A study on seismic vulnerability of Inca constructions

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Abstract

To evaluate the vulnerability of the historical Inca stone constructions, a simple model is developed to investigate their seismic behavior. The analytical model consists of multiple rigid body system subjected to dynamic action under the condition of friction and impact. First, a sinusoidal wave is used as input motion to investigate the dynamic behavior of the model. Then an actual earthquake record is selected and scaled to the corresponding peak ground acceleration to simulate the seismic behavior of a typical stone masonry wall. Results show that a uniform friction coefficient between stone blocks permits the stone wall to work as a single rigid body although the mathematical model is a multi rigid body system. However if a small difference or variation of the friction coefficient is introduced among the blocks, the displacements of adjacent blocks become different, originating impact actions between blocks resulting in a separation of blocks or failure of the wall.

1 Introduction

About 500 years ago, the Inca culture reached the peak in its development, which was just before the arrival of Spaniard conquistadors. By the time, the Incas had integrated a vast empire that stretched from the Maule river in Chile to the northern Ecuador along the western side of the Andes mountain range. This territory, as in present days, had continuously been exposed to natural disasters such as excessive rainfalls, earthquakes, landslides, floods, etc. In spite of such impending disasters, the Incas were able to develop techniques of construction to

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withstand such natural forces. The awe-inspiring cities and road networks that remain intact to this day serve as witness to their acumen in construction.

The structural system of their construction involves the use of adobe (sun-dried clay bricks), roughly shaped stones laid with mud mortar and finely shaped stones. They also used mud and clay as mortar for surface finishing. Finely shaped stone masonry was used for important building like temples, administrative structures and king's residences. In this type of construction, the adjacent stones are carefully shaped and fit snugly against each other without the use of mortar.

The Inca's stone structures have survived earthquakes that have occurred in the region. Attempt is made in this paper to investigate the behavior of typical Inca stone masonry wall component under the action of the earthquakes. For this purpose, multiple rigid body model is developed to simulate the seismic behavior of stone masonry walls. The model takes into account the friction between stone blocks and the impact between adjacent blocks.

2 Multiple rigid body model with friction and impact

For this research, stone wall of finely shaped blocks is selected to investigate its dynamic behavior. A typical example of this type of construction can be observed in Figure 1, where stone blocks are fitted snugly without mortar.

Figure 1: Typical stone masonry Inca wall.

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To obtained and analyze the general seismic behavior of the Inca's stone masonry walls, a simple model that consists of five blocks was developed. The proposed model can be observed in Figure 1. The model considers friction coefficient or frictional forces acting in the horizontal joints between blocks. In the vertical joints of adjacent blocks it is considered that forces due to impact are developed.

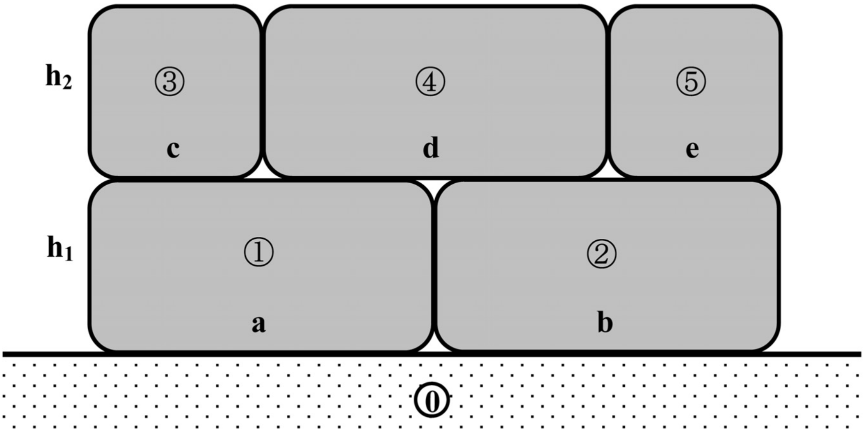

Figure 2: Multiple rigid body model.

For this system, the equation of motion can be expressed as is shown in the following equation:

$$[M]\{a\}+\{f\}=-\{m\}a_g \tag{1}$$

where $[M]$ is the mass matrix, $\{a\}$ is the acceleration vector, {f} is the force vector due to the friction between blocks, and a_g is the ground or input acceleration. The mass matrix is a diagonal matrix with the masses of each block as elements.

