Cumulative damage diagnosis of smart structures under dynamic loading

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Abstract

Smart structures must modify their response mechanisms according to changeable loading conditions. Therefore, it is very important to estimate a damage effect of a dynamic loading and to predict a fatigue life of structures. An estimation of cumulative damage under dynamic loading is described in the paper. Basis of the method consists in specification of an equivalent amplitude of the harmonic cycle with zero stress and strain mean values. At the same time, a relative damage that is accumulated in the course of the equivalent cycle must be equal to the relative damage of the actual cycle that has been identified using the rainflow method. This procedure allows to determine the response value under either stress or strain control, without identification of the complete time loading history, using a rainflow decomposition. The method is acceptable for structural materials with different tension and compression response properties, too.

1 Introduction

An optimum design of a control system, which can modify system parameters according to operating conditions, is very important for smart structures. That depends above all on the design strategy and service conditions. Usually, namely in aerospace and transport engineering, an operational safety and reliability is the decisive criterion. That is dependent on a damage accumulation process during a time loading
Therefore, the cumulative damage diagnosis is a very important assumption for the structure design because the ultimate goal is to control the system parameters so that the fatigue life is maximized.

In order to predict a cumulative damage and residual life of structures, we must know stress-strain response properties of the material. The most of hypotheses are based on a dependence of a relative cumulative damage upon amplitudes of closed cycles with respect to corresponding mean values. Under residual stresses occurrence, an equivalent amplitude of a closed cycle must be calculated. The equivalent amplitude is calculated according to cyclic stress-strain response properties because it is dependent on a total plastic deformation which corresponds to the stress parameters.

For specification of all required stress and strain parameters of an actual closed cycle, we need to know the complete time loading history. But that is not very convenient, especially if high cycle fatigue properties would be investigated. Therefore, a suitable simplified interpretation of a time history is very desirable in this case. Such an interpretation can be made using a rainflow decomposition, which can decrease time and memory demands many times. The rainflow decomposition procedure is independent upon material properties, and it can be applied for any material and for any loading character.

In order to create an effective method, the procedure must be optimally formulated. Therefore, stress-strain response relationships, e.g. a cyclic deformation curve, must be estimated and then analytically expressed and transformed, so that the procedure would be quick and performed on-line, in a real time. Such an approach enables a continuous monitoring of a fatigue damage with respect to a residual stresses influence.

2 Cumulative damage evaluation

In order to evaluate the fatigue damage under dynamic loading, the cumulative representation of a relative damage of a closed cycle is accepted. The problem could arise, if the mean values of cycles are changing, because the relative damage of individual cycles cannot be simply added in this case, neither if the cycles have been identified according to the rainflow method (Čačko¹). The relative damage of a loading cycle with some definite amplitude and mean value under block loading need not correspond to the relative damage under pure harmonic loading. Moreover, the relative damage is significantly different in the case of a random loading.
The effect of a mean stress/strain must be effectively substituted by considering of cycles with some equivalent amplitude \( \sigma_a^* \) (or \( \varepsilon_a^* \)) and zero mean value of \( \sigma \) (or \( \varepsilon \)) instead of the cycles with the actual amplitude \( \sigma_a \) (stress control) or \( \varepsilon_a \) (strain control) and mean value \( \sigma_m \) (or \( \varepsilon_m \) respectively). At the same time, of course, the damage effect of both cycles must be identical.

The equivalent amplitude can be derived using various criteria that are based on the damage parameter specification. However, it can be applied only for a harmonic or block loading, where a start-up procedure before the cycle creation is supposed in accordance with the cyclic strain curve.

That is why we must use some next independent parameter for specification of the equivalent amplitude under random loading. If a stabilized stress/strain curve is introduced, we could proceed from the requirement of equal strength safety. Then we may use the Haigh diagram.

The most serious difficulty is, that we must know complete load history until the actual loading cycle will occur. That is very inconvenient and frequently also practically uncontrollable. Therefore, it was necessary to find a new method to interpret the load history more properly. As a very perspective method seems to be the decomposition of closed hysteresis loops according to the rainflow method. For this purpose, the continual rainflow counting described in Čačko\(^2\) can be very effectively used.

### 3 Operational stress-strain response under random loading

The problem of the loading mean stress/strain effect is usually solved in such a way that we specify some equivalent amplitude \( \sigma_a^* \) in dependence of a stress mean value \( \sigma_m \) and strain mean value difference \( \Delta \varepsilon_m \) (Čačko\(^3\)). We can derive the amplitude \( \sigma_a^* \) from a triaxial Haigh diagram (Figure 1) \( \sigma_A = f(\sigma_M) \) for different \( \Delta \varepsilon_m \), where \( \Delta \varepsilon_m \) denotes the shift of strain mean value of the actual cycle from the mean value of the corresponding cycle on the cyclic curve. Supposing that the actual cycle is equivalent to the cycle with \( \sigma_m = 0 \) and \( \Delta \varepsilon_m = 0 \), we can express the equivalent amplitude as follows.
The relationship (1) can be linearized using a description of the triaxial diagram. Thus, we obtain the approximate relationship

\[ \sigma_a^* = \sigma_a + \psi_\sigma \sigma_m + \psi_\varepsilon \Delta \varepsilon_m, \]  

where \( \psi_\sigma = \cotg \varphi_\sigma \) and \( \psi_\varepsilon = \cotg \varphi_\varepsilon \).

