Active noise control by modal state feedback using piezo-electric actuators

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Abstract

The use of structural control by modal state feedback for noise reduction of plate structures is described in this paper.

The Finite Element Method supplies a numerical description of the noise radiating structure and the effects of the attached piezo actuators. Technical expenditures are kept low by minimizing the number of sensors and actuators used in the control system. To achieve a high control performance, a method is derived to find convenient locations for these elements.

The noise reduction experiments are conducted with a clamped rectangular aluminium plate. Control outputs are computed on basis of a modal state observer considering 10 modes in connection with a linear quadratic optimal controller. This algorithm is programmed on a digital signal processor and the control forces are generated by piezo-electric actuators directly applied on the panel surface. The developed active noise control achieves excellent results for the reduction of the sound pressure levels at the resonance frequencies of the plate.

Introduction

The increasing attention on environmental problems makes noise control an important task of mechanical engineering. Radiation of noise has several sources. Vibrational excitation of mechanical devices can cause sound radiation from their surfaces. Plate and shell structures in automotive, aviation and space applications vibrating in resonance

ow particularly high noise levels. A reduction of disturbing noise can be achieved by two basic concepts: on one hand a reduction at the origin of radiation and on the other hand at the locality of noise reception. In the past passive methods for noise reduction have been used widely. Common examples are the use of passive damping or additional stiffening of the construction. These methods cause an increase of weight, which cannot be accepted in special cases of aviation and space applications. These purposes require a method for active noise control which makes exact knowledge of system dynamics necessary.

Several proposals for active noise control are already existing. A common practice is the use of anti-noise which utilizes the interference of sound waves. Since it does not affect the origination of sound, this method is rather ineffective. For this reason the use of structural control for reduction of sound radiation is investigated.

Piezo-Electric Actuators

During the experiments presented here, small piezo-electric actuator patches have been used which are attached to the plate surface. Since their size is rather small compared to the plate dimensions they are able to control many mode shapes. Large distributed actuators like those described by Tzou[6] are unsuitable for noise control because they can only control the lowest modes and thus only a small frequency range. The experimental setup uses opposite faced actuator pairs on the plate surface. The actuator pairs are electrically driven in a way that the upper element extends and the lower one contracts. This results in bending of the basic material (i.e. the aluminium plate). Crawley[1] has shown for beam structures that this effect can be modelled as a pair of bending moments which act in opposite directions at the ends of the actuator.

Based on this description for beams an equation is derived in [5] which describes the effect of piezo-electric actuator pairs attached to a plate surface. Assuming the actuators are perfectly bound to the plate surface, a constant strain appears in the contact surface between piezo ceramic and aluminium plate. This is equivalent to moments per unit length acting along the edges of the actuator patch which are proportional to the piezo driving voltage U. These moments per unit length are inserted into the finite element model of the vibrat-

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e actuator edges. Their dependance on the control vector $\mathbf{u}(t)$ is expressed by the control locaction matrix $\mathbf{D}$.

Modelling of Vibrating Structure

The development of the active noise control system requires a finite element model of the vibrating structure leading to the differential equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}_D\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{D}\mathbf{u}(t) + \mathbf{E}\mathbf{f}(t) \quad . \tag{1}$$

The matrices $\mathbf{D}$ and $\mathbf{E}$ define the locations of control and excitation forces $\mathbf{u}(t)$ and $\mathbf{f}(t)$. To recieve a manageable size of the finite element model the method of modal reduction is used. The solution of the eigenvalue problem gives the eigenfrequencies ω_i and the eigenvectors ϕ_i of the structure which are arranged in the modal matrix $\mathbf{\Phi}$. The introduction of modal coordinates $\mathbf{q}(t)$ by the transformation $\mathbf{x}(t) = \mathbf{\Phi}\mathbf{q}(t)$ results in the modal differential equation

$$\ddot{\mathbf{q}}(t) + \mathbf{\Delta}\dot{\mathbf{q}}(t) + \mathbf{\Omega}\mathbf{q}(t) = \mathbf{\Phi}^T\mathbf{D}\mathbf{u}(t) + \mathbf{\Phi}^T\mathbf{E}\mathbf{f}(t) \quad . \tag{2}$$

The diagonal matrix $\mathbf{\Omega}$ contains the eigenvalues ω_i^2. Under assumption of modal damping the matix $\mathbf{\Delta}$ is also a diagonal matrix.

