Analytical formulation of fragilities
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Abstract

In this paper, formulations for developing fragility curves for reinforced concrete frame structures are presented. Fragility curves describe the conditional probabilities of a structure being in different damage states at given levels of earthquake ground motion. The ground motion, in this paper, is characterized by spectral acceleration. Gaussian stationary models with modulating functions are used for the generation of earthquake time histories. Identification of the different damage states is based on the modified Park and Ang damage index. The modified Takeda model is used to describe the hysteretic behavior of the components of a structure in nonlinear time domain analysis. Example fragility curves are developed for low rise reinforced concrete frames.

1 Introduction

Reliable damage estimation and rehabilitation decisions require sufficient information on the degree of structural damage. Relationships between earthquake ground motion severity and structural damage along with seismic site hazard analysis can be used to predict structural damage. The motion-damage relationships obtained in the form of probability distributions of damage at specified ground motion intensities are usually expressed by means of fragility curves and damage probability matrices. Currently available motion-damage relationships for different types of buildings are in the form of damage probability matrices (ATC-13). These matrices are based on expert opinion since actual damage data are not available to develop these functions empirically.
Fragility curves and damage probability matrices describe the conditional probabilities of sustaining different degrees of damage at given levels of ground motion. Thus, the development of fragility curves and damage probability matrices requires the characterization of the ground motion and the identification of the different degrees of structural damage. Earthquake ground motion amplitude, frequency content, and strong motion duration are some of the characteristics important for determining structural response and damage. In this paper, the spectral acceleration is used to characterize the ground motion.

Monte Carlo simulation techniques are used for evaluating the fragility curves. The Latin Hypercube sampling scheme is used to reduce the number of simulation cycles. At a given ground motion parameter, an ensemble of ground motions is required for evaluating the conditional probabilities of the different degrees of damage. Due to the paucity of recorded ground motions for the same ground motion parameter, artificial ground motions are generated. Gaussian stationary models with modulating functions are used for ground motion simulation. There are several damage models which characterize the damage to reinforced concrete structures. In this paper, the modified Park and Ang damage index is used. Example fragility curves are developed for low-rise reinforced concrete frames.

2 Ground Motion Parameters

It is difficult to determine a single parameter that best characterizes the ground motion. Recorded time histories, even at the same site, show variations in details. Earthquake ground motion amplitude, frequency content, strong motion duration and the number of peaks in the time history above a certain amplitude are some of the characteristics important for determining structural response and damage. In this paper, spectral acceleration ($S_a$) is used to characterize the ground motion.

The spectral acceleration provides an estimate of the maximum elastic force that can be developed in an elastic SDOF system. In this paper, the average spectral acceleration ordinate in the period range 0.1-0.45 seconds is another parameter used to characterize ground motion for low-rise (1-3 stories) reinforced concrete frames. Spectral acceleration is a ground motion parameter which includes some of the building characteristics in its definition.

There are several measures of strong-motion duration of an earthquake. Trifunac and Brady's strong motion duration, defined as the time interval to accumulate 90 percent of the energy of the earthquake, is used in this paper.

3 Damage Measures for Reinforced Concrete Structures

There are many damage models which characterize the state of reinforced concrete structures after earthquakes. Most of the definitions of damage
consider damage to individual elements and are based on ductility ratio or dissipated energy (Bannon et al.2).

Damage indices for reinforced concrete structures usually employ the concepts of ductility and dissipated energy. Examples of these damage indices include Park and Ang10, Chung, Meyer and Shinozuka9, and DiPasquale and Cakmak4. The modified form of the Park and Ang9 damage index (Kunnath et al.7) is expressed by the following equation:

\[
D_L = \frac{\theta_m - \theta_r}{\theta_u - \theta_r} + \frac{\beta}{M_y \theta_u} E_T
\]

where

- \(\theta_m\) = maximum rotation attained during load history
- \(\theta_u\) = ultimate rotation capacity of section
- \(\theta_r\) = recoverable rotation at unloading
- \(\beta\) = strength degrading parameter
- \(M_y\) = yield moment of section
- \(E_T\) = dissipated hysteretic energy

The first term in the expression for the damage index represents the damage due to maximum deformation experienced during cyclic loading, and the second term represents the damage due to cumulative hysteretic energy dissipation. The damage index, \(D_L\), is zero when there is no damage and is 1.0 for collapse.

