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Structural analysis of Hagia Sophia: a historical perspective

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ABSTRACT

An ongoing structural study by a group of American and Turkish engineers is aimed at deriving a better understanding of the structure and determining the current earthquake worthiness of Justinian's Hagia Sophia. This paper discusses possible design antecedents and aspects of the building's structural history as well as the creation of numerical models of its primary structure that account for both short- and long-term, linear and non-linear material behavior.

INTRODUCTION

Begun in 532 as the principal church of the Byzantine Empire (and converted to a royal mosque after the fall of the Empire in 1453), Hagia Sophia in Istanbul held the record as the world's largest domed building for some 800 years. For the dual role that the building was to assume in both ecclesiastical and imperial liturgies, the architects, Anthemius of Tralles and Isidorus of Miletus, combined a traditional longitudinal basilican plan (a large rectangular hall having a high central space flanked by lower side aisles) with an immense central dome.

Given the close correspondence in scale between the original dome of Hagia Sophia and that of the early second-century Roman Pantheon, it is likely that the Pantheon provided the principal structural model for this translation of Roman concrete into Byzantine, largely-brick construction. While archeological evidence for the original dome that collapsed in 558 is unavailable, sixty-century descriptions indicate that the dome interior was likely profiled from the same spherical surface as the pendentives (thus creating a "pendentive dome"). Such a dome would have had an interior radius of 23 meters, making it some six meters lower than the present dome, which agrees with the height difference between the first and second dome cited by ancient chroniclers [1]. No further direct evidence for a reconstruction of the first dome exists, but a comparison of the cross-sections of the Hagia Sophia (taken through diagonal piers and the Pantheon (figure 1) suggests that Justinian's builders were referring

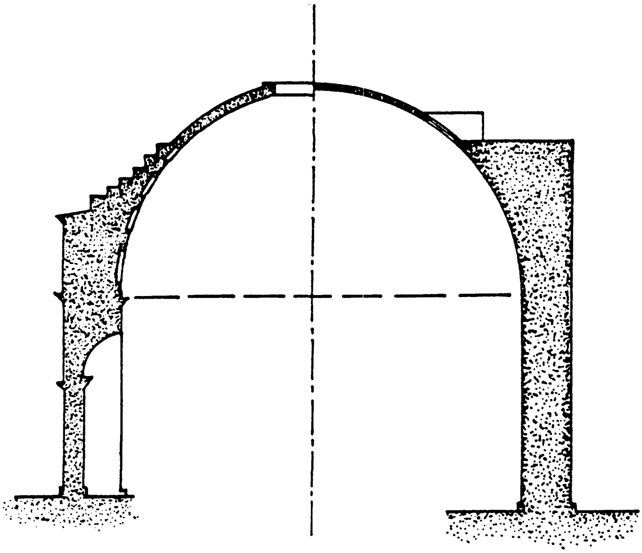

Figure 1. Comparative sections: Pantheon (left) and Hagia Sophia (partial section along diagonal).

consciously to the Pantheon in their design. The lower portions of both domes are similarly massive and provide support to lighter "shells" above, although in the Hagia Sophia this massiveness is concentrated in regions above the four piers. Both "shells" also subtend angles of about 90 degrees and are of almost the same span. Yet where the dome of the Pantheon rests on continuous, massive niched walls, four enormous arches and a like number of pendentives direct the weight of Hagia Sophia's superstructure to four great supporting piers, allowing the tympanum walls below the arches to be pierced with windows that light the central expanse of the church (figure 2). For the architectural antecedents that may have led Anthemius and Isidorus to realize these large glazed surfaces in combination with a great dome, one must first also look back to other buildings in Rome.