For reduction of the model size the vector of modal displacement $\mathbf{q} = [\mathbf{q}_c, \mathbf{q}_r]^T$ is divided into components of controlled modes $\mathbf{q}_c(t)$ and uncontrolled residual modes $\mathbf{q}_r(t)$. Only the controlled modes $\mathbf{q}_c(t)$ are referred furthermore in the reduced model of the system, the residual modes $\mathbf{q}_r(t)$ are neglected. By introduction of the modal state vector $\mathbf{z}_c = [\mathbf{q}_c, \dot{\mathbf{q}}_c]^T$ the reduced modal differential equation (2) is transferred into state space

$$\begin{aligned} \dot{\mathbf{z}}_c(t) &= \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}_c & -\mathbf{\Delta}_c \end{bmatrix}}_{\mathbf{A}_c} \mathbf{z}_c(t) + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}_c^T\mathbf{D} \end{bmatrix}}_{\mathbf{B}_c} \mathbf{u}(t) + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}_c^T\mathbf{E} \end{bmatrix}}_{\mathbf{H}_c} \mathbf{f}(t) \\ &= \mathbf{A}_c\,\mathbf{z}_c(t) + \mathbf{B}_c\,\mathbf{u}(t) + \mathbf{H}_c\,\mathbf{f}(t) \quad . \end{aligned} \tag{3}$$

The use of a digital control system requires the transformation of the time-continuous state space equation (3) into its time-discrete form

$$\mathbf{z}_{k+1} = \bar{\mathbf{A}}\mathbf{z}_k + \bar{\mathbf{B}}\mathbf{u}_k + \bar{\mathbf{H}}\mathbf{f}_k \quad . \tag{4}$$

Discrete state space control requires complete knowlegde of the state vector $\mathbf{z}_k$ which cannot be measured completely. Thus it is necessary to estimate the modal vibrational state from the measured data $\mathbf{y}_k = \mathbf{C}\mathbf{z}_k$ by a *modal state observer* based on the reduced system model. The reconstructed state vector $\tilde{\mathbf{z}}_k$ is continuously corrected by measured data obtained from the real system which are fed back via the observer feedback matrix $\bar{\mathbf{L}}$

$$\tilde{\mathbf{z}}_{k+1} = \bar{\mathbf{A}}\tilde{\mathbf{z}}_k + \bar{\mathbf{B}}\mathbf{u}_k + \bar{\mathbf{L}}[\mathbf{y}_k - \mathbf{C}\tilde{\mathbf{z}}_k] \quad . \tag{5}$$

The matrix $\bar{\mathbf{L}}$ is found by minimizing the variance of the observer error. The consideration of the variances of system disturbances and measurement noise by the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ yields in the following iteration algorithm:

$$\bar{\mathbf{L}}_k = \bar{\mathbf{A}}\bar{\mathbf{N}}_k\mathbf{C}^T(\mathbf{V}_2 + \mathbf{C}\bar{\mathbf{N}}_k\mathbf{C}^T)^{-1} \tag{6}$$

$$\bar{\mathbf{N}}_{k+1} = (\bar{\mathbf{A}} - \bar{\mathbf{L}}_k\mathbf{C})\bar{\mathbf{N}}_k\bar{\mathbf{A}}^T + \mathbf{V}_1 \quad . \tag{7}$$

The application of the state observer completes the state vector whose information is necessary to make use of a linear-quadratic optimal controller. To design this type of control algorithm a performance index

