formulated by La Borderie et al. [7]
for concrete:

\[ D_j^S = 1 - \frac{1}{1 + A_H < Y_j^M - Y_{0H} >^B_H} \]  

(2)

where \( A_H, B_H \) are model parameters \( (H = C \text{ or } T) \), \( Y_j^M \) are 'static damage forces' [4] associated with the elastic strain in the Maxwell's spring. Macauley brackets are employed in eqn(2) to make static damage vanish if any damage force does not exceed a certain threshold value \( Y_{0H} \). Note that under increasing stresses both the static and the viscous damage variables can increase, which makes the model rate-sensitive; under increasing stress, however, the effects of the static damage variable are prevailing on those of the viscous one.

Actually, ‘static’ and ‘viscous’ damage variables are meant to describe a single physical phenomenon (the decay in mechanical properties of the material, associated to microcracking, etc.), which evolves differently according to the load history: this is accounted for through eqns(1) and (2). The use of a single incremental evolution law of the type of eqn(1) for any load history was found by the authors to give unrealistic predictions.

To qualitatively illustrate the response of the model in the simulation of a uniaxial compression test at constant stress, in Fig. 2 some numerical creep curves are plotted. It is important to stress what follows: (i) provided that creep failure is reached, these curves exhibit the classical S-shape corresponding to the sequence of primary, secondary and tertiary creep; (ii) the time to failure decreases as the stress intensity increases, in agreement with experiments.

2.2 Parametric analysis of the model predictions

The influence of the main parameters defining the model response will be now briefly discussed. This parametric study can help in the identification of the values of the parameters that allow for best fitting experimental results.

2.2.1 Threshold values for static damage

\( Y_{0C} \) and \( Y_{0T} \) are the values of the static damage driving force beyond which static damage starts evolving in compression or tension, respectively (eqn(2)). These values indirectly affect the model response in the simulation of tests at constant stress; in fact the initial value of the viscous damage variable (to be employed in the integration of eqn(1)) is supposed to be equal to the value of the static damage variable, eqn(2), reached at the beginning of a step at constant stress.

Fig. 3 shows the time evolution of the axial and of the transversal strains in the simulation of a uniaxial creep test with different values of the static damage thresholds. Both \( Y_{0C} \) and \( Y_{0T} \) affect the values of the elastic strains at the beginning of the phase at constant stress, since lower threshold values result in higher static damage and higher elastic strains, but this influence is hardly visible in the plots. \( Y_{0C} \) affects the global form of the axial creep curves and tertiary creep starts earlier with lower values of \( Y_{0C} \). \( Y_{0T} \) has a smaller influence on the
time evolution of the transversal strains, and the creep time to failure is nearly independent on $Y_{OT}$. With the highest value of $Y_{OT}$ chosen in Fig. 3, damage is not activated before the beginning of the creep behaviour.

![Figure 2: Simulation of uniaxial creep tests at different stresses.](image1)

![Figure 3: Influence of the damage thresholds values, $Y_{OC}$ and $Y_{OT}$, on the strain evolution in time during the simulation of a uniaxial creep test.](image2)

2.2.2 Static damage parameters

Static damage is meant to describe the decrease in mechanical properties of masonry as the applied stress increases. The evolution laws for the static damage variables in tension and compression are given by eqn(2) and are defined, respectively, by $A_T$, $B_T$ and by $A_C$, $B_C$. For sake of clarity, the attention is focused here on the first couple of parameters; remarks similar to those made in the sequel regarding $A_T$ and $B_T$ apply, however, to $A_C$ and $B_C$, respectively.

Increasing values of $A_T$ and decreasing values of $B_T$ are matched by: (i) an increase in static damage, that is, in the initial value of the viscous damage variables; (ii) an increase in the elastic strains at the beginning of the phase at constant stress; (iii) an increase in the secondary creep rate; (iv) a decrease in the duration of both the secondary and the tertiary creep phases and, accordingly, (v) a shorter creep time to failure (see Fig. 4).