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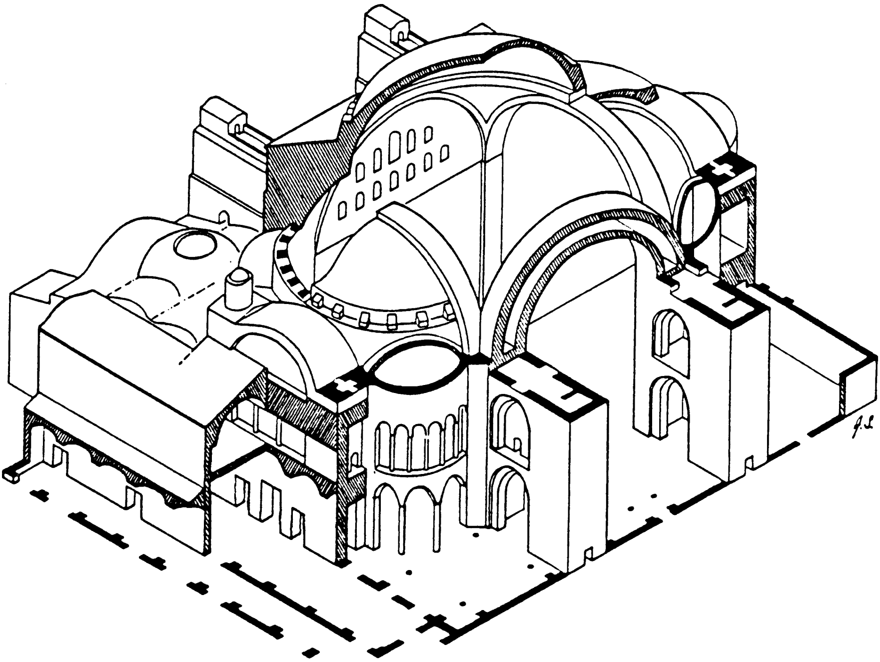

Figure 2. Analytical drawing of the Hagia Sophia structure.

The fourth-century so called Temple of Minerva Medica (also known as the Pavilion in the Licinian Gardens) best exemplifies the later progression of large-domed Roman buildings to more skeletal construction. The supporting structure for the Minerva Medica dome has the form of a decagon (25 meters across its sides) with tall windows incorporated in the walls set above the nine projecting apses and the entrance (figure 3). Apparently the supporting piers exhibited structural distress early on because they were reinforced with additional, projecting masonry even before the building was completed. The outside profile of the nearly semi-circular dome, whose crown rose 29 meters above the floor, resembled that of the Pantheon. Yet the use of brick ribs in this dome represents a later advance in construction. Rather than fully erecting the ribs in a skeletal, first stage of assembly, they were likely raised together with the concrete infil of the dome; and at various stages of the construction process, the unfinished truncated dome was capped by continuous rings of large tiles. This type of ribbing does not seem to have been considered as a structural element by the Roman builders since similar ribs observed in contemporaneous buildings usually end abruptly before reaching the crown [2].

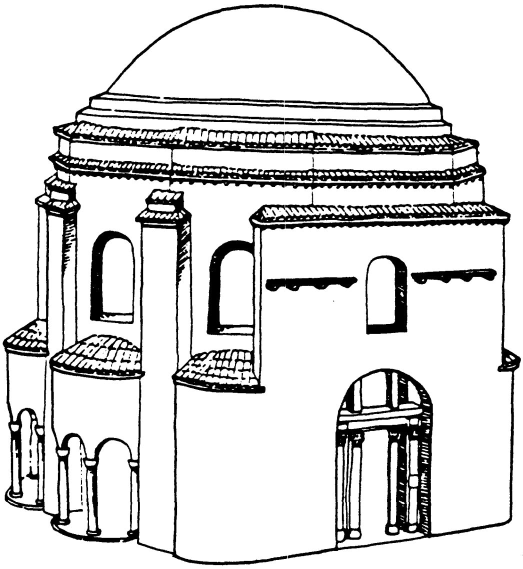

Figure 3. Temple of Minerva Medica (Pavilion in the Licinian Gardens); reconstruction (after MacDonald).