The force vector due to the friction is evaluated considering the equilibrium of the dynamic forces for the multiply rigid body system and is giving by the following equation:

$$\{f\}=\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{Bmatrix}=\begin{Bmatrix} F_{10}\operatorname{sgn} v_1 - F_{31}\operatorname{sgn}(v_3-v_1) - F_{41}\operatorname{sgn}(v_4-v_1) \\ F_{20}\operatorname{sgn} v_2 - F_{42}\operatorname{sgn}(v_4-v_2) - F_{52}\operatorname{sgn}(v_5-v_2) \\ F_{31}\operatorname{sgn}(v_3-v_1) \\ F_{41}\operatorname{sgn}(v_4-v_1) + F_{42}\operatorname{sgn}(v_4-v_2) \\ F_{52}\operatorname{sgn}(v_5-v_2) \end{Bmatrix} \tag{2}$$

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here, the F_{ij} represents the friction force between the upper block i and the lower block j. In the analysis it is considered that the forces become active only in the case that the input acceleration is larger that the friction factor multiply by the acceleration of gravity. That is, the friction force acts in the dynamic equation only when the input force is larger that the static friction force. Furthermore, the sign of the friction forces depends on the sign of the relative velocities between blocks. These friction forces F_{ij} are calculated by considering the weight of the blocks and the assumed friction coefficient μ. The weight of each block is calculated from the dimension of blocks and for a given specific weight of the material γ, in this case stone material. Therefore, the following expression are used to estimate these frictional forces:

$$\begin{aligned} F_{10} &= [at(h_1 + h_2)\gamma]\mu_{10} \\ F_{20} &= [bt(h_1 + h_2)\gamma]\mu_{20} \\ F_{31} &= [cth_2\gamma]\mu_{31} \\ F_{41} &= [(a-c)th_2\gamma]\mu_{41} \\ F_{42} &= [(b-e)th_2\gamma]\mu_{42} \\ F_{52} &= [eth_2\gamma]\mu_{52} \end{aligned} \tag{3}$$

The equation of motion (1) is solved numerically by means of the Newmark integration scheme. In this case for the step i+1 the expression for the equation of motion is given by:

$$[M]\{a\}_{i+1} + \{f\}_{i+1} = -\{m\}a_{g_{i+1}} \tag{4}$$

Then, the following equations for displacement, velocity and acceleration are used for the iteration scheme:

$$\begin{aligned} \{d\}_{i+1} &= \{d\}_i + \Delta t\{v\}_i + \frac{\Delta t^2}{2}\{a\}_i \\ \{v\}_{i+1} &= \{v\}_i + \frac{\Delta t}{2}(\{a\}_i + \{a\}_{i+1}) \\ \{a\}_{i+1} &= [M]^{-1}\left[-\{m\}a_{g_{i+1}} - \{f\}_{i+1}\right] \end{aligned} \tag{5}$$

In addition to the friction forces that are used in the equation of motion, the effect of impact forces between adjacent blocks are considered. For this initial analysis, elastic impact is assumed, therefore the coefficient of restitution is equal to 1. That is, the conservation of energy and momentum of each block during the dynamic action is considered.

It is assumed that the average friction coefficient is of the order of 0.30. Also, to compare with other values, friction coefficient of 0.20 was used. Then, to analyze the dynamic behavior of the model, a sinusoidal wave is assumed as input motion.

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The maximum amplitude of the sinusoidal wave was taken as 200, 300, 400, and 500 gals. In addition, the Miyagi earthquake record is used as input motion to obtain the response characteristics of the model.

3 Analysis results

Figure 3 presents a comparison of maximum displacement responses of the wall model for different frequencies of the sinusoidal input motion. In this case, uniform or constant friction coefficient is considered. Figure 3(a) are results for friction coefficient μ = 0.20 and Figure 3(b) presents results for μ = 0.30. Different curves are plotted for correspondent different maximum amplitude of the input motion.

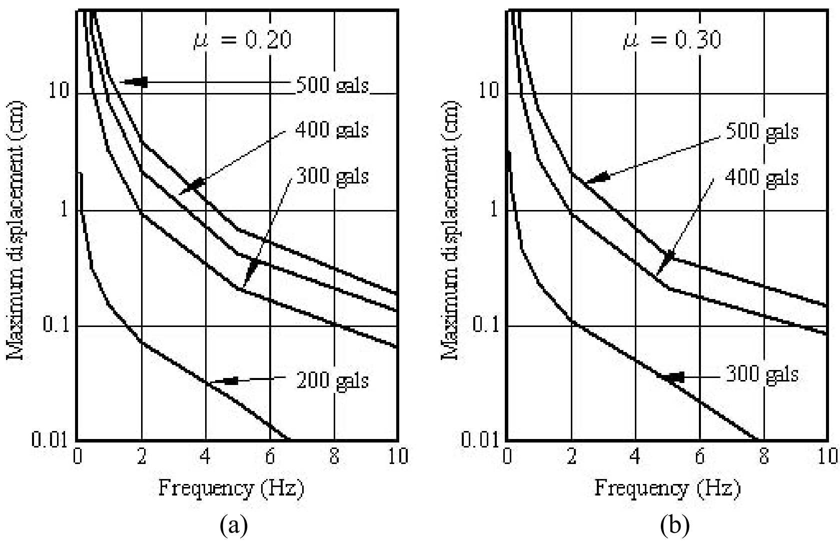

Figure 3: Maximum responses for sinusoidal input motion.

