Numerical analysis of states of strain and stress of material during machining with a single abrasive grain

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Abstract

In papers [4, 7], a mathematical model was derived of a single-grain machining process in conditions of centreless continuous grinding. This model comprises discrete equations of the object's motion and heat transfer together with initial and boundary conditions. In this paper, an effective solution of these equations has been presented. The manner of a partial linearization of the incremental motion equations has been shown, which allows for an application of an explicit method of integration, which makes use of an approximation with the method of central differences. As a result of this, it is possible to calculate vectors of displacements and temperature at the end of a given step on the basis of values concerning the previous step. For this method, an adequate algorithm of the solution of motion equations has been developed with the assumptions of Rayleigh's proportional damping. Its application has been developed in the system of the ANSYS finite element method, which allows for a complex time analysis of the states of displacements, strains and stresses occurring in the set (which consists of an object, an abrasive grain and a binding agent, both for spatial and flat states). Numerical computations of the process of the object's material strain have been conducted with the use of two methodologies. The first one requires an introduction of boundary conditions for displacements in the contact area determined in modeling investigation, while the second requires a proper definition of the contact zone through the introduction of finite elements of TARGET and CANTACT types, without the necessity of introducing boundary conditions. It is evident from the analyses conducted that forecasting of the physical properties of the surface layer in the centreless continuous grinding processes may occur on the basis of analyses of thermal phenomena and the state of strains in the machining zone. An analysis of machining with a model abrasive grain is the first stage towards a proper design and control of this complex technological process.

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1 Introduction

A theoretical analysis of machining in the process of centreless grinding (Fig. 1) shows factors which have influence upon the irregularity of the object's rotations or have an indirect influence upon the forces occurring in this process. The change of the object's angular velocity when it is passing through the space between grinding wheels results in changes in the machining layer. At the same time, there occurs a feedback between these changes and the forces participating in the process. This constitutes one of the difficulties in an analytical description of the process of centreless grinding with the use of a continuous method.

The first stage of procedures aimed at the description of the process of centreless grinding with the use of a transversal method is an analysis of machining with a model abrasive grain (blade).

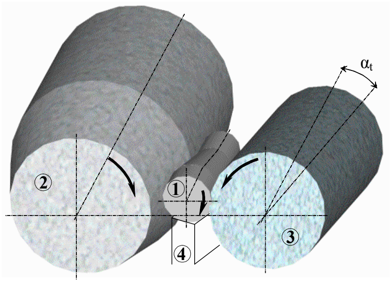


Figure 1: Scheme of cylindrical centreless grinding process by continuous method: 1 – object, 2 – grinding wheel, 3 – regulating wheel, 4 – support.

The first stage of procedures aimed at the description of the process of centreless grinding with the use of a transversal method is an analysis of machining with a model abrasive grain (blade).

The grain shape constitutes one of the crucial factors, as the grain's mechanical strength depends of it [2]. The grain shape is a notion difficult to define. Abrasive grains are characterized by irregular shapes; hence their mean equivalent or statistical size is determined. In a number of theoretical research works, a model in the form of a polyhedron, ball, pyramid or cone was assumed as the grain shape. In paper [8], a microscope analysis of abrasive grains used was conducted, on the basis of which their shape was determined as isometric. A detailed analysis of the images of shape received led to the finding that there occurs a vast majority (59 per cent) of geometric figures in the form of pentagons. The characteristics of the abrasive material allows for the construction of a hypothetical model of an abrasive micro-grain being a constituent of the model of the centre less grinding process model. It was stated that the analyzed solid which was closest to the real form is a spatial form consisting of two regular pentagons intersecting at right angle along the line connecting two opposite vertices of the pentagrams (Fig. 2). Ten triangles and one rectangle can be distinguished in such a solid.

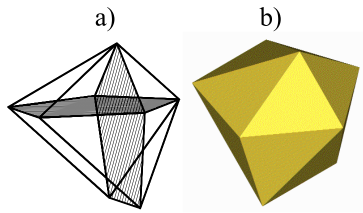


Figure 2: Hypothetical abrasive grain: a) geometric structure, b) view of grain assumed in this paper.

A mathematical model of process of machining with a single abrasive grain in the conditions of centreless continuous grinding, for thermo-elastic, thermo-visco-plastic material model [6], is composed of the following [4,7]: a discrete equation of the object's motion and a discrete equation of heat transfer together with initial and boundary conditions.