Despite its much reduced scale, the church of SS. Sergius and Bacchus in Istanbul, begun ca. 527, may afford an almost contemporaneous transition between Minerva Medica and Hagia Sophia. Recalling the tructure of the Roman pavilion, support for the 16-sided "pumpkin dome" of Sergius and Bacchus is provided by eight piers, 16 meters across its sides. Unlike Minerva Medica, however, the church is essentially rectangular in plan, resembling more the exterior form of Hagia Sophia (figure 4). The dome, rising 22 meters above the floor, is pierced with eight diminutive windows that also offer a foretaste of the fenestration pursued in Hagia Sophia.

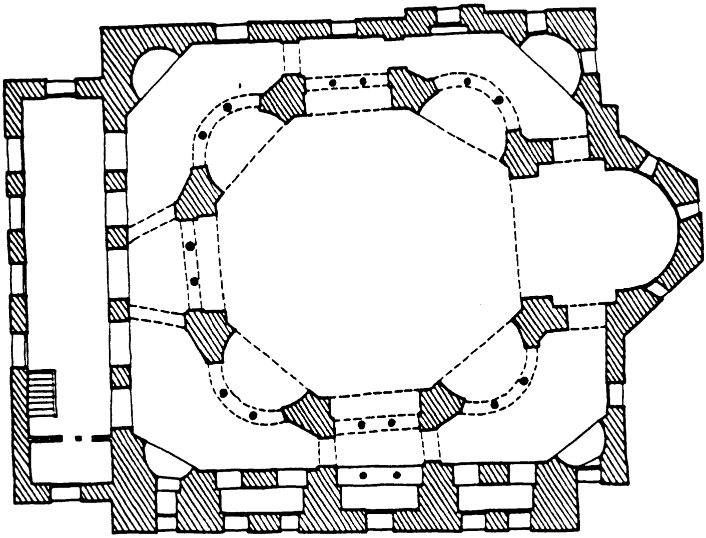

Figure 4. Church of SS. Sergius and Bacchus in Istanbul, begun ca. 527: plan.

As it is a much larger building than the relatively diminutive SS. Sergius and Bacchus, the recently rediscovered and excavated palace-church of St. Polyeuktos in Istanbul may prove to be the most important precursor of Hagia Sophia. Securely dated to 524-537, some ten years before the construction of Justinian's great church, surviving substructures suggest a square-planned central-domed building whose linear dimensions are two-thirds those of the Hagia-Sophia [3]. Even though the form of the superstructure of St. Polyeuktos illustrated in figure 5 is speculative, the extensive, excavated remains of the church may serve to reveal construction details present also in Hagia Sophia.

Intended to serve as the most visible symbol of the emperor's prestige in the imperial capital, and following on the destruction of its predecessor by a mob, it was deemed necessary that the great church be completed as swiftly as possible. Building proceeded in more or less horizontal layers until the erection, in ca. mid 535, of the main arches, 31 meters in span, and springing some 25 meters above the floor, to support the dome. Flying centering (as illustrated in figure 6) was probably used for assembly of the arches, and in all likelihood this centering would not have been

