A one-dimensional coupled hydrodynamic and salinity model of the Clyde Estuary

J. M. Crowther & F. Chen

1Department of Energy & Environmental Technology, Glasgow Caledonian University, Glasgow, Scotland, U.K.
2Proudman Oceanographic Laboratory, Bidston Observatory, Birkenhead, England, U.K.

Abstract

We report the development of a one-dimensional mathematical model of the hydrodynamics and salinity of the Clyde Estuary from Greenock to the tidal weir in Glasgow. The predicted tidal levels and the shapes of the tide curves agree well with tide gauge records at Rothesay Dock and the Broomielaw. The predictions of cross-sectionally averaged salinities along the Clyde Estuary also agree well with measured data. The accuracy of the model has been increased, relative to the usual one-dimensional model formulations, by dividing the lower estuary into a compound channel with three lateral compartments. The conveyance of the compound channel is then calculated for each longitudinal element of the one-dimensional model.

1 Introduction

The Clyde estuary is a wide shallow estuary (see Fig. 1a, 1b) with a narrow dredged channel, five tributaries and two main sewage treatment works (STW’s). It was modified by a series of major engineering works (Riddell, [1]) and was an important factor in the economic development of the City of Glasgow. The shallow natural estuary was deepened by excavation, and constricted by lateral dykes to increase natural scouring of bed material. Dredging was later introduced to deepen the channels and maintain an adequate depth for shipping. In the late 1940’s, Thom [2] constructed a physical model to investigate the effect on tidal levels of a proposed reclamation of land in the estuary but his model appeared to be overdamped and failed to predict the falling tide levels. In 1965, the Clyde Port Authority
Figure 1a: Clyde Estuary and Firth of Clyde

Figure 1b: Upper Clyde Estuary
commissioned the construction of a new, and much larger scale, physical model to
examine the feasibility of a Maritime Industrial Development Area (Collar and
Mackay, [3]). Thomson [4] presented the characteristic hydraulic features of the
Clyde Estuary, and showed that the new model, in contrast to Thom's, appeared to
be slightly underdamped, showing an enhanced response to the tidal harmonics.

Attention then turned to numerical models and Ellis [5] used the method of
characteristics and a roughness coefficient of 0.01925 to simulate the water level
and flow velocity from Greenock to the tidal weir. The next attempts to model the
Estuary were made by Curran [6] who developed a tidally averaged, one-
dimensional model for the prediction of dissolved oxygen concentrations. Later
Wallis, Crowther et al. [7] developed a one-dimensional time-dependent
mathematical model of the Upper Clyde Estuary from Rothesay Dock to the tidal
weir, using a constant Darcy-Weisbach friction factor of 0.03. Meanwhile,
Falconer et al. [8] investigated the risks of flooding if dredging operations were to
cease, using a hydrodynamic one-dimensional model, with 229 cross-sections to
represent the river, upper estuary and tidal reaches of the major tributaries. Bed
roughness values, expressed in terms of Manning's n, averaged 0.019 downstream
of the tidal weir, and between 0.023 and 0.036 upstream of the weir. At the same
time, Crowther, Bennett et al. [9] developed an extended model of the Upper
Clyde Estuary by the combination of their earlier hydrodynamic model with a
sediment transport and bacterial survival model. The model described in this paper
extends the one-dimensional hydrodynamic model and salinity model to cover the
36 km between the City of Glasgow and the Greenock tide gauge.

2 The mathematical model

The conventional simplifications and assumptions associated with a one-
dimensional hydrodynamic model are to ignore the effects of wind shear stress,
Coriolis forces, and cross-sectional density gradients. It is also assumed that the
lateral inflows exchange no momentum with the main flow. Velocity and salinity
variables are interpreted as averages over the respective cross-sections.

2.1 Governing equations

The hydrodynamic model is based on conservation of mass:

\[ (W + W_s) \frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\rho_L L}{\rho} \]  

(1)
conservation of momentum (de St. Venant equation):

\[
\frac{\partial Q}{\partial t} + \frac{\partial \left[ Q^2 / A \right]}{\partial x} + gAS_f + gA \frac{\partial h}{\partial x} + \frac{gAR}{2 \rho} \frac{\partial \rho}{\partial x} = 0
\]  \hspace{1cm} (2)

conservation of salt:

\[
\frac{\partial (AS)}{\partial t} + \frac{\partial (QS)}{\partial x} = \frac{\partial \left[ AD \frac{\partial S}{\partial x} \right]}{\partial x} + LS_L
\]  \hspace{1cm} (3)

The equation of state is:

\[
\rho = \rho_o (1 + \lambda S), \quad \rho_L = \rho_o (1 + \lambda S_L),
\]  \hspace{1cm} (4)

\[
\lambda = 0.00075
\]

The friction slope is given by the combination of the Darcy-Weisbach equation, and the Manning formula:

\[
S_f = \frac{n^2 Q |Q|}{A^2 R^{4/3}}
\]  \hspace{1cm} (5)

In these equations ‘x’ represents the distance from the tidal weir to the grid point and ‘t’ denotes time. The unknown quantities are:

- \(h(x,t)\) water level (m OD),
- \(Q(x,t)\) discharge (m^3 s^{-1}),
- \(S(x,t)\) salinity (parts per thousand),
- \(S_L\) inputs of salinity (parts per thousand),
- \(S_f\) friction slope.

