Structural models and the prediction of default probabilities

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Abstract

In this paper, the three main structural models of default, the Merton model, Longstaff and Schwartz model and Leland and Toft model, are compared in terms of the real default probabilities they produce. I find that none of the models can accurately predict the default probabilities in all cases. Merton as well as Leland and Toft models underpredict default probabilities in all cases. The Longstaff and Schwartz model, although in some cases it produces EDFs that are close to the observed ones, suffers from important limitations. The model tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility. Consistent with previous studies, I find that structural models tend to underestimate the default probabilities in early years.

*Keywords: credit risk, structural models, real default probabilities, expected default frequencies.*

1 Introduction

In this paper, I will examine the differences in real default probabilities produced by different structural models. Three main structural models, Merton model, the Longstaff and Schwartz model and the Leland and Toft model, will be compared. The first model is the original structural model, where default can only occur at maturity when the market value of assets equals the total book value of liabilities. Longstaff and Schwartz model accounts for coupon paying bonds and stochastic interest rates. In this case, default happens when the market value of assets falls below a lower threshold point that is a fraction of the total value of debt. Leland and Toft model, accounts for taxes and bankruptcy costs and assumes that the default barrier is determined endogenously. A detailed
description of these models can be found at Merton [10], Longstaff and Schwartz [9], Leland and Toft [8] as well as Dimou [4].

During the last years, there has been considerable work on the comparison of different structural models in terms of the term structure of credit spreads they generate. Nevertheless, apart from Leland [7] there has been no additional academic research on the comparison of the term structure of real default probabilities or Expected Default Frequencies (EDFs) produced by structural models. In this paper, I am building on Leland’s work. In his study, he allows the models to differ only in terms of the default barriers they assume, by keeping all other parameters constant and identical across models.

Although, his study provides useful insights on the impact of the different default boundaries on the EDFs, it does not answer the main question of how other differences in the assumptions of the structural models can affect their prediction of real default probabilities. The main difference of this study compared to Leland’s is that instead of allowing the models to differ only in terms of the specification of the default barrier, I allow the models to differ also in the value and the volatility of assets they produce. This is particularly important, since these two variables are unobservable. Therefore, it is useful to investigate whether by using the above mentioned models to derive these values will have any impact on the term structure of EDFs.

The following analysis answers two main questions.

1. How different and accurate are the Expected Default Frequencies (EDFs) produced by different models for BBB, BB and B rated bonds, as we move from the original Merton model to more sophisticated models that incorporate more realistic economic considerations and define the value of assets, volatility of assets and default barrier in a different way?

2. How the assumption of different equity volatilities in each rating class can affect the level and accuracy of EDFs produced by different models?

The main motivation behind this paper is the lack of research concerning the real default probabilities produced by different structural models. There are three reasons why this is a concern. First of all, with the new Basel Accord banks will be able to use internal models to assign default probabilities on individual borrowers and hence to internally assess their economic capital. Therefore, there is a great need for the empirical testing and validation of the existing credit risk models in order to assess their ability to adequately help banks detect credit rating changes in due course. Moreover, as mentioned above, there have been many studies on the structural models and the credit spreads they produce. Although, these studies produced mixed results on the ability of structural models to produce spreads that are in line with the observed ones (Anderson and Sundaresan [1]), the main consensus is that the structural models underpredict credit spreads, which may suggest that structural models are inadequate for measuring credit risk. Nevertheless, recent studies suggest that credit risk is only one of the components that explain yield spreads (Huang and Huang [6]). This suggests that structural models might be more adequate in the prediction of
default probabilities rather than in the calculation of credit spreads. Last but not least, while credit spreads are central to the pricing of defaultable bonds, default probabilities are equally important, since they are central to calculation of credit value at risk, as in CreditMetrics or KMV Credit Monitor framework.

In this paper, except in Merton case, the primary focus is on the default probabilities of coupon paying finite maturity bonds. The main assumption is that one firm has one risky bond. This implies that the firm has only one class of risky debt. The last assumption helps us to focus on the questions mentioned above and not on the issues arising from the seniority of bonds.