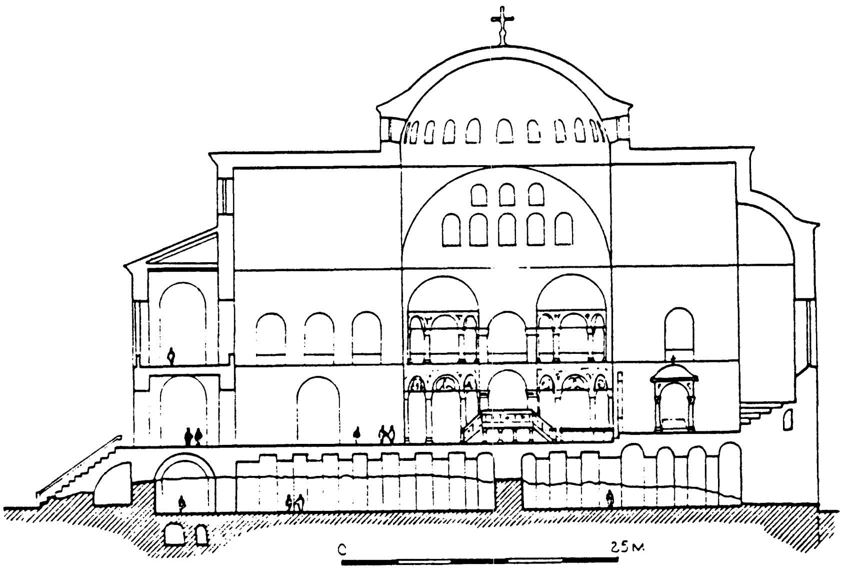

Figure 5. Reconstruction of St. Polyeuktos: sectional elevation (Harrison).

adequately tied to prevent enormous horizontal forces from impinging upon the upper portions of the main piers which then proceeded to tilt outward (the average, outward defection of the piers at the level of the springing now measures 45 cm). Before continuing with the church's construction, the exterior pier buttresses were reinforced and enlarged to their present height. The piers must have then seemed secure because the dome was raised in time to allow the vast building project to be completed in 537. Nonetheless, this first central dome fell in 558 after being subjected to two major earthquakes: the first of these in August 553, and the second in December 557. A second dome having a higher profile than its predecessor was then erected in 558-562. Despite two partial collapses after earthquakes in the tenth century, and again in the fourteenth, the general form of the second dome today remains essentially unchanged from that of 562. But structural repairs associated with these incidents, as well as other adversities, have involved the placement of additional buttressing around the entire structure.

The present study is aimed at deriving a better understanding of the structural history of Hagia Sophia over its one-and-a-half-millennium

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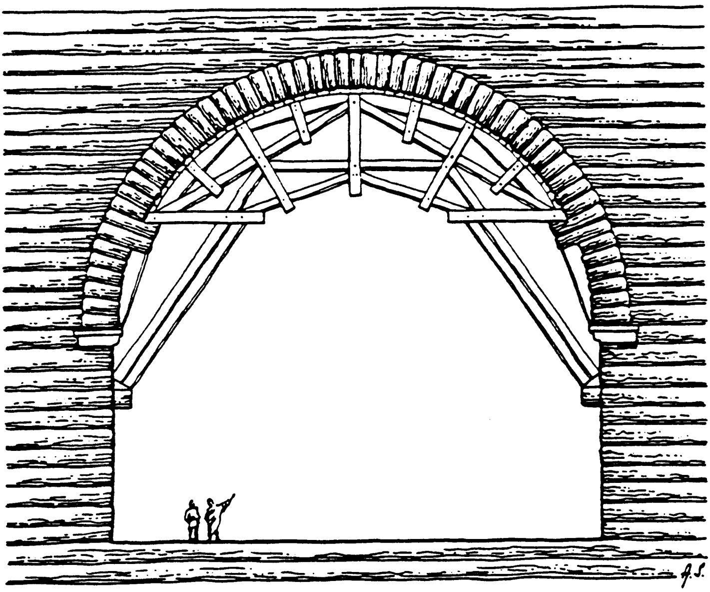

Figure 6. "Flying centering" used in masonry arch construction.

life, including the strategies employed for its design and construction, and to determine the monument's current earthquake worthiness (and if necessary, to recommend possible structural amelioration). To accomplish these ends, concurrent efforts are being undertaken at Princeton and at the building site which include: 1) observation of the building fabric, especially deformation, fracture, and distortion emanating from environmental loadings (including earthquake) and the structural interventions associated with the numerous campaigns of restoration, 2) creation of numerical models to account for both short- and long-term non-linear material behavior, including the consequences of cracking and effects of component deformation during the initial sequence of construction as well as subsequent structural modification, 3) determination, from physical and chemical tests, of the properties of the building materials, particularly the time-dependent behavior of early mortars, 4) monitoring of measurements from accelerometers placed on the actual building structure under the action of vibrations produced by earthquakes, and 5) determination of the form and magnitude of the likely dynamic loadings (e.g., strong ground motions) to which Hagia Sophia will be subject in future years. Progress in the first two areas is reported herein, and progress in the other areas is the subject of papers following.