The known quantities are:

- \(W(x,h)\) width of the cross-section at the surface (m),
- \(R(x,h)\) hydraulic radius of the cross-section (m),
- \(A(x,h)\) area of the cross-section (m^2),
- \(n(x,h)\) Manning’s roughness value (s m^{-1/3}),
- \(W_d(x,h)\) the average width of static storage: docks, tributaries (m),
- \(L(x,t)\) lateral inflow per unit length from tributaries/sewage (m^2 s^{-1}),
- \(\rho(x,t)\) the density (kg m^{-3}).
2.2 Boundary conditions

The boundary conditions for the model at the seaward limit, Greenock, are expressed as follows:

\[ h = h_0(t) \quad S = S_0(t) \]  

The boundary conditions at the landward limit, the tidal weir, are as follows:

\[ Q = Q_1(t) \quad S = S_1(t) \]

The water levels \( h_0(t) \) in this study were taken from the records of an automatic tide gauge at Greenock, operated by the Clyde Port Authority. The flows \( Q_1(t) \) were obtained from the Daldowie gauging station run by the Clyde River Purification Board (C.R.P.B.). \( S_0(t) \) is the salinity at the seaward boundary and \( S_1(t) \) the salinity at the upstream boundary.

2.3 Initial conditions of the hydrodynamic model

The model starts at high water, for which the initial conditions are assumed to be:

\[ h(i, t_0) = h(t_0), \quad Q(i, t_0) = Q(1, t_0), \quad \text{and} \quad i = 1, 2, ..., n_s \]  

for all \( x \) at \( t = t_0 \) where \( t_0 \) is the model start time, \( i \) is the section number, and \( n_s \) is the total number of sections. The initial conditions for the solute models, such as the salinity model, are usually calculated from the longitudinal survey data, as will be discussed later.

2.4 Lateral inflows

The hourly average lateral inflows to the Clyde estuary from the Rivers Kelvin and Leven, and the Cart system were supplied by the C.R.P.B. from their gauging stations. The flow rates of the final effluents of Shieldhall STW and Dalmuir STW were provided by Strathclyde Regional Council.

2.5 The friction coefficients

In previous versions of the model, Darcy Weisbach friction factors were used which were constant in time but which varied in space from 0.009 at Greenock, linearly increasing to 0.026 at the head of the estuary. The predicted heights, when compared with the field data, showed a similarity but the hump on the rising tide curve was too small (See Fig. 2). Accordingly a different approach was tried in the present model. Rewriting the Manning formula (Equ. 5), the following expression is obtained for the magnitude of the flow rate in terms of a conveyance factor times the square-root of the friction slope:
If the cross-section has a compound shape, as at the mouth of the estuary, it is common practice to divide the cross-section into several distinct subsections (Cunge & Holly, Jr., [10]). Assuming the same friction slope for each subsection, the whole cross-section conveyance factor can be found by summing all the subsection factors. If section i is divided into m subsections (vertical slices), then the overall conveyance factor and flow rate at this section can be expressed as follows:

$$K_i = \sum_{j=1}^{m} \frac{1}{n_{ij}} A_{ij} R_{ij}^{2/3}$$  \hspace{1cm} (9)

$$Q_i = \sum Q_{ij} = \sum_{j=1}^{m} \frac{1}{n_{ij}} A_{ij} R_{ij}^{2/3} S_{j}^{1/2}$$  \hspace{1cm} (10)

The total discharge Q can also be expressed in terms of Equ. 8 with the overall area and hydraulic radius, and an effective Manning’s n:

$$Q_i = \frac{1}{n_i} A_i R_i^{2/3} S_i^{1/2}$$  \hspace{1cm} (11)

from which we obtain the following identity:

$$n_i = \frac{A_i R_i^{2/3}}{K_i}$$  \hspace{1cm} (12)

The overall conveyance $K_i$ is calculated from Equ. 9 and the effective roughness can be found using Equ. 12, noting that cross-sectional dimensions change with the water level. The effective roughness is then substituted in Equ. 5.

In this project the distance from Greenock to Bowling Harbour was divided into 34 sections, each section was divided into three transverse subsections (vertical slices) according to depth, corresponding to the main channel and the extensive mud banks. A constant $n_{ij} = 0.018$ was used in all three subsections in Equ. 11. Between Bowling and the tidal weir the cross-section was not subdivided and Manning’s n was assumed to have a constant value (0.019) for different water
levels, because the dredged channel is narrow and has a rather simple cross-section.