In the present paper, due to strong non-linearities of the machining process, a method of central differences has been applied, the so-called explicit method. The explicit method is the most versatile method of direct integration, as it allows for an effective analysis of non-linear systems with arbitrary forcings, with a relatively low cost of computations [1, 3]. For this method, an algorithm of solution has been developed as well as an application in ANSYS programme, which allows for a complex time analysis of the states of displacements, strains and stresses occurring in the object consisting of the object, an abrasive grain and a binding agent, both for spatial and flat states. Sample results of numerical computations have been presented.

2 Algorithm of solving the equations

2.1 Partial linearization of incremental equations of motion

Assuming that the time step increment Δt is adequately small, one can conduct a partial linearization of an incremental non-linear equation of motion [4,7], consisting in neglecting an increment of the matrix of the system's rigidity $[{}^{\tau}_{t}\Delta\mathbf{K}_{T}]$ and an increment of the vector of internal loads $\{{}^{\tau}_{t}\Delta\mathbf{F}(\cdot)\}$. An equation obtained is non-linear in relation to the vector of an increment of displacements of the points of kinematic pairs $\{{}^{\tau}_{t}\Delta\mathbf{r}\}$ and its time derivatives $\{{}^{\tau}_{t}\Delta\ddot{\mathbf{r}}\}$, $\{{}^{\tau}_{t}\Delta\dot{\mathbf{r}}\}$:

$$[{}^{t}_{t}\mathbf{M}]\{{}^{\tau}_{t}\Delta\ddot{\mathbf{r}}\}+[{}^{t}_{t}\mathbf{C}_{T}]\{{}^{\tau}_{t}\Delta\dot{\mathbf{r}}\}+[{}^{t}_{t}\mathrm{K}_{T}]\{{}^{\tau}_{t}\Delta\mathbf{r}\}=\{{}^{t}_{t}\mathbf{F}\}+\{{}^{\tau}_{t}\Delta\mathrm{R}\}+\{{}^{t}_{t}\mathrm{R}\}. \tag{1}$$

Using further the principle of incremental decomposition:

$$\{{}_{t}^{\tau}\Delta\ddot{\mathbf{r}}\}=\{{}_{t}^{\tau}\ddot{\mathbf{r}}\}-\{{}_{t}^{t}\ddot{\mathbf{r}}\},\quad \{{}_{t}^{\tau}\Delta\dot{\mathbf{r}}\}=\{{}_{t}^{\tau}\dot{\mathbf{r}}\}-\{{}_{t}^{t}\dot{\mathbf{r}}\},\ \{{}_{t}^{\tau}\Delta \mathrm{R}\}=\{{}_{t}^{\tau}\mathrm{R}\}-\{{}_{t}^{t}\mathrm{R}\}, \tag{2}$$

equation (3), at moment t, can be written in the following form:

$$[\mathbf{M}]\{{}_{t-\Delta t}{}^{t}\ddot{\mathbf{r}}\}+[\mathbf{C}_{\mathrm{T}}]\{{}_{t-\Delta t}{}^{t}\dot{\mathbf{r}}\}+[{}_{t-\Delta t}^{t-\Delta t}\mathrm{K}_{\mathrm{T}}]\{{}_{t-\Delta t}{}^{t}\Delta\mathbf{r}\}+\{{}_{t-\Delta t}^{t-\Delta t}\mathbf{F}_{\mathrm{T}}\}=\{{}_{t-\Delta t}{}^{t}\mathrm{R}\} \tag{3}$$

where the following has been defined:

$$\{{}_{t-\Delta t}^{t-\Delta t}\mathbf{F}_{\mathrm{T}}\}=[\mathbf{M}]\{{}_{t-\Delta t}^{t-\Delta t}\ddot{\mathbf{r}}\}+[\mathbf{C}_{\mathrm{T}}]\{{}_{t-\Delta t}^{t-\Delta t}\dot{\mathbf{r}}\}-\{{}_{t-\Delta t}^{t-\Delta t}\mathbf{F}\}. \tag{4}$$