The analysis of the paper will be as follows. The following section provides the parameter values that will be used for the estimation of the real default probabilities for the three models. Section 3 describes the formulas used for the estimation of the real default probabilities as well as the iterative technique used to estimate the two unobservable variables, the value and the volatility of assets. The estimation of these variables is vital for the prediction of default probabilities. Section 4 shows the results and compares the term structure of EDFs produced by the models with the observed real default probabilities reported by Moody’s for the BBB, BB and B rated bonds. Section 5 provides all the main conclusions, limitations of the analysis as well as routes for further research.

2 Parameter values

At this point it is useful to determine the values of the parameters needed in the estimation of real default probabilities. Our aim is to select empirically reasonable parameter values. Since, the term structure of EDFs produced by the models will be compared with the default probabilities provided by Moody’s for bonds over the period of 1970-1997, the chosen parameter values should represent that period. In most of the parameter choices, I follow Huang and Huang [6] and Leland [7].

1. Value of equity, $V_e$: The value of equity for all cases is taken to be 100. Although this value should change as we move to different rating categories, there is no publicly available data on the average value of equity for BBB, BB and B rated firms.

2. Riskless interest rate, $r$: this is assumed to be 8%, which is the historical average of Treasury bills for the period of 1973-1998.

3. Asset risk premium, $\lambda$: This is assumed to be 4%, as in Leland [7]. Normally, the asset risk premium is derived from the equity risk premium. Since, the equity risk premium is different for each rating category, we should have assumed different asset risk premiums for the BBB, BB and B rated bonds that we examine here. However, I did not pursue that since equity risk premia is difficult to be determined with accuracy, unless a large historical database is available.

4. Volatility of equity, $\sigma_e$: Unfortunately, there is no publicly available information on the equity volatilities of an average firm in different
credit ratings. In order to be able to choose some reasonable parameters, I collected, from Bloomberg, 20 BB rated US bonds and 16 B rated US bonds that are currently traded. All these bonds are non perpetual non convertible and non callable bonds, with fixed coupon. In the majority of the firms, I collected daily prices for the last seven years and calculated the annualized volatility using the standard deviation approach and the Exponential Moving average approach. The advantage of the latter approach is that it places geometrically declining weights on past observations, thus assigning importance to recent observations and produces smoother time series. Nevertheless, the results of using the two different methods for the calculation of the annualized volatility are relatively the same. Detailed presentation of these results can be found at Dimou [4]. Although, these results offer a good guidance for the equity volatility parameters that will be used, they are not taken from bonds between 1970-1998. One implication of this lack of data is that we choose to test the different models by assuming different equity volatilities for each rating class. These equity volatilities are empirically reasonable to assume. Hence, for BBB bonds we assume 25% and 30% equity volatility, for BB bonds we choose 35% and 40% equity volatility and in the case of B rated bonds we assume 45% and 50% equity volatility.

5. Expected return on assets, $\mu : 12\%$, which is the sum of risk-free rate and the asset risk premium.

6. Payout rate, $\delta : 6\%$ as in Huang and Huang [6] paper.

7. Face value of debt, $P$: In our analysis, we use the average leverage ratios reported by Huang and Huang [6] for BBB, BB and B rated, which are 43.3%, 53.53% and 65.70% respectively. This leverage ratio is the ratio of the market value of debt divided by the market value of assets. It is useful to highlight the fact that the most adequate measure of the face value of debt would be the average book value of liabilities for each rating category, or alternatively a ratio of the face value of debt divided by the market value of assets. Although this is a limitation in our analysis, data on the average book value of liabilities is not publicly available and these parameter values are reasonable to assume. Hence, the face value of debt is assumed to be 43.3, 53.53 and 65.70 in the case of BBB, BB and B rated bonds.

8. Coupon, $C$: The coupon is calculated for each rating category as a 10% of the principal value.

9. Corporate tax rate, $\tau : 15\%$, as in Leland [7].

10. Debt maturity, $T : 10$ years (only in Merton case the value and the volatility of assets is calculated from 1 to 10 years, since EDF at time $t$ is the default probability of a zero-coupon bond maturing at time $t$).