Figure 1: The triaxial Haigh diagram
Rainflow decomposition of a random loading signal

In order to identify the strain mean value of the actual hysteresis loop, we can dismiss already closed hysteresis loops during cyclic process in the \(\sigma - \varepsilon\) diagram, and we can only consider hitherto unclosed loops and open branches. At the same time, the first unclosed loop is called as a branch of the zero inclusion, the second unclosed loop that is included in the first one is called as a branch of the first inclusion, etc. The actual \(N\)-th unclosed loop is then called as a branch of the \((N-1)\)-th inclusion.

The ordinates \(\sigma_1, \sigma_2, \ldots, \sigma_N\) (Figure 2) represent the origins of hitherto unclosed hysteresis loops. If we know the form of a cyclic curve (OSSRC): \(\sigma = \Phi_\pm(\varepsilon)\), for \(\varepsilon > 0\), we can specify the branches \(\sigma_1 \rightarrow \sigma_2, \sigma_2 \rightarrow \sigma_3, \ldots, \sigma_{N-1} \rightarrow \sigma_N\) as follows:

\[
\sigma = (1 + \kappa) \frac{\varepsilon}{1 + \kappa}. \tag{3}
\]

for loading \(\frac{d\sigma}{dt} > 0\), and

\[
\sigma = \left(1 + \frac{1}{\kappa}\right) \frac{\varepsilon}{1 + \frac{1}{\kappa}}. \tag{4}
\]

for unloading \(\frac{d\sigma}{dt} < 0\), where \(\kappa = -\frac{R^-}{R^+}\), whereby \(R^+ > 0\) is a yield stress in tension and \(R^- < 0\) is a yield stress in compression.

If \(R^+ = -R^-\) (i.e. \(\kappa = 1\)), it holds (for \(\varepsilon > 0\)):
\(\Phi_+(\varepsilon) = -\Phi_-(\varepsilon) \equiv \Phi(\varepsilon)\) and then the relationships, can be unified using the form

\[
\sigma = \pm 2 \Phi\left(\frac{\varepsilon}{2}\right); \text{ for } \varepsilon > 0, \text{ respectively.} \tag{5}
\]
The experimental results suggest that cyclic curve can be expressed in the form: \( \Phi(\varepsilon) = \pm K |\varepsilon - \varepsilon_e|^{n} \), for \( \varepsilon \geq 0 \), where \( \varepsilon_e \) is the elastic strain, and \( K \) and \( n \) are definite constants. Then, eq (5) can be adapted into the form

\[
\sigma = \pm 2K \left( \frac{|\varepsilon - \varepsilon_e|}{2} \right)^n ; \text{ for } \varepsilon \geq 0 , \text{ respectively.} \tag{6}
\]

Then we could step by step identify the corresponding values: \( \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N \)

Let us introduce

\[
\Delta \varepsilon_m = \varepsilon_m - \varepsilon_0 , \tag{7}
\]
Using the denotation: $\varepsilon_u$ - the maximum strain; $\varepsilon_d$ - the minimum strain and $\varepsilon_a$ - the strain amplitude of the hysteresis loop, it follows:

$$\varepsilon_{mN} = \varepsilon_u - \varepsilon_a$$
respectively
$$\varepsilon_{mN} = \varepsilon_d + \varepsilon_a.$$ \hspace{2cm} (8)

and

$$\varepsilon_{m0} = \varepsilon_u - \varepsilon_a$$
respectively
$$\varepsilon_{m0} = \varepsilon_d + \varepsilon_a.$$ \hspace{2cm} (9)

Because it holds $\varepsilon_{aN} = \varepsilon_{a0}$ in accordance with the requested condition, from eq (7) we obtain:

$$\Delta \varepsilon_{mN} = \varepsilon_u - \varepsilon_0$$
respectively
$$\Delta \varepsilon_{mN} = \varepsilon_d - \varepsilon_0.$$ \hspace{2cm} (10)

With respect to: $\varepsilon_u = \varepsilon_N$ (respectively $\varepsilon_d = \varepsilon_N$) and $\varepsilon_0 = \Phi^{-1}_+(\sigma_N)$ (respectively $\varepsilon_0 = \Phi^{-1}_-(\sigma_N)$), we further obtain the final relationship as follows

$$\Delta \varepsilon_{mN} = \varepsilon_N - \Phi^{-1}_\pm(\sigma_N),$$ \hspace{2cm} (11)

where the sign $(\pm)$ holds for $\sigma_{mN} > 0$ and $(\pm)$ for $\sigma_{mN} < 0$.