2.2 Solution of the problem of dynamics with the use of the explicit method

Because vector $\{{}_{t-\Delta t}^{t-\Delta t}\mathbf{F}_{\mathrm{T}}\}$ can be represented as [3]

$$\{{}_{t-\Delta t}{}^{t}\mathbf{F}_{\mathrm{T}}\}=[{}_{t-\Delta t}^{t-\Delta t}\mathrm{K}_{\mathrm{T}}]\{{}_{t-\Delta t}{}^{t}\Delta\mathbf{r}\}+\{{}_{t-\Delta t}^{t-\Delta t}\mathbf{F}_{\mathrm{T}}\}, \tag{5}$$

equation (5) can finally be written in the following form:

$$[\mathbf{M}]\{{}^{t}\ddot{\mathbf{r}}\}+[\mathbf{C}_{\mathrm{T}}]\{{}^{t}\dot{\mathbf{r}}\}=\{{}^{t}\mathbf{R}\}-\{{}^{t}\mathbf{F}_{\mathrm{T}}\}. \tag{6}$$

In the explicit method a differential approximation of derivatives of partial displacements is accepted in accordance with equations [1}:

$$\{{}^{t}\dot{\mathbf{r}}\}=\frac{1}{2\Delta t}(\{{}^{t+\Delta t}\mathbf{r}\}-\{{}^{t-\Delta t}\mathbf{r}\}),\quad \{{}^{t}\ddot{\mathbf{r}}\}=\frac{1}{\Delta t^{2}}(\{{}^{t+\Delta t}\mathbf{r}\}-2\{{}^{t}\mathbf{r}\}+\{{}^{t-\Delta t}\mathbf{r}\}). \tag{7}$$

Introducing to equation (6) dependencies (7), the following system of equations is obtained:

$$\begin{aligned}&\left(\frac{1}{(\Delta t)^{2}}[\mathbf{M}]+\frac{1}{2\Delta t}[\mathbf{C}_{\mathrm{T}}]\right)\{{}^{t+\Delta t}\mathbf{r}\}=\\&=\{{}^{t}\mathbf{R}\}-\{{}^{t}\mathbf{F}_{\mathrm{T}}\}+\frac{2}{(\Delta t)^{2}}[\mathbf{M}]\{{}^{t}\mathbf{r}\}+\left(\frac{-1}{(\Delta t)^{2}}[\mathbf{M}]+\frac{1}{2\Delta t}[\mathbf{C}_{\mathrm{T}}]\right)\{{}^{t-\Delta t}\mathbf{r}\},\end{aligned} \tag{8}$$

from which, while knowing vectors $\{{}^{t}\mathbf{r}\}$ and $\{{}^{t-\Delta t}\mathbf{r}\}$, displacements $\{{}^{t+\Delta t}\mathbf{r}\}$ are calculated at the moment t+Δt.

Determining as $[\widetilde{\mathbf{M}}]$ the effective matrix of masses and $\{{}^{t}\widetilde{\mathbf{Q}}\}$ effective vector of loads:

$$[\widetilde{\mathbf{M}}]=\frac{1}{(\Delta t)^{2}}[\mathbf{M}]+\frac{1}{2\Delta t}[\mathbf{C}_{\mathrm{T}}], \tag{9}$$

$$\{{}^{t}\widetilde{\mathbf{Q}}\} = \{{}^{t}\mathbf{R}\} - \{{}^{t}\mathbf{F}_{T}\} + \frac{2}{(\Delta t)^{2}}[\mathbf{M}]\{{}^{t}\mathbf{r}\} + \left(\frac{-1}{(\Delta t)^{2}}[\mathbf{M}] + \frac{1}{2\Delta t}[\mathbf{C}_{T}]\right)\{{}^{t-\Delta t}\mathbf{r}\}, \quad (10)$$

then equation (8) accepts the following simple form:

$$[\widetilde{\mathbf{M}}]\{{}^{t+\Delta t}\mathbf{r}\} = \{{}^{t}\widetilde{\mathbf{Q}}\}. \quad (11)$$

If matrices of masses [**M**] and damping [$\mathbf{C}_T$] are diagonal then it is possible to disengage the system of equations and an analysis on the level of the element is possible in accordance with the following dependence:

$${}^{t+\Delta t}r_{i} = \frac{1}{p_{ii}}\,{}^{t}\widetilde{Q}_{i}, \quad (12)$$

where:

$$p_{ii} = \frac{1}{(\Delta t)^{2}}M_{ii} + \frac{1}{2\Delta t}\mathbf{C}_{Tii} > 0, \quad (13)$$

$${}^{t}\widetilde{Q}_{i} = {}^{t}R_{i} - {}^{t}F_{Ti} + \frac{2}{(\Delta t)^{2}}M_{ii}\,{}^{t}r_{i} - \left(\frac{1}{(\Delta t)^{2}}M_{ii} - \frac{1}{2\Delta t}C_{Tii}\right){}^{t-\Delta t}r_{i}. \quad (14)$$

are the i-th element of the diagonal effective matrix of masses $[\widetilde{\mathbf{M}}]$ and the i-th component of the effective vector of system loads $\{{}^{t}\widetilde{\mathbf{Q}}\}$ respectively.