11. Fraction of default costs, $\alpha : 30\%$ as in Leland [7].

12. Bankruptcy costs, $K$: These are assumed to be 30. As in Leland [7], we assume that although, Andrade and Kaplan [2] find that bankruptcy
costs are 10-20% for firms that had undergone highly leveraged buyouts previously, this underestimates the bankruptcy costs. This is due to the fact that high leverage is taken by firms with low default costs.

13. Stochastic interest rate parameters, \(\alpha\), \(\beta\), \(\eta^2\), \(\rho\). These parameters are taken to be equal to 0.06, 1, 0.001 and -0.25 respectively, as in Longstaff and Schwartz [9].

3 Estimation of the real default probabilities

The value of assets is denoted as \(V_a\) and the volatility of assets as \(\sigma_a\).

3.1 Calculation of default probabilities

For the modified Merton model, the real default probability is given using the following equation:

\[
EDF = N\left[ - \frac{\ln\left(\frac{V_a}{P}\right) + \left(\mu - \delta - \frac{\sigma_a^2}{2}\right)t}{\sigma_a \sqrt{t}} \right] = N\left(-DD\right)
\]

(1)

It is worthwhile to mention that where this model is differentiated with respect to the original model is the way the Expected Default Frequency is calculated. The real default probability for this model is calculated using the Distance to Default (DD) measure that includes the actual drift \(\mu\) (equals the asset risk premium plus the risk-free interest rate) instead of the risk-free interest rate. This is crucial, since failure to include the actual drift \(\mu\), will result in a risk-neutral probability of default.

For the Longstaff and Schwartz as well as the Leland and Toft model the default probability can be determined as follows (Leland [7]):

\[
EDF = N\left( - \ln\left(\frac{V_a}{V_b}\right) - \left(\mu - \delta - 0.5*\sigma_a^2\right)*t / \sigma_a \sqrt{t} \right) + e^{-2\ln\left(\frac{V_a}{V_b}\right)(\mu - \delta - 0.5\sigma_a^2)}\sigma_a^2 N\left( - \ln\left(\frac{V_a}{V_b}\right) + \left(\mu - \delta - 0.5*\sigma_a^2\right)*t / \sigma_a \sqrt{t} \right)
\]

(2)

where \(V_b\) denotes the default boundary level.

The reason why we cannot use the above equation for the Merton model is because the above equation denotes the cumulative default probability and as it is explained before Merton model does not give the possibility of default before the maturity day.

It is clear that in order to determine the EDFs, except of the parameter values given in the previous section we need to calculate three more parameters: the default boundary, the value of assets, the volatility of assets.
3.1.1 Calculation of default barrier
For Merton and Longstaff and Schwartz models the determination of the default barrier value is straightforward and can be generated using the parameter values given in Section 2. For Merton model, it is equal to the total value of liabilities \( P \) and for Longstaff and Schwartz (LS) model it is equal to 0.6 the total value of liabilities. For the Leland and Toft (LT) model the default barrier is determined endogenously and will be calculated together with the value and the volatility of assets. The equation can be found in Leland and Toft [8].

3.1.2 Calculation of value and volatility of assets
As it is described above, the term structure of EDFs produced by the three models for three categories of firms, BBB, BB and B rated will be estimated. Additionally, in each rating category I investigate two cases of equity volatility. This means that there are six cases: BBB firms with 25% equity volatility, BBB firms with 30% equity volatility, BB rated firms with 35% equity volatility, BB firms with 40% equity volatility, B rated firms with 45% equity volatility and B rated bonds with 50% equity volatility.

Due to the fact that Merton model does not allow for an early bankruptcy, since default can only happen at maturity, the value and the volatility of assets will be estimated in each case 10 times, for this model. Hence, each point in the tables, for the Merton model, will represent the probability that a zero-coupon bond with maturity \( t = 1 \) to \( t = 10 \) will default. This is not the case for the LS and LT models, where the value and volatility of assets is estimated once for each case assuming a ten year debt maturity. This together with the fact that I am using different formulas for the calculation of Merton’s and LS-LT’s default probabilities, explain why EDFs produced by Merton model are not directly comparable to the EDFs produced by LS and LT models.

The estimation of the value and the volatility of assets can be done by solving the eqns (3) and (4) simultaneously:

\[
V_e = V_a - B. \tag{3}
\]

The value of equity, \( V_e \), is calculated for all models, where B is the value of a risky bond and is defined differently across the structural models. Both the values of the risky bonds for all models as well as the values of equity are presented analytically in Merton [10], Longstaff and Schwartz [9] and Leland and Toft [8].