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left[\mathbf{z}_k^T\mathbf{Q}\mathbf{z}_k + \mathbf{u}_k^T\mathbf{R}\mathbf{u}_k\right] \tag{8}$$

under consideration of the system equation (4) is minimized. The matrices $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices of the system state and the control vector. The solution of this optimization problem gives the iterative Riccati equation

$$\bar{\mathbf{P}}_{k+1} = \mathbf{Q} + \bar{\mathbf{A}}^T\bar{\mathbf{P}}_k[\mathbf{I} - \bar{\mathbf{B}}(\mathbf{R} + \bar{\mathbf{B}}^T\bar{\mathbf{P}}_k\bar{\mathbf{B}})^{-1}\bar{\mathbf{B}}^T\bar{\mathbf{P}}_k]\bar{\mathbf{A}} \tag{9}$$

to compute the Riccati matrix $\bar{\mathbf{P}}$. The linear-quadratic control law

$$\mathbf{u}_k = -[(\mathbf{R} + \bar{\mathbf{B}}^T\bar{\mathbf{P}}\bar{\mathbf{B}})^{-1}\bar{\mathbf{B}}^T\bar{\mathbf{P}}\bar{\mathbf{A}}]\mathbf{z}_k = \mathbf{G}\mathbf{z}_k \tag{10}$$

finally supplies the requested control gain matrix $\mathbf{G}$.

The derivation of the control algorithm was done under assumption that all events in the control loop happen synchronous. In real,

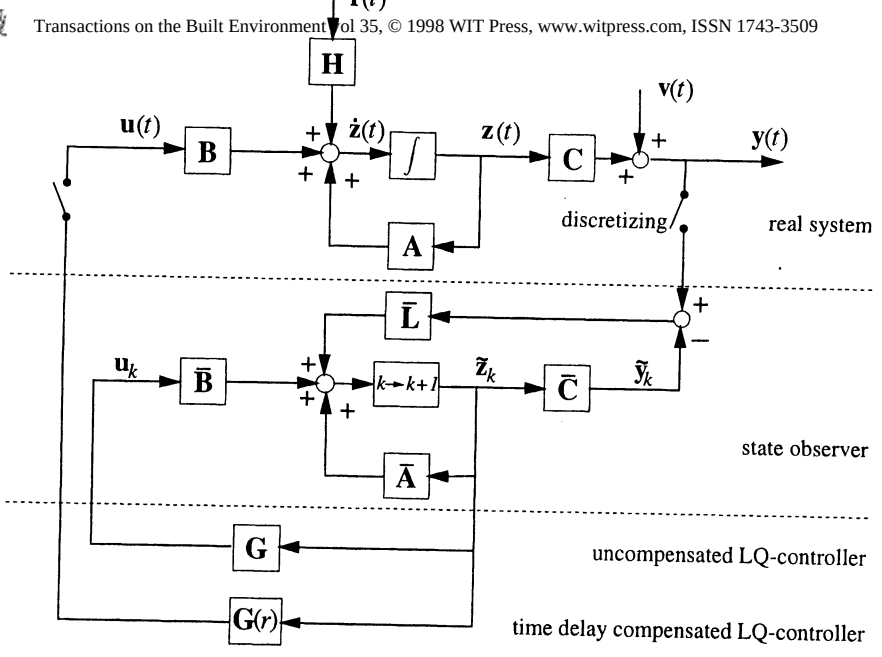


Figure 1: Control system with time delay compensation

time delays are caused by computation time, other activities of the real-time system, and actuator dynamics. To prevent instabilities the effect of time delay must be compensated. Based on a modified form of the state equation (4)

$$\mathbf{z}_{k+1} = \bar{\mathbf{A}}\mathbf{z}_k + \bar{\mathbf{B}}\mathbf{u}_{k-r} + \bar{\mathbf{H}}\mathbf{f}_k \tag{11}$$

which considers a time delay τ of r scans, a compensation can be found (see Soong[4]). The solution of the linear-quadratic optimization problem leads to the contol law

$$\mathbf{u}_{k-r} = \mathbf{G}(r)\mathbf{z}_{k-r} \tag{12}$$

with the time delay compensated feedback gain matrix

$$\mathbf{G}(r) = [\bar{\mathbf{B}}^T\bar{\mathbf{P}}\bar{\mathbf{B}} + \mathbf{R}]^{-1}\bar{\mathbf{B}}^T\bar{\mathbf{P}}\bar{\mathbf{A}}\bar{\mathbf{T}}^{r+1} \tag{13}$$