OBSERVATION

A fount of material on the building fabric is at hand through the Dumbarton Oaks publication of the comprehensive survey completed by Robert Van Nice in the 1970s [4]. Van Nice also compiled an extensive archive that was intended to form the basis of a comprehensive text. Even though the text was never produced, the archive has recently been cataloged by the Dumbarton Oaks Library and is accessible to scholars. Additional information concerning the building's structural history is avilable from the recent ext of Rowland Mainstone [5] and from the proceedings of the international colloquim held at Princeton University in 1990 [6]. Most observations of the building structure by earlier investigators have been corroborated by the writers in their visits to the site. Probably the most important structural detail still to be ascertained concerns the actual foundation support conditions for the main piers. An exploration, with appropriate instrumentation, beneath the building floor is planned for the coming year.

MODELING

Two parallel types of numerical models are being formulated. The first, based on SAP 90 software, provides a purely linear elastic representation of the structure. The second, using the program FENDAC being developed by Colby Swan at Princeton, can account for the non-linear elasto-plastic behavior of masonry. Much of the effort to date has concentrated on the linear, SAP 90 models, a few of which are described in the following.

Linear elastic models (including elastic models rendered non-linear by allowing cracking or weakening at specified tensile stress levels), although insensitive to values of elastic moduli, can provide essential information about overall stress distributions where the prototype is essentially composed of a single material (for example, to highlight regions of tension where cracking is likely to occur [7]). The structure of Hagia Sophia, on the other hand, incorporates at least three major classes of materials: stone, brick, and mortar, the later containing brick dust and fragments that impart to its pozzolanic characteristics, with a relatively long curing time [8]. In this case, criteria for modeling integrity are based on matching deformations: 1) predicted static deformations should agree in both form and magnitude with those observed in the prototype; and 2) natural frequencies and mode shapes computed by the models should match those determined from the on-site measurements. Because of the long curing time of the Byzantine mortar, however, both criteria cannot be simultaneously satisfied using the same material characteristics.

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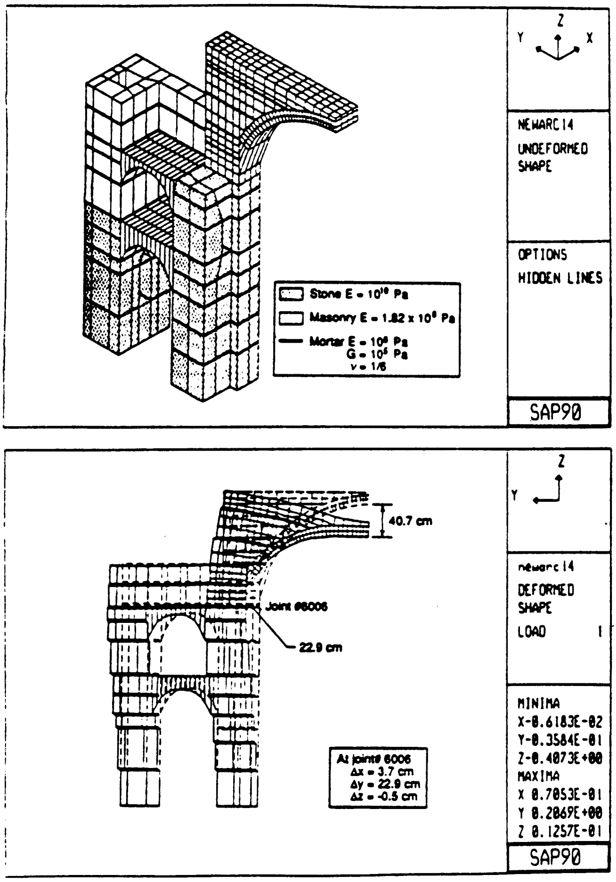

Figure 7. NEWARC 14: Partial finite element model of the Hagia Sophia structure under dead-weight loading.