2.2.3 Viscous damage parameters

Viscous damage accounts for the decay in mechanical properties of the masonry both at constant and at increasing stress, although its effect is less remarkable in the latter case. The evolution law for the viscous damage variables is expressed by eqn(1) and depends on $a_{1T}$, $a_{2T}$ in tension and on $a_{1C}$, $a_{2C}$ in compression. Again, only the influence of the first couple of parameters on the model response is investigated here, according to the importance of tensile cracks in the macroscopic creep behaviour of the material.
Figure 4: Influence of the static damage parameters (a) $A_T$ and (b) $B_T$ on the time evolution of the transversal creep strains during the simulation of a uniaxial test at constant stress.

Increasing values of $a_{1T}$ are matched by a faster growth of the viscous damage variables, that sooner reach their maximum value 1 (corresponding to complete failure) – see Fig. 5. Accordingly, the strain vs. time plots have a steeper slope in the secondary phase, the duration of this phase is shorter and so is the creep time to failure. Similar remarks apply to the effects of $a_{2T}$ upon the creep response of the model. The parameters that define the increase in compressive viscous damage, $a_{1C}$ and $a_{2C}$, are expected to qualitatively have the same influence as $a_{1T}$ and $a_{2T}$.

Figure 5: Influence of the parameters $a_{1C}$ and $a_{2C}$ on the time evolution of viscous damage during the simulation of a uniaxial test at constant stress.
3 Experimental tests and numerical simulations

The bell-tower of Monza Cathedral, about 70 m high, dates back to the end of XVI century. Because of the passing-through crack pattern existing in two of the bearing-load walls of the tower and of a diffuse damage crack pattern at 11 m height, an extensive on-site and laboratory investigation has been recently carried out to assess the safety of the Tower. Details on the Tower and the experimental survey can be found in [2]. Here, the attention is focused on the results of some laboratory tests on specimens cut from two large pieces of wall recovered from the crypt of the Cathedral during the opening of a door. The crypt is likely to have been built during the same period and with the same technique as the Tower. Prisms of about 200×200×500 mm were cut and subjected to three series of uniaxial compression tests. Initially, monotonic tests were carried out to have a first indication on the compressive strength of the masonry. Then cyclic tests were performed, during which cycles of ±0.15 MPa at 1 Hz were applied at increasing stress levels. Finally, compression tests were also carried out applying the load in subsequent steps (equal to 0.25 MPa each) and keeping it constant for a given time interval (about 1.5 hours). Since each test lasted more than one day, the samples were unloaded before night, for safety reasons, and reloaded the day after. Figures 6 and 7 show the strain vs. time plots obtained during three of the tests. Creep strains can be clearly observed while the load is kept constant, with the appearance of tertiary creep during the application of the last load step.

Because of the experimental scatter, a great number of creep tests at different stress levels, with measurements of the degraded elastic properties of the material with increasing deformation, would be necessary. This in order to reliably identify the values of the model parameters suitable for the tested material. To overcome this lack of information, the following procedure was used to estimate the parameter values according to the only three creep tests available.

First, the elastic properties \( (E^0, E^*, v) \) and the threshold values of the damage forces \( (Y_{OC}, Y_{OT}) \) for the material taken from the crypt of the Monza Cathedral were assessed, according to the monotonic compression tests performed. Also, relating the secondary creep rate with the stress intensity, the relaxation time of Maxwell’s spring, \( \tau^N \), could be obtained. These values are reported in the third column of Table 1. Then, a simulation of the experimental tests was made using the identified values and setting the values of the other parameters equal to those obtained from similar tests performed on samples taken from the ruins of the Pavia tower (second column of Table 1). These values can be assumed as sufficiently reliable for the material of the Pavia tower; in fact, when employed in a structural analysis, they gave a good estimate of the time to failure of the Tower. In this way, the axial strain vs. time plot of Fig. 6a was obtained.

To increase the predicted time to failure, three numerical tests were made, by decreasing \( A_C \) (from 800 to 380), or by increasing \( a_{2C} \) (from 0.45 to 0.55); in the latter case, it was found that better results are obtained if, at the same time, \( a_{27} \) is reduced (from 0.35 to 0.25). Decreasing \( A_C \) slackens the evolution of damage in accelerated creep tests between two subsequent load steps. Changing \( a_{2C} \) and \( a_{27} \) makes the transversal viscous damage variable to evolve more quickly.
than the axial one, so that the phenomenon of dilatancy when approaching failure can be captured (which is not the case with the original choice of the parameter values). In this way, the curves shown in Fig. 6b were obtained. For further analyses, it was decided to keep the values $A_C = 380$, $a_{2C} = 0.55$, $a_{2T} = 0.25$ that give approximately the same time to failure as the experimental test of intermediate duration. The final set of identified values is reported in the third column of Table 1.