At the same time, during the start of computations for $t=t_0$ knowledge of vector $\{{}^{t_0-\Delta t}\mathbf{r}\}$ is necessary. For this purpose, while knowing vector $\{{}^{t_0}\dot{\mathbf{r}}\}$, first the vector of accelerations $\{{}^{t_0}\ddot{\mathbf{r}}\}$ is calculated from equation (8) rearranged into the following form:

$$\{{}^{t_0}\ddot{\mathbf{r}}\} = [\mathbf{M}]^{-1}\left(\{{}^{t_0}\mathbf{R}\} - \{{}^{t_0}\mathbf{F}_{T}\} - [\mathbf{C}_{T}]\{{}^{t_0}\dot{\mathbf{r}}\}\right), \quad (15)$$

the desired vector $\{{}^{t_0-\Delta t}\mathbf{r}\}$ is calculated from equations (7) rearranged into the following form:

$$\{{}^{t_0-\Delta t}\mathbf{r}\} = \{{}^{t_0}\mathbf{r}\} - \Delta t\{{}^{t_0}\dot{\mathbf{r}}\} + \frac{(\Delta t)^{2}}{2}\{{}^{t_0}\ddot{\mathbf{r}}\}. \quad (16)$$

3 Results of numerical analyses of the process of machining with a single grain

Numerical calculations of the process of machining with a pentagonal representative grain were conducted for three depths of machining: g = 0.005 mm, 0.01 mm and 0.0125 mm. Numerical calculations of the process of straining the object's material were conducted with the use of two methodologies. The first one requires an introduction of boundary conditions for displacements determined in model research in the contact area, while the second one requires a proper definition of the contact zone through an introduction of finite elements of TARGET and CANTACT type, without the necessity to introduce boundary conditions. A vertical displacement of the grain was diversified and for individual machining depths it was 0.51475 mm, 0.5375 mm and 1.01125 mm. The temperature of the material machined was changes, as well (T = 20, 200, 400, 600, 800, 1000 and 1200 °C).

3.1 Modeling investigation

Modeling investigation was conducted in order to determine the course of straining of the object's surface layer and in order to determine boundary conditions for displacements necessary for numerical analyses of the process of machining with a single abrasive grain. Properly prepared modeling clay was used in the modeling investigation. The sample of the model had a rectangular shape with the following dimensions: length L=148 mm, height h=76 mm, width S=10 mm. It is evident from these that deformations occur on a very small depth. In the scale of a single element, the intensity of viscous and plastic strains is very large and is $\varepsilon_i^{(VP)} = 3 \div 5$. Such large strains are, apart from friction, the reason of an accumulation of a high temperature, which occurs in the surface layer of an object subject to centreless grinding.

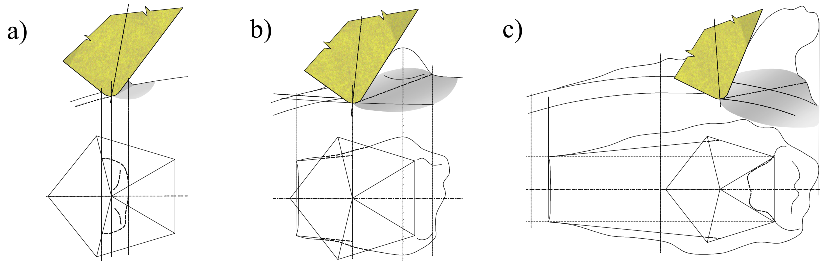


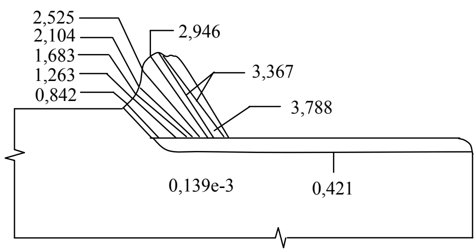
Figure 3: Chip creation process in continuous centreless grinding process [7]: a) phase I – grain sinking, b) phase II – start of grinding and chip creation, c) phase III – grain coming out.