One example is the model designated as NEWARC 14 (figure 7). Mechanical properties of the constituent model materials are here determined from an inverse analysis that focused on the northeast main pier – whose outward deflection at the point of springing was estimated to be 18 cm during initial construction, just prior to erection of the first great dome [9] – the associated pier buttress, and adjacent great arches. The main piers are formed of stone, either limestone or a local granite, of up to about a meter in length and 45 cm thick. Mortar layers between stones are relatively thin, probably no more than several centimeters.

For modeling, the stones are represented by elements whose thickness averages about 3 meters, interspersed with 25 cm of mortar. The pier buttress is assumed to incorporate similar stone and mortar layers up to the level of the first connecting arch, above which it is composed of brick masonry containing a large cavity for the existing stairwell. As shown in figure 8, good results were achieved from the model by adopting the indicated values of Poisson's ratio and elastic and shear moduli.

Using these properties for early material behavior, a full model of Hagia Sophia was then constructed in three stages, allowing those portions of the structure in each stage to deform and weaken (at prescribed levels of tension) before the portions of subsequent stages were added. Stages of the modeling procedure used to take into account the cracking and weakening of masonry in regions experiencing appreciable tension are illustrated in figures 8 & 9 that display stress levels in a model of the original (pendentive-domed) Hagia Sophia structure. The uncracked model, figure 8, indicates fairly extensive regions where tension exceeds 1.4 MPa. Figure 9 demonstrates the redistribution of stress after several iterations where the elastic modulus in highly-stressed tensile regions has been reduced to 10^9 Pa, reflecting local disintegration. The extent of the regions subject to high tensile stress has been reduced, and accompanying this, the maximum compressive stress is found to be higher than in the uncracked model. Although this model accounts for geometric non-linearity, it remains elastic. Nevertheless, it helps to reveal some of the characteristic behavior of the prototype that will influence future non-linear modeling.

CONCLUSION

Further work on the project remains to be undertaken, yet some new perceptions about the structure of this magnificent monument are already coming light. The first concerns the role of the slow-curing pozzolanic mortar during the initial construction process. The mortar allowed the development of early, large deformations, but its inherent plasticity would also have helped reduce possible cracking. A second insight derives from the predisposition in the numerical models of the east and west great arches that provide support to the central dome to warp out-of-plane under gravity loading. With additional out-of-plane motion caused by an earthquake (in this regard note also that the lowest vibration mode

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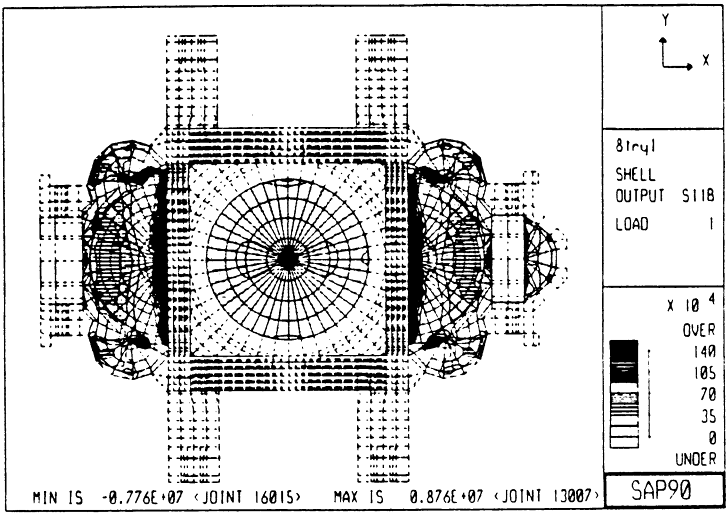