3 Numerical simulation

The Clyde Estuary was represented by 72 grids and 71 elements with the zero of the x coordinate defined to be at the tidal weir. The element length, \( \Delta x_i = x_i - x_{i-1} \), varies with each element and was chosen following some useful rules of thumb recommended by Samuels [11]. A Preissman finite difference scheme was used to discretize Equations 1 and 2. Functions and their x-derivatives were evaluated at the mid-point of each interval and a weighted average taken between values at successive time steps. Time derivatives were taken at the mid-point of the time and space intervals. For the salinity (Equ. 3), a linear Galerkin finite element method was used to treat the x-derivatives (Wallis, Crowther et al., [7]; Williams and Nassehi, [12]). This is easier to implement on a non-uniform grid than a finite difference scheme in which the equations are second order in x. The time derivatives in the diffusion equations are treated by finite differences in a manner similar to that of the hydrodynamic equations. The solutions were stable with a 15-minute time step. The output consists of the values of the height, velocity and salinity at every cross-section at time intervals specified by the user.

3.1 Verification of the hydrodynamic model

The hydrodynamic model was verified using five data sets. Three data sets (two spring cases and one neap case) were used to verify the predicted water heights against the tidal curves measured by two automatic tide gauges (Rothesay Dock, 25 km from Greenock, and Broomielaw, 35 km from Greenock). Two data sets were used to compare predicted velocities with the measured field data. The results for a model run started at 13:00 on 16/08/89 and lasting 36 hours under spring tide conditions with a double peak phenomenon is shown in Figs. 2 and 3.
Figure 2: Spring tide measured (-----) and predicted (----) water levels at Rothesay Dock

Figure 3: Spring tide measured (-----) and predicted (----) water levels at the Broomielaw

A model run started at 20:00 on 11/08/89 and lasting 36 hours under neap tide conditions with harmonic phenomena present is shown in Figs. 4 and 5.
The fourth model run started at 1:30 on 13/07/87 and lasted 22 hours. The fifth model run started at 18:30 on 25/02/88 and lasted 22 hours. The velocities of the predicted and measured data for these two cases are shown on Figs. 6 and 7. The predicted velocities agree reasonably well with the field measurements.
4 One-dimensional salinity model

The field salinity measurements indicated a high degree of stratification in the Clyde Estuary associated with a pronounced salt wedge penetration. The salinity distribution found in the survey of 7th of August 1991 is displayed in Fig. 8.
It is obviously difficult for a one-dimensional model to represent such a distribution because it predicts only average cross-sectional salinity.

4.1 Boundary conditions

The boundary conditions for the salinity model at the seaward site, Greenock, can be reasonably fixed about 31.0 parts per thousand. Surveys often end at the Broomielaw (grid 69 in this model) where the river becomes shallow, and therefore the salinity at the tidal weir, the landward boundary of the model, is unknown. An assumption was made of a steady-state distribution of salinity near the tide weir. A simple formulation of Fick's diffusion law was used to estimate the salinity at grid 70 by the following equation:

\[ QS = DA \frac{\partial S}{\partial x} \]  

(13)

The velocity and the area of grid 69 (where the salinity is known) are predicted by the hydrodynamic model. \( \Delta x \) is known and \( \Delta S \) can be found by using the above formula. Consequently, the salinity at grid 70 can be found. This process is repeated to find the salinity at grids 71 and 72. Linear interpolation was applied to find the salt concentrations at other times.
4.2 Initial conditions

The crew of the C.R.P.B. survey vessel, Endrick II, took samples at three depths (1, 4, and 9 m below surface) for 9 sites along the estuary between Greenock and the Broomielaw, Glasgow. The initial conditions for the salinity model were established from true cross-sectional averages which were found using the predicted water levels from the hydrodynamic model to take account of the actual times of sample collection and the depths of water then existing. It was assumed that there was no lateral variation of salinity at a given depth.

4.3 Dispersion coefficients

The dispersion coefficients used in this salinity model are constant in time but vary in space. They were adapted from Curran [6] (taking his 90% exceedence case) with some modifications. The dispersion coefficients used in this model are shown in Fig. 9.

![Dispersion Coefficients](image)

**Figure 9:** Variation of dispersion coefficient with distance.

4.4 Verification of salinity model

The set of data chosen was for longitudinal surveys on three consecutive days (6th - 8th Aug 1991). The C.R.P.B. took samples at 9 sites along the estuary as usual. The 6/8/91 survey data was used for initial conditions and the model run started at 15:15 and lasting 25 hours. The survey data of 06/08/91 and 07/08/91 were converted from three depths into cross-sectional averages. The results were then compared with the second day's survey data (See Fig. 10).
The predicted salinity data at the same time as the survey time on the above date and at the specific grid nearest to the sample point are represented by crosses in Fig. 14.

The comparison of salinity distribution along the Clyde Estuary shows that the predicted data agree well with the field data.

A sensitivity analysis of a 25% and 50% increase, and a 25% and 50% decrease of the dispersion coefficients shows that the predicted salinities are relatively insensitive to the exact value of the dispersion coefficient.

5 Conclusions

Though the bed depth data for the extensive mud banks and sub-channel in the 19km downstream part of the estuary is poor, the hydrodynamic model predicts accurately the hydraulic aspects (heights, velocities and so on). The comparisons of this model with the field data are satisfactory and the model can be as a foundation for other models, e.g. dissolved oxygen model, bacterial model. It was noted that the water levels are sensitive to the exact shape and accuracy of the measured tidal level at Greenock.

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