It contains the Riccati matrix $\bar{\mathbf{P}}$ from equation (9) and a time transition matrix $\bar{\mathbf{T}}$ given as

$$\bar{\mathbf{T}} = (\bar{\mathbf{A}}^T\bar{\mathbf{P}})^{-1}(\bar{\mathbf{P}} - \mathbf{Q}) \tag{14}$$

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the time delay compensated gain matrix $\mathbf{G}(\tau)$ is used to compute the control outputs transmitted to the actuators, whereas the state observer needs the uncompensated control vector (see Fig. 1).

Optimal Sensor and Actuator Placement

Practical and economical reasons set a limit for the number of sensors and actuators used. This requires a method to determine optimal positions for these elements. Convenient positions for sensors can be found by consideration of the mode shapes (eigenvectors) of the vibrating structure. For a single mode i an observability index ψ_i is defined depending on the components of its eigenvector $\boldsymbol{\phi}_i$. To measure several modes with a single sensor their observability indices are superponed to build a total observability index ψ. The maximum of ψ deliveres the optimal position for the sensor.

The search for optimal actuator positions is done assuming an actuator is conveniently positioned at a location where the system response due to a disturbance is up to a maximum. To control a single mode i the surface strains of its mode shape in x- and y-direction is to be considered when defining a controllability index ξ_i. By superposition of the modal indices ξ_i the total controllability index ξ for control of several modes is found. The consideration of ξ supplies an irregular arrangement of the piezo elements. Regular layouts such as those along an axis of symmetry (see Falangas[2]) are not convenient since certain modes (e.g. the (2,2)-mode) cannot be controlled. An unfavourable arrangement of actuators and sensors deteroriates the performance of the control system. The chosen positions of sensors and actuators can be seen in Fig. 2.

Experimental Evaluation

The verification of the theoretical analysis was conducted with the experimental setup shown in Fig. 2. The rectangular aluminium panel (600 mm × 400 mm × 3 mm) is clamped in a heavy frame construction. The plate vibrations are measured by two non-contacting inductive displacement transducers and these signals are fed to a digital signal processor (DSP) running at a sampling frequency of 6000 Hz. The state observer programmed on the DSP utilizes the two inputs to reconstruct the entire vibrational state of the first 10 plate modes.