The global damage index is defined as the weighted average of the local damage indices of each element. The weighting function for each element is proportional to the energy dissipated in the element. The global damage index is given by the following equation:

\[
D_T = \sum_{i} \lambda_i D_{iL}
\]

where

- \(\lambda_i = \frac{E_i}{\sum E_i}\)
- \(E_i\) = energy dissipated at location i.
- \(D_{iL}\) = \(D_L\) at location i.

In addition to the overall damage index, story-level damage indices can also be obtained in a similar manner. In this paper, the story-level damage indices are used for identifying the different damage states of the building.
4 Fragility Formulation

A fragility curve for a particular damage state is obtained by computing the conditional probabilities of being in that damage state at various levels of ground motion. A plot of the computed conditional probabilities versus the ground motion parameter gives the fragility curve for that damage state. The conditional probabilities are given as follows:

\[ p_{ik} = P[D = d_i | Y = y_k] \]  

where

- \( p_{ik} \) = probability of being in damage state \( d_i \) given the ground motion is \( y_k \)
- \( D \) = damage random variable defined on the damage state vector \( D = \{d_0, d_1, ..., d_n\} \)
- \( Y \) = ground motion random variable

The damage states are discussed briefly in the next section. The ground motion parameter used to characterize the variable \( Y \), in this paper, is the spectral acceleration \( S_a \). An alternate representation of fragilities is given by the probabilities of exceeding a specified damage state given a ground motion level. This definition of fragility can be evaluated from equation 3 as follows:

\[ p_{ik} = P[D \geq d_i | Y = y_k] = \sum_{j=i}^{n} p_{jk} \]  

The fragility formulations defined in equations 3 and 4 are determined by Monte Carlo simulation. From the simulations, the mean and variance of the modified Park and Ang global damage index are estimated for an ensemble of time histories corresponding to a given level of the ground motion. A large number of simulation cycles are needed to achieve an acceptable level of confidence in the estimated probabilities. The Latin Hypercube sampling technique is used to reduce the number of simulation cycles.

Artificial ground motion is generated to obtain ensembles of time histories for the development of the fragility curves. Autoregressive moving average models are used for the generation of artificial time histories when the ground motion is characterized by root mean square acceleration. Gaussian stationary models with modulating functions are used for the generation of time histories when spectral acceleration is used to characterize the ground motion.

4.1 Damage states

The different damage states of the building can be identified based on the global damage indices of the overall structure discussed earlier. Gunturi reviews the ranges of the Park and Ang global damage index for four different damage states: Minor, Moderate, Severe and Collapse. The ranges
of the modified Park and Ang damage index for different damage states, used in this paper, are presented in Table 1.

### Table 1. Modified Park-Ang damage index for different damage states

<table>
<thead>
<tr>
<th>Damage State</th>
<th>Range of the modified Park-Ang index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor</td>
<td>0.1 - 0.2</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.2 - 0.4</td>
</tr>
<tr>
<td>Severe</td>
<td>0.4 - 1.0</td>
</tr>
<tr>
<td>Collapse</td>
<td>&gt; 1.0</td>
</tr>
</tbody>
</table>

#### 4.2 Random variables for analysis

In damage analysis, the random variables to be considered are the resistance and the load imposed on the structure. The different parameters which affect the resistance of the structure include the compressive strength of concrete, yield strength of reinforcing steel, hysteretic behavior, damping ratio, physical dimensions of the different components and amount of reinforcing steel. In this paper, the compressive strength of concrete and the yield strength of steel are the only parameters treated as the strength random variables.

A normal probability distribution for concrete strength and a lognormal probability distribution for steel strength are used in this paper following Galambos et al. Concrete strength has a mean of 1.14 times the nominal concrete strength and a coefficient of variation of 0.14. Steel strength has a mean of 1.05 times the nominal strength and a coefficient of variation of 0.11.

The uncertainty associated with dead and live loads is considerably smaller compared to the uncertainty in seismic load. In this paper, only the earthquake load and consequently the ground motion is modeled as a stochastic Gaussian process.