From Ito’s lemma, we can extract a formula that connects volatility of equity to the volatility of assets. Hence, the second equation used is:

\[
\sigma_e = \sigma_a \times \frac{V_a}{V_e} \times \frac{\partial V_e}{\partial V_a}. \tag{4}
\]

It is clear that in the above formula that the partial derivative of the value of equity to the value of assets will be different across models and will be calculated from the first equation. For the special case of Merton model, since the value of equity can be represented by a call option, the partial derivative of the value of equity to the value of assets is equal to the \( N(d_1) \), where \( d_1 \) is given by
\[ d_1 = \frac{\log \left( \frac{V_a}{P} \right) + \left( r + \frac{\sigma_a^2}{2} \right) T}{\sigma_a \sqrt{T}}. \]  

(5)

In the case of Merton and LS models we have two equations and two unknowns and in LT model we solve three equations with 2 unknowns, where the third equation is the default barrier. In all cases, having as inputs all the parameter values described in the previous section, the value and the volatility of assets are calculated using an iterative technique in Matlab. In particular, Newton – Raphson iterations are used in order to derive these values. The results from the iterative technique are provided at Dimou [4], while the tables of the estimated probabilities of default are given in the next section together with their interpretation.

4 Results

In the case of BBB bonds when equity volatility is assumed to be 25% all models underestimate the observed EDFs. On the other hand, in the case of a firm with higher equity volatility, Longstaff and Schwartz (LS) model tends to overestimate the EDFs, while Merton and Leland and Toft (LT) model still underpredict the Moody’s real default probabilities. Hence, LS model tends to react a lot in an increase in equity volatility, keeping all other variables constant.

Table 1: Real default probabilities for BBB bonds.

<table>
<thead>
<tr>
<th>Year</th>
<th>Merton 25% equity volatility</th>
<th>Longstaff and Schwartz 25% equity volatility</th>
<th>Leland and Toft 25% equity volatility</th>
<th>Merton 30% equity volatility</th>
<th>Longstaff and Schwartz 30% equity volatility</th>
<th>Leland and Toft 30% equity volatility</th>
<th>Moody’s 30% equity volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.12%</td>
</tr>
<tr>
<td>2</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.02%</td>
<td>0.00%</td>
<td>0.39%</td>
</tr>
<tr>
<td>3</td>
<td>0.01%</td>
<td>0.02%</td>
<td>0.00%</td>
<td>0.06%</td>
<td>0.24%</td>
<td>0.00%</td>
<td>0.75%</td>
</tr>
<tr>
<td>4</td>
<td>0.05%</td>
<td>0.08%</td>
<td>0.00%</td>
<td>0.35%</td>
<td>0.78%</td>
<td>0.02%</td>
<td>1.26%</td>
</tr>
<tr>
<td>5</td>
<td>0.22%</td>
<td>0.23%</td>
<td>0.01%</td>
<td>0.72%</td>
<td>1.62%</td>
<td>0.07%</td>
<td>1.70%</td>
</tr>
<tr>
<td>6</td>
<td>0.40%</td>
<td>0.46%</td>
<td>0.03%</td>
<td>1.58%</td>
<td>2.65%</td>
<td>0.16%</td>
<td>2.19%</td>
</tr>
<tr>
<td>7</td>
<td>0.62%</td>
<td>0.77%</td>
<td>0.06%</td>
<td>2.20%</td>
<td>3.80%</td>
<td>0.29%</td>
<td>2.74%</td>
</tr>
<tr>
<td>8</td>
<td>1.16%</td>
<td>1.13%</td>
<td>0.11%</td>
<td>2.83%</td>
<td>5.00%</td>
<td>0.46%</td>
<td>3.29%</td>
</tr>
<tr>
<td>9</td>
<td>1.49%</td>
<td>1.53%</td>
<td>0.16%</td>
<td>4.32%</td>
<td>6.21%</td>
<td>0.65%</td>
<td>3.91%</td>
</tr>
<tr>
<td>10</td>
<td>1.81%</td>
<td>1.95%</td>
<td>0.22%</td>
<td>5.02%</td>
<td>7.41%</td>
<td>0.86%</td>
<td>4.53%</td>
</tr>
</tbody>
</table>