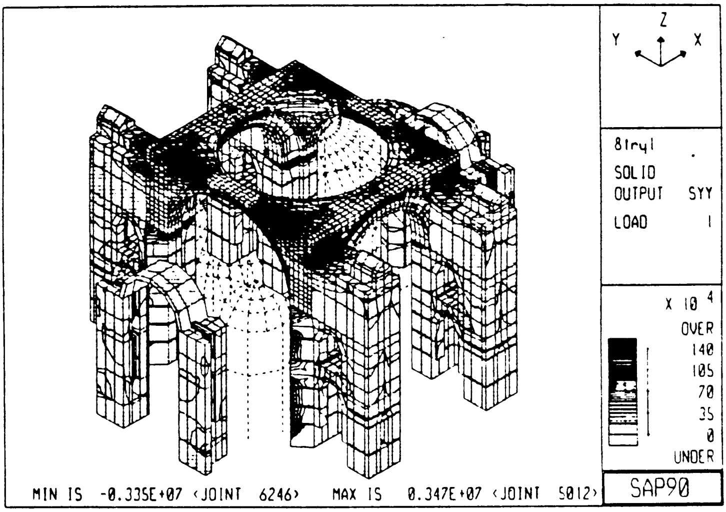

Figure 8. Stress distribution in portion of an uncracked finite element model of the original Hagia Sophia structure.

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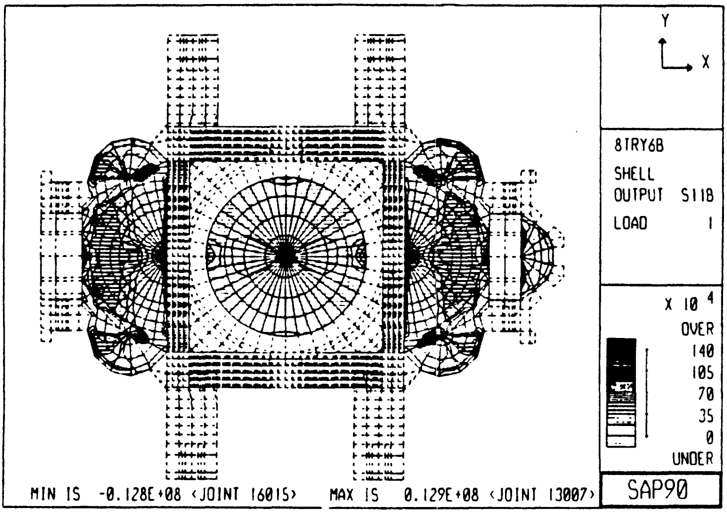

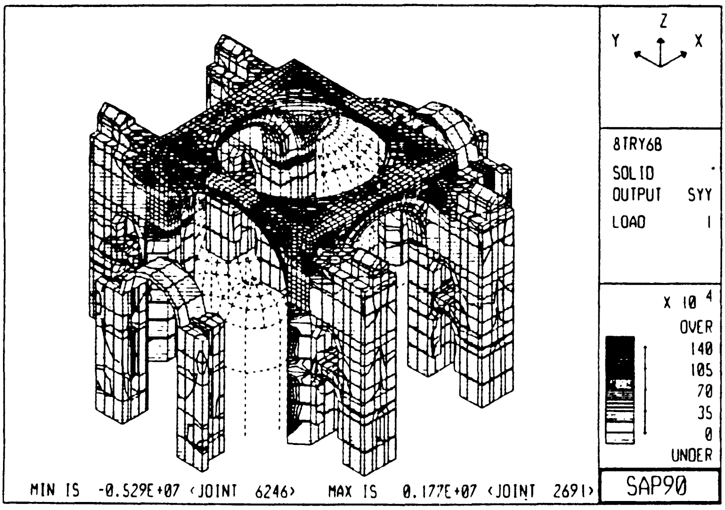

Figure 9. Stress distribution in portion of finite element model of the original Hagia Sophia structure with elastic modulus reduced in highly-stressed tension regions.

is east-west [10]), the basis for the collapse of adjacent portions of the central dome (in the east and west) at different times throughout the building's history begins to be clearer.

Perhaps most important for historical interpretation of the Hagia Sophia structure is our finding that the changing of the first to the second dome configuration had only small effect on relieving the total outward thrusts on the main piers. This new understanding gross counter to almost every modern historical explication of the second dome form (e.g., reference 11).

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