The firm site records of Loma Prieta, Morgan Hill and Whittier Narrows earthquakes are used for obtaining the parameters of the lognormal distributions of the dynamic amplification factors at different periods. The dynamic amplification factors are used to obtain response spectra corresponding to a given average ordinate of spectral acceleration in the period range 0.1-0.45 sec.

The ground motion can have different strong motion durations for a particular value of spectral acceleration. Thus, the strong motion duration is also considered to be a random variable. A small correlation is observed between spectral acceleration and strong motion duration. Therefore a lognormal probability density independent of the spectral acceleration is assumed for the strong motion duration when generating time histories for a given spectral acceleration. The parameters of the distribution are: $\lambda = 2.396$ and $\xi = 0.331$.

#### 4.3 Ground motion simulation

The development of the fragility curves and damage probability matrices
requires that ensembles of time histories be available at various levels of the ground motion. It is difficult to find even a small number of recorded time histories with the same ground motion parameter. Thus, the alternative is to simulate time histories to provide such ensembles of ground motion. Gaussian stationary models with modulating functions are used to generate artificial time histories. Such models have been proposed, among many others, by Liu, and Shinozuka and Deodatis. In this paper, the program SIMQKE is used to generate time histories corresponding to a given response spectra. A trapezoidal envelope function is used to incorporate the nonstationarity of the motion.

5 Simulation Results

The computer program IDARC2D is used for damage analysis. This program evaluates the modified Park and Ang index as defined by equations 1 and 2. A hypothetical building is selected for the purposes of generic fragility formulation. The building consists of four similar frames: two each along the length and width of the building. The elevation of one of the frames is shown in Figure 1. Table 2 summarizes the properties of beams and columns in the frames. The building chosen does not have plan or elevation irregularities. Studies are currently under way to determine the modifications to the fragility curves due to plan or elevation irregularities.

The results of damage analysis at given ground motion levels are presented as fragility curves in Figure 2. One hundred artificial ground motions are generated at each value of the ground motion parameter. The statistics of the modified Park and Ang damage index, obtained at each value of the ground motion parameter, are used to compute the probabilities of the different damage states. Smooth fragility curves are obtained by arbitrarily fitting lognormal distribution functions to the simulation results. The simulation results and the fitted curves are shown as discrete points and smooth curves respectively, in Figure 2. A considerable amount of computational effort is required to obtain the statistics. Procedures for obtaining the fragility curves with reduced computational effort are currently being investigated.

Figure 1. Elevation of the frames resisting the lateral load.
Table 2. Properties of beams and columns in the frames.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Beams</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>16&quot; x 30&quot;</td>
<td>25&quot; x 25&quot;</td>
</tr>
<tr>
<td>Main Reinforcement</td>
<td>3 in² (B) 6 in² (T)</td>
<td>7.8 in² (S) 11.4 in² (F)</td>
</tr>
<tr>
<td>Concrete Strength</td>
<td>4 ksi</td>
<td>4 ksi</td>
</tr>
<tr>
<td>Steel Strength</td>
<td>60 ksi</td>
<td>60 ksi</td>
</tr>
</tbody>
</table>

T=top; B=bottom; S=1st story; F=2nd story

Figure 2 Fragility curves for the two story building

6 Conclusions

This paper demonstrates that generic fragility curves can be generated using simulation methods. The validity of these curves still remains to be assessed. Ideally, such curves should be developed from observed damage data. However, such data are very limited and usually not in the proper format to be able to generate the curves. With the limited data, partial verification of the generic fragilities is possible. In addition, variations in the fragility curves due to elevation and plan irregularities need to be determined. The program EDARC2D\textsuperscript{a}, however, used for obtaining the fragility curves cannot be used to perform a three dimensional non-linear dynamic analysis needed to assess the effect of plan and elevation irregularities. Studies are currently being conducted to determine the effect of elevation and plan irregularities on the fragility curves. Procedures for obtaining the fragility curves with reduced computational effort are also being investigated.

In the course of developing the fragility curves presented in this paper, it was found that the global damage index based on the modified Park and Ang local damage index is linearly related to the interstory drift. Thus, the contribution of the hysteretic energy dissipation part of that index is captured by the interstory drift.
Acknowledgments

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References