For the BB rated bonds when the volatility of a firm is 35%, LS model produces a term structure that is quite in line with the observed one. Only in the first years it underpredicts the default probabilities. Nevertheless, when we increase the equity volatility LS models overpredicts EDFs. On the other hand, Merton and LT model still consistently produce low probabilities of default. The reason why LT model produces such low default probabilities is that the model produces quite high asset values while generating low asset volatility. Moreover,
LT model does not generate highly different term structures as we increase the equity volatility keeping all other variables constant. This is due to the fact that the asset volatility increase generated by the model is offset by the decrease in the default barrier. Hence, the net effect is quite small. In contrast, an increase in the volatility, ceteris paribus, causes the term structure of EDFs produced by LS model to increase substantially, leading to an overprediction of default probabilities. This is due to the fact that LS model tends to produce quite high asset volatility values.

On the other hand, we cannot directly compare Merton and LT model and conclude that Merton model produces higher default probabilities. This difference should be expected from the methodology used to estimate the default probabilities of these models. In Merton case each point at the table represents the probability that a zero coupon bond will default at time t. Hence, the firm will default if at each time the value of assets falls below the total value of debt. On the other hand, in LT model the default barrier at each point in time is less than the full amount of debt, producing a lower probability of default. Having established that no direct comparison can be made between these models, the only conclusion we can derive is that both models underestimate default probabilities.

Table 2: Real default probabilities for BB bonds.

<table>
<thead>
<tr>
<th>Year</th>
<th>Merton 35% equity volatility</th>
<th>Longstaff and Schwartz 35%</th>
<th>Leland and Toft 35%</th>
<th>Merton 40% equity volatility</th>
<th>Longstaff and Schwartz 40%</th>
<th>Leland and Toft 40%</th>
<th>Moody's</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.12%</td>
<td>0.00%</td>
<td>1.34%</td>
</tr>
<tr>
<td>2</td>
<td>0.08%</td>
<td>0.93%</td>
<td>0.00%</td>
<td>0.30%</td>
<td>2.31%</td>
<td>0.01%</td>
<td>3.71%</td>
</tr>
<tr>
<td>3</td>
<td>0.71%</td>
<td>3.31%</td>
<td>0.00%</td>
<td>1.62%</td>
<td>6.50%</td>
<td>0.09%</td>
<td>6.21%</td>
</tr>
<tr>
<td>4</td>
<td>1.68%</td>
<td>6.42%</td>
<td>0.14%</td>
<td>3.84%</td>
<td>11.18%</td>
<td>0.33%</td>
<td>8.77%</td>
</tr>
<tr>
<td>5</td>
<td>3.37%</td>
<td>9.68%</td>
<td>0.36%</td>
<td>6.80%</td>
<td>15.69%</td>
<td>0.75%</td>
<td>11.44%</td>
</tr>
<tr>
<td>6</td>
<td>5.59%</td>
<td>12.84%</td>
<td>0.67%</td>
<td>8.84%</td>
<td>19.83%</td>
<td>1.30%</td>
<td>13.72%</td>
</tr>
<tr>
<td>7</td>
<td>7.04%</td>
<td>15.80%</td>
<td>1.04%</td>
<td>12.20%</td>
<td>23.56%</td>
<td>1.95%</td>
<td>15.53%</td>
</tr>
<tr>
<td>8</td>
<td>8.25%</td>
<td>18.54%</td>
<td>1.47%</td>
<td>15.10%</td>
<td>26.91%</td>
<td>2.64%</td>
<td>17.44%</td>
</tr>
<tr>
<td>9</td>
<td>10.89%</td>
<td>21.07%</td>
<td>1.92%</td>
<td>16.70%</td>
<td>29.94%</td>
<td>3.36%</td>
<td>19.19%</td>
</tr>
<tr>
<td>10</td>
<td>13.50%</td>
<td>23.38%</td>
<td>2.39%</td>
<td>19.75%</td>
<td>32.67%</td>
<td>4.08%</td>
<td>20.88%</td>
</tr>
</tbody>
</table>

In the case of B rated bonds, LS model overpredicts EDFs in both cases while Merton and LT models suffer from the same underprediction problem.