The modeling investigation conducted allowed for the observation of the size of the zone of plastic strains and of the mechanism of slip and shearing occurring during the chip creation process. Fig. 3 presents the process of chip creation during the machining under discussion. Three phases of the abrasive grain work are distinguished: I – the phase of cutting the grain into the material, during which there initially occur elastic strains – which are transformed into plastic strains, as well as friction between the grain and the material, II – the phase of further sinking of the grain into the material to the depth which exceeds the minimum depth a_{min}, at which there occurs machining and creation of a chip as a result of plastic flow, the zone of plastic strain is developed and there occurs an agreed line of a slip, which is determined by the angle of slip, III – the phase of a distinct chip creation (upsetting) and pushing it out in as a result of the slip and shearing.

A characteristic element of centreless grinding is an additional participation of friction in each of these phases, which makes such a mechanism of chip creation different from conventional machining. Higher temperature during machining is a consequence of this. From this, a different behavior of the material is evident, which has a higher temperature as a result of heating the previous grains by work. The problem remains with the determination of the minimum machining depth a_{min} for materials machined. An application of a representative abrasive grain in the research into vision-plasticity allowed for a development of models of grinding with a single grain, which were used in numerical analyses.

3.2 Calculations for assigned boundary conditions

At this point, selected results of a numerical analysis of the states of displacement, strains and stresses in the surface layer were presented during machining with a single grain in the assigned boundary conditions in the contact area for displacements designated in modeling investigations. The numerical analysis was conducted with the help of an application developed in ANSYS program. A list of sample results of calculations for machining with depth g=0.005 mm is presented in Fig. 4.



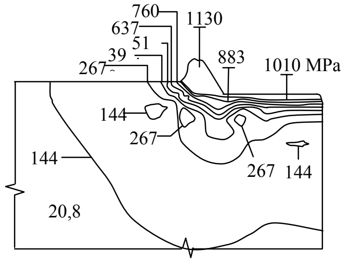


Figure 4: Intensities of strain (a) and stress (b) for g = 0.005 mm, at T = 20°C.

Fig. 5 presents a comparison of the results of numerical calculations of maximum intensities of stresses $\sigma_{i\,max} = f(T)$ in the surface layer of the object ground with the values of yield stress $\sigma_y = f(T)$ [5] steel NC4 experimentally determined. It is evident from it that the values of the intensities of stresses $\sigma_{i\,max}$ can be forecast on the basis of curve σ_y.

3.3 Calculations at the partial knowledge of boundary conditions

The contact area was modeled with the help of finite elements of CONTACT and TARGET types. Element CONTA172 has the same property of geometry as the surface of a solid body, with which it is associated.

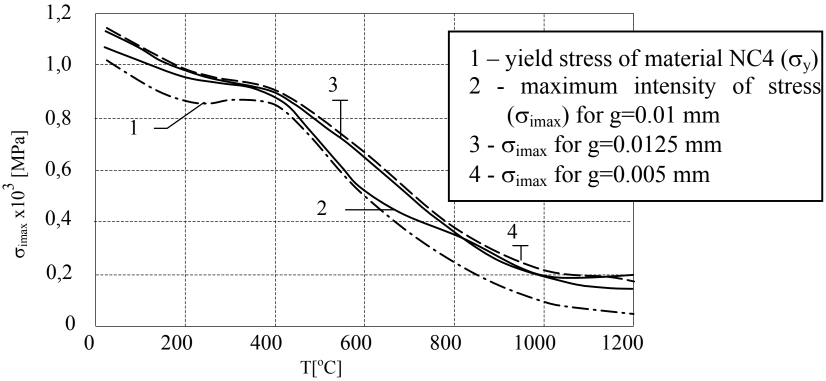


Figure 5: Comparison of maximum intensity of stress in the surface layer of the object machined with various depths with yield stress of the material depending of the temperature.

The results of pressures for the case of machining steel NC4 with a model pentagonal grain with the depths of 0.005 mm and 0.01 mm is presented in Fig. 6, however the state of intensities of strain and stress in Fig. 7.

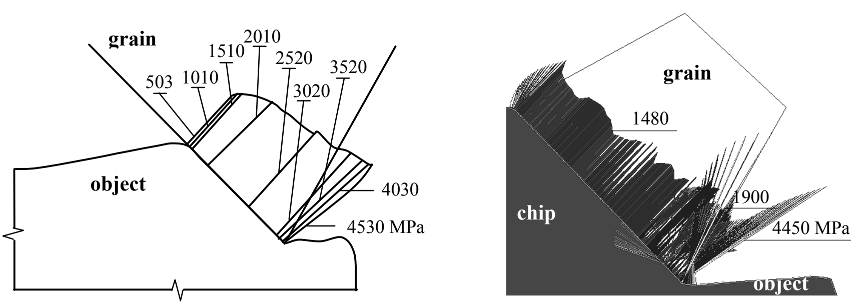


Figure 6: Distribution of pressures on grain for g = 0.005 mm (a) and g = 0.01 mm (b), at T = 20°C.