Table 1. Values of the model parameters for Pavia and Monza tower.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Pavia</th>
<th>Monza</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^M$ (MPa)</td>
<td>3500</td>
<td>2800</td>
</tr>
<tr>
<td>$\tau^M$ (sec)</td>
<td>690000</td>
<td>110000</td>
</tr>
<tr>
<td>$E^K$ (MPa)</td>
<td>3000</td>
<td>20000</td>
</tr>
<tr>
<td>$\tau^K$ (sec)</td>
<td>7500</td>
<td>7500</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>$A_C$</td>
<td>800</td>
<td>380</td>
</tr>
<tr>
<td>$B_C$</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td>$A_T$</td>
<td>9000</td>
<td>9000</td>
</tr>
<tr>
<td>$B_T$</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>$Y_{0C}$ (MPa)</td>
<td>0.002</td>
<td>0.00001</td>
</tr>
<tr>
<td>$Y_{0T}$ (MPa)</td>
<td>0.00002</td>
<td>0.00000001</td>
</tr>
<tr>
<td>$a_{1C}$ (sec$^{-1}$)</td>
<td>2.2</td>
<td>8</td>
</tr>
<tr>
<td>$a_{2C}$</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>$a_{1T}$ (sec$^{-1}$)</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>$a_{2T}$</td>
<td>0.35</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 6: Comparison between the results of accelerated creep tests on samples taken from the crypt of the Monza Cathedral and the predictions of the theoretical model: (a) same damage parameters as for the Pavia tower; (b) different choices of $A_C$, $a_{2C}$, $a_{2T}$ to increase the time to failure.
Finally, a problem that first rose in the structural analysis of the Pavia tower had to be tackled. Using the value for $\tau^M$ identified from accelerated creep tests on material samples also in a structural analysis leads to the prediction of an unrealistically short time to failure of the structure. Thus, for future applications, it was decided to set $\tau^M$ equal to a very high value (1000 years), so that the estimate of the time to failure of the tower will not be affected by this choice. In Fig. 7 the numerical results obtained with the two values of $\tau^M$ are compared with the experimental ones: note that, although the secondary creep rate is in general underestimated, the time to failure of the material is not very sensitive to the choice made for $\tau^M$.

![Graph](image)

Figure 7: Comparison between the results of accelerated creep tests on samples taken from the crypt of Monza Cathedral and the predictions of the theoretical model with different values of the relaxation time of Maxwell’s spring.

4 Concluding remarks. Future perspectives

The model proposed is a first step toward the prediction of the effects of damage induced by sustained loads in heavy masonry buildings. It must be emphasized that literature regarding this subject is extremely scarce. On the other hand, the necessity of accounting for creep-induced damage when the safety of ancient masonry structures has to be assessed is now widely recognized. The presented model is capable of accounting for different aspects of the behaviour of masonry (and of most brittle materials) subjected to persistent loads, such as (i) damage-induced anisotropy, (ii) development of primary, secondary and tertiary creep, (iii) decrease in creep time to failure with increasing stress level.

Some inconsistencies which are described in the following have still to be eliminated from the model. According to the rheological origin of the model, it predicts primary creep only at the first step at constant stress and not at the subsequent ones, contrary to experimental evidences (see Figs. 6 and 7). The relaxation time of Maxwell’s spring, $\tau^M$, as identified by creep tests to failure on a ma-
terial element, cannot be employed in structural analyses [8], otherwise an unrealistically short time to failure for the analyzed building would be predicted. This contradicts the assumption that $\tau^m$ is a material parameter, that is, independent on the load conditions. The model presented is suitable for masonry with macroscopically isotropic properties in the undamaged state, but might be improved to take into account the initial orthotropy of masonry made by a regular arrangement of brick or stones. An extension of the model to heterogeneous anisotropic materials has already been planned.

Using this model, in conjunction with a commercial general-purpose finite-element program for non-linear analyses, has recently allowed the authors to perform a stress analysis of the belltower of Monza Cathedral, which has given a realistic picture of the crack pattern across the Tower and an estimate for the residual life of the Tower [8].

References