It is important to note that initially we made the assumption that the equity value is equal to 100 and does not change as we move to different rating categories. By assuming a higher equity value the results would have changed. A more scaled representation of equity assigning a higher value to the BBB rated firms and moving to lower values for B rated firms would probably allow LS model to better predict EDFs in two cases. In the case, of BBB rated bonds with high volatility and B rated bonds with low equity volatility. Unfortunately, the limitations of data do not allow more accurate investigation.
Last but not least, it is important to highlight the fact that in early years all of the models, even those that perform quite well at longer time horizons, substantially underpredict default probabilities.

Table 3: Real default probabilities for B rated bonds.

<table>
<thead>
<tr>
<th>Year</th>
<th>45% equity volatility</th>
<th>50% equity volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Merton</td>
<td>Longstaff and Schwartz</td>
</tr>
<tr>
<td>1</td>
<td>0.05%</td>
<td>2.04%</td>
</tr>
<tr>
<td>2</td>
<td>1.40%</td>
<td>10.84%</td>
</tr>
<tr>
<td>3</td>
<td>4.57%</td>
<td>19.84%</td>
</tr>
<tr>
<td>4</td>
<td>10.60%</td>
<td>27.37%</td>
</tr>
<tr>
<td>5</td>
<td>12.54%</td>
<td>33.54%</td>
</tr>
<tr>
<td>6</td>
<td>16.65%</td>
<td>38.65%</td>
</tr>
<tr>
<td>7</td>
<td>20.68%</td>
<td>42.94%</td>
</tr>
<tr>
<td>8</td>
<td>24.36%</td>
<td>46.60%</td>
</tr>
<tr>
<td>9</td>
<td>27.94%</td>
<td>49.76%</td>
</tr>
<tr>
<td>10</td>
<td>31.41%</td>
<td>52.52%</td>
</tr>
</tbody>
</table>

5 Conclusion

This paper has examined the ability of three structural models of corporate bonds to predict Expected Default Frequencies (EDFs). I contrasted Merton model, Longstaff and Schwartz model and Leland and Toft model.

I find that none of the models can accurately predict the default probabilities in all cases. Longstaff and Schwartz model can in the case of BB bonds with 35% equity volatility produce EDFs that are close to the EDFs reported by Moody’s. I believe that under more realistic assumptions about equity value this model would be able to produce similar results in the cases of BBB with 30% volatility. The model however, tends to overestimate the default probabilities of riskier bonds as well as the default probabilities of bonds with the same rating but higher equity volatility. This is due to the fact, that the predicted asset volatility of the model is quite high.

On the other hand, Merton and Leland and Toft model tend to underpredict the EDFs in all cases. The main problem with Leland model is that it does not produce sufficiently high asset volatility values. Moreover, unlike Longstaff and Schwartz model, Leland and Toft model does not generate highly different term structures as we increase the equity volatility keeping all other variables constant. This is due to the fact that the asset volatility increase generated by the model is offset by the decrease in the default barrier, producing a quite small net effect. Last but not least, all models underpredict EDFs in the first years with the exception of the Longstaff and Schwartz model in the B rated bonds. In this case, the model highly overpredicts the term structure of EDFs in all years. Nevertheless, this might be due to the choice of parameters. In case that different parameter values were assumed, the whole term structure would have shifted.
down. In this case, as in the other models, Longstaff and Schwartz model would still underestimate the default probabilities in the first years.

These results are not encouraging for the ability of the structural models to predict default probabilities. Nevertheless, this study suffers from important limitations. Its main limitation is the lack of publicly available data on the value and volatility of equity. Further research on a firm by firm basis would be extremely useful in order to test more accurately the ability of structural models to predict EDFs.

Moreover, in the future I intend to include into the analysis the models of Collin-Dufresne and Goldstein [3] and Geske [5]. The former is an extension of the LS model. It will be useful to see if the assumption of stationary leverage will solve the overestimation problems of LS models. The latter will produce a term structure that will be more comparable with the rest of the models since it will allow for early default.

References