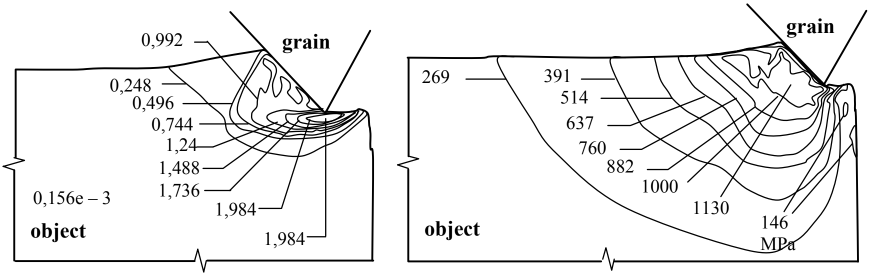


Figure 7: Intensities of strain (a) and stress (b) for g = 0.005 mm (a) and g = 0.01 mm (b) at T = 20°C.

It was found that the greatest pressures occur on the vertex of the grain and they decrease along the plane of rubbing, while for machining depths analyzed of 0.005 mm and 0.01 mm close values of maximum pressures were obtained: $0.443 \cdot 10^{10}$ MPa and $0.445 \cdot 10^{10}$ MPa respectively. An increase of the depth of machining from 0.005 mm to 0.01 mm did not result in any substantial changes in stresses. In all the cases analyzed, a zone of the adhesion of the material machined to the abrasive grain was observed, while the maximum stresses occur not directly at the grain but in the chip creation zone. The maximum plastic strains $\varepsilon_i^{(VP)} = 3{,}292$ occur in the object's surface layer being formed, at its surface.

4 Conclusions

1. The results of numerical calculations prove the fact that is it possible to conduct a correct analysis of the deformation process of the surface layer in the machining zone during centreless transversal grinding, both on the basis of boundary conditions assigned and without the knowledge of boundary conditions in the contact area. For the first method, it was necessary to carry out visual-plastic investigations. The limitations of defining boundary conditions imposed in the computational program are laborious, however. Another disadvantage of this method is also a lack of possibility to determine stresses in the contact area. These inconveniences are eliminated by the other method of calculations, which consists in an introduction of additional contact finite elements (CONTACT and TARGET). An additional advantage of using the second method is making the distribution of pressures in the contact area, which is not possible through experiments.
2. On the basis of the results obtained, it was found that there was a substantial heterogeneity of the displacement and deformation of the material machined, while the range of the fields of displacements and deformations depends of the shape of the grain accepted. It was observed that together with an increase of the rounding radius of the abrasive grain's vertex, there is an increase of the relative depth of the deposition of these fields. This is of a key importance in the selection of grinding wheels and the control of the properties of the surface layer formed in the process of centreless transversal grinding. This gives a possibility of an effective interference with designing the technological process and of an adaptation of the technological quality to proper operating conditions of the object.
3. Both in modeling investigations and simulative ones, conducted in accordance with two methodologies, it was found that the maximum intensity of the strain of the material may be as high as $\varepsilon_i^{(VP)} = 5$, which exceeds greatly deformations quoted in literature. Also, the value of stresses can be as high as $1.460 \cdot 10^{11}$ MPa, while the value of pressures on the tool point can reach even $0.791 \cdot 10^{11}$ MPa and it decreases while the material heats. In the temperature of the material of $T = 1000$ °C, stresses are only $1.9 \cdot 10^{8}$ MPa. A substantial influence of the

friction coefficient was shown on both the value of pressures in the contact zone and on the intensities of stresses.
4. The value of maximum stresses in the surface area, obtained with the use of the methodology applied has the same pattern as the yield stress of material and is greater from it by ca. 100 MPa. Therefore, the values of stresses in the surface layer formed can be forecast on the basis of the values of yield stresses.

An implementation of the model developed and the application developed in ANSYS system for the design of the centreless grinding process will allow for the solution of important problems, such as forecasting of the state of deformations and stresses in the surface area of objects subject to centreless grinding, as well as for an improvement of the product quality with an increased efficiency of machining.

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