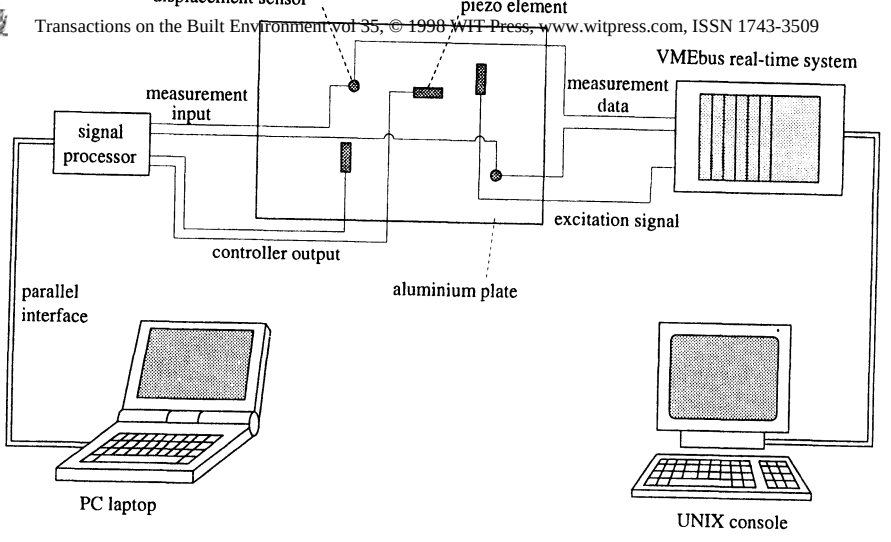


Figure 2: Sketch of experimental setup

Based on this estimation a linear-quadratic controller computes the outputs for two piezo electric actuator pairs to achieve a reduction of the plate vibration and its sound radiation. The radiated sound pressure is measured by a microphone directed to the panel surface.

Structural vibration is excited with a third piezo actuator pair. Different excitation functions are realized with the help of a software running on a VMEbus real-time computer system. Most important are sinusoidal excitations for measurement of frequency response. Furthermore the VMEbus system samples data for storage on hard disk and later evaluation on a workstation.

The performance of the noise control system is evaluated by comparison of uncontrolled and controlled structural response. Frequency response is measured in a range of $f = 50...700$ Hz which encloses the first 10 natural frequencies of the plate. In Fig. 3 the responses $\hat{y}_1(\omega)$ of the uncontrolled and controlled system at the displacement transducer No. 1 are compared. It can be seen that the controller influences a frequency range up to 600 Hz, corresponding to the highest controllable 10th mode. This interval contains the highest resonance peaks, for higher excitation frequencies the peaks decrease. The evaluation of response of channel 1 shows, that the state controller distinctly attenuates the dominating resonance vibrations up to the seventh mode. Most notable effects are achieved for the modes 2, 5, 6, and

which are attenuated between 12 dB and 15 dB. The reduction of modes 9 and 10 is rather low since modelling errors become more apparent for higher modes. This causes an inaccurate estimation of the state vector which results in a decrease of control performance in this frequeny range.

Nr.	frequency [Hz]	mode	vibration [dB]	SPL [dB]
1	110.3	(1,1)	−3.57	−3.31
2	186.2	(2,1)	−11.99	−12.13
3	281.1	(1,2)	−4.05	−5.47
4	306.1	(3,1)	−7.41	−16.88
5	348.8	(2,2)	−15.25	−13.83
6	452.3	(4,1)	−13.88	−10.45
7	456.5	(3,2)	−15.22	−0.65
8	532.1	(1,3)	+0.99	−0.97
9	598.0	(2,3)	−1.87	−3.10
10	606.4	(4,2)	−4.39	−4.00
11	651.0	(5,1)	+2.86	+0.12
12	694.1	(3,3)	−0.63	−4.30

Table 1: Attenuations of vibration amplitude $\hat{y}_1$ and sound pressure level L_p

The measurement of the sound pressure level (SPL) L_p reflects the results of the vibration measurement (see Fig. 4). Particular reductions of the SPL are achieved at the modes 2, 4, 5, and 6. Attenuations between 12 dB and 17 dB have been measured. Most to be noticed are the reductions at the 5th and 6th natural frequencies whose uncontrolled radiation is extremely high. The attenuation of the remaining controlled modes is smaller (3...5 dB). The experiments show that a large reduction of the resonance amplitudes yields in a high attenuation of the radiated SPL. Table 1 gives an overview of the achieved attenuations of vibration amplitudes at the displacement transducer No. 1 and the SPL at the resonance frequencies.

Conclusions

The rapid development of the computing performance of digital signal processors made it possible to realize noise reduction systems using structural control. This was verified by experiments conducted with