For the start cycle, using an inversion of the cyclic curve, we get:

$$\varepsilon_1 = \Phi^{-1}_\pm(\sigma_1),$$
where $(+)\,\text{holds for } \frac{d\sigma}{dt} > 0 \text{ and } (-)\,\text{for } \frac{d\sigma}{dt} < 0.$

Regarding (3) and (4), for the branch of the first loop we then obtain:

$$\varepsilon_2 = \varepsilon_1 + \left(1/\kappa^{\mp 1}\right) \Phi^{-1}_\pm\left(\frac{\sigma_2 - \sigma_1}{1/\kappa^{\mp 1}}\right),$$

for the second branch.
and generally for the integer $k \geq 1$:

$$
\varepsilon_{2k} = \varepsilon_{2k-1} + \left(1 + K^{\pm 1}\right) \Phi_{\pm}^{-1}\left(\frac{\sigma_{2k} - \sigma_{2k-1}}{1 + K^{\pm 1}}\right),
$$

(12)

$$
\varepsilon_{2k+1} = \varepsilon_{2k} + \left(1 + K^{\pm 1}\right) \Phi_{\pm}^{-1}\left(\frac{\sigma_{2k+1} - \sigma_{2k}}{1 + K^{\pm 1}}\right),
$$

where the upper signs hold for $\sigma_I > 0$, and the lower ones for $\sigma_I < 0$.

For the most of structural materials, we suppose that $K = 1$ and $\Phi_-(\varepsilon) = -\Phi_+(\varepsilon)$, i.e. the cyclic curve for the entire range can be expressed as $\sigma = \Phi(\varepsilon)$. Then, it follows from (12), using $\varepsilon_1 = \Phi^{-1}(\sigma_1)$

$$
\varepsilon_N = \Phi^{-1}(\sigma_1) + 2 \sum_{i=2}^{N} \Phi^{-1}\left(\frac{\sigma_i - \sigma_{i-1}}{2}\right).
$$

(13)

After substitution (13) into (11), for the loop of the $N$-th inclusion it holds

$$
\Delta \varepsilon_m = \Phi^{-1}(\sigma_1) - \Phi^{-1}(\sigma_N) + 2 \sum_{i=2}^{N} \Phi^{-1}\left(\frac{\sigma_i - \sigma_{i-1}}{2}\right).
$$

(14)

If the cyclic curve is analytically expressed by eq (6), we can put down an inverse relationship in the form

$$
\varepsilon = \varepsilon_e \pm 2 \left(\frac{|\sigma|}{2K}\right)^{\frac{1}{n}} = \frac{\sigma}{E} \pm 2 \left(\frac{|\sigma|}{2K}\right)^{\frac{1}{n}}; \text{ for } \sigma > 0,
$$

(15)

where $E$ is the elasticity modulus.

After substitution into (14), we obtain
\[
\Delta \varepsilon_m = \frac{\sigma_l}{E} \pm \left( \frac{\sigma_l}{K} \right)^n - \frac{\sigma_N}{E} \mp \left( \frac{\sigma_N}{K} \right)^n + \\
+ \frac{1}{E} \sum_{i=2}^{N} (\sigma_i - \sigma_{i-1}) \pm 2 \sum_{i=2}^{N} \left( \frac{\sigma_i - \sigma_{i-1}}{2K} \right)^n
\]

and after adaptation, finally

\[
\Delta \varepsilon_m = K \left[ \pm \frac{\sigma_l}{n} \mp \frac{\sigma_N}{n} \pm 2 \frac{n-1}{n} \sum_{i=2}^{N} \frac{\sigma_i - \sigma_{i-1}}{n} \right],
\]

where for \( \sigma_l > 0, \sigma_N > 0 \) and \( \sigma_i > \sigma_{i-1} \), it holds the upper corresponding sign, otherwise the lower one.

### 5 Conclusions

The computational method has been verified for composite materials on the basis of an aluminium foam. The structure parameters significantly effect the relative damage, and therefore the service life can be extended using the optimum system parameters control.

Hitherto, only few experimental verification results have been obtained, owing to big theoretic demands of the method and to a necessity to specify different characteristics for each case. Nevertheless, the already performed results fully confirm the presented method. Prediction of the residual life in dependence of service conditions enables to control actuators in order to maximally suppress a damage effect of an operating loading.

It can be expected that the method will be used in progressive CAD/CAE technologies, and it will create a very effective tool for an optimum design of smart structures.
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References