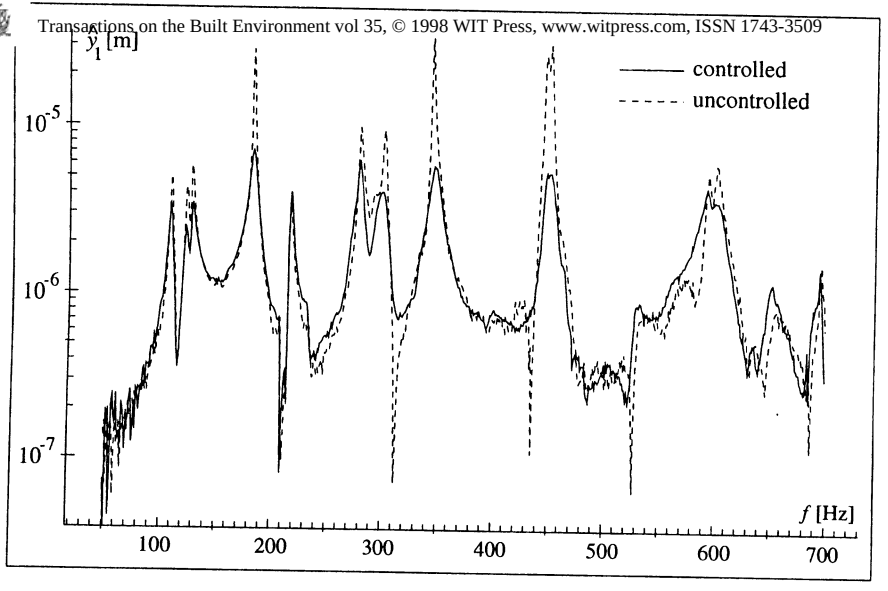


Figure 3: Vibration response $\hat{y}_1(\omega)$ at displacement sensor No. 1 with 10 controlled modes

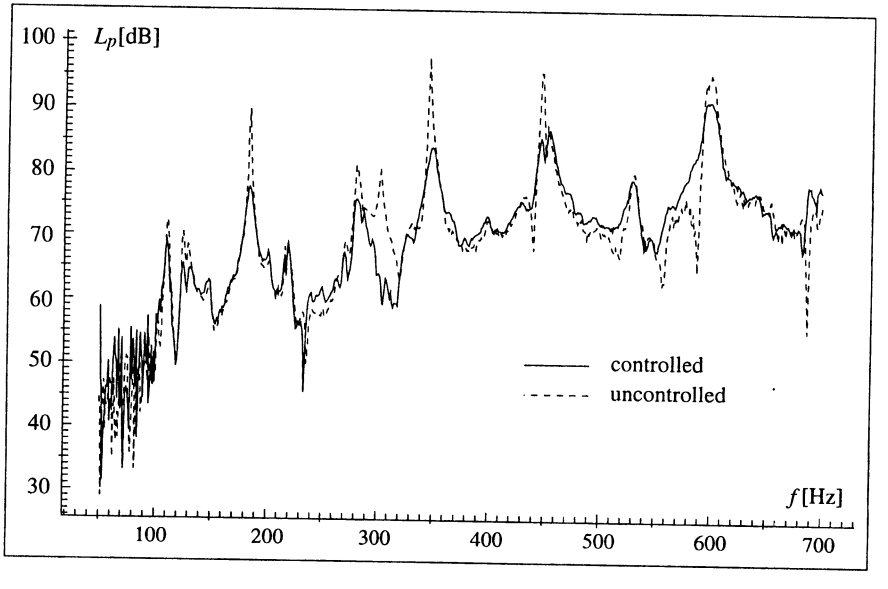


Figure 4: Sound pressure level L_p at the microphone with 10 controlled modes

clamped rectangular aluminium plate. The measured data of its structural vibrations serve as control inputs which are fed to a digital signal processor. The programmed modal state observer estimates the unknown states of 10 modes which are needed to compute the control outputs with a linear quadratic optimal control algorithm. The outputs are transmitted to piezo-electric actuators attached to the plate surface. The developed active noise control achieves large reductions of the vibration amplitudes and the radiated SPL at the resonance frequencies. In a frequency range of 0...650 Hz vibration reductions up to 15 dB and attenuations of the SPL up to 17 dB have been measured. Since digital signal processors and piezo-electric actuators are commercially available components, the developed control system is suitable for a wide range of future applications.

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