Since it is assumed that the frictional force acts only if the inertial force equals the static friction resistance, the dynamic response is obtained only in the case that the input force is equal or larger than the friction coefficient multiplied by the acceleration of gravity ($ma_g \geq \mu g$). Then for friction coefficient μ = 0.20, results for sine wave amplitude of 200 gals to 500 gals are presented, while for friction coefficient μ = 0.30 results from 300 gals are given. It is clear that the maximum response depends on the assumed friction coefficient. Moreover, larger maximum responses are obtained for small frequencies of the input motion, becoming smaller for high frequencies. Therefore, in case of actual earthquake motion, those that contain low frequencies components may affect the stone walls more severely than earthquakes with high frequency content. From the analysis, it was perceived that adjacent blocks move on together and therefore no impact forces were detected in

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this case of uniform or constant friction coefficient. However, small relative displacement between blocks of the lower row and upper row was observed.

In the case that variable friction coefficient is considered, the maximum responses become larger than the case of uniform friction coefficient. In this case, due to the differences of the friction coefficients, relative displacements are produced between adjacent block originating therefore impact forces that results in larger maximum displacements. In this analysis small variation of the friction coefficient was considered. These values were chosen as follows: $\mu_{10} = 0.29$, $\mu_{20} = 0.31$, $\mu_{31} = 0.28$, $\mu_{41} = 0.30$, $\mu_{42} = 0.30$ and $\mu_{51} = 0.28$. The criteria to select these values were to have different friction coefficients in adjacent blocks, and to have the smallest coefficients for the upper blocks of the left end and right ends. Under these conditions, the model was subjected again to the sinusoidal input motion, and results of maximum displacements for different frequencies of input motion are presented in Figure 4. Figure 4(a) shows results for the upper central block 4 and Figure 4(b) shows results for the upper left block 3. This block 3 gives the larger maximum displacements in comparison with the displacement of other blocks.

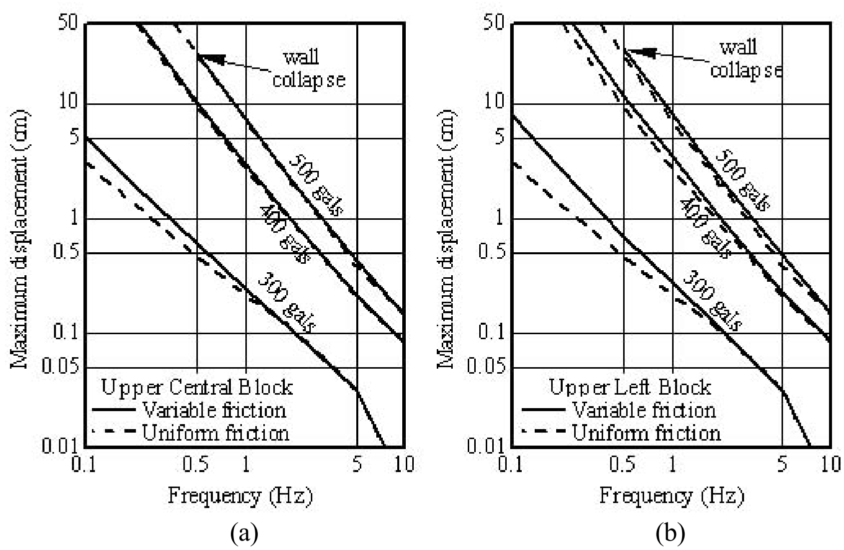

Figure 4: Maximum displacement responses in case of variable friction.

The maximum displacement responses for the case of variable friction coefficients (solid lines in Figure 4) are larger that those for the case of uniform friction (dashed lines). This difference is remarkable for low frequencies of input motion, while for high frequencies the responses for both cases are almost equal. In addition collapse of the wall occurs when variable coefficient is considered since upper blocks may fall down when the relative displacement with lower block exceeds half of the block length. In the figure this collapse is pointed out for the case of 500 gals of amplitude of the sinusoidal input motion.

In case of actual input motion, the differences between uniform friction and variable friction is also observed in the maximum displacement response and it is more remarkable if final condition of the wall is observed. For uniform or constant friction coefficient, no relative displacement is observed between adjacent blocks, while for variable friction relative displacements were detected resulting in separation of blocks. In Figure 5, the final condition after the action of Miyagi input motion amplified by 5, is presented. It is clear the separation of adjacent blocks. When uniform friction is employed for analysis, no separation of adjacent blocks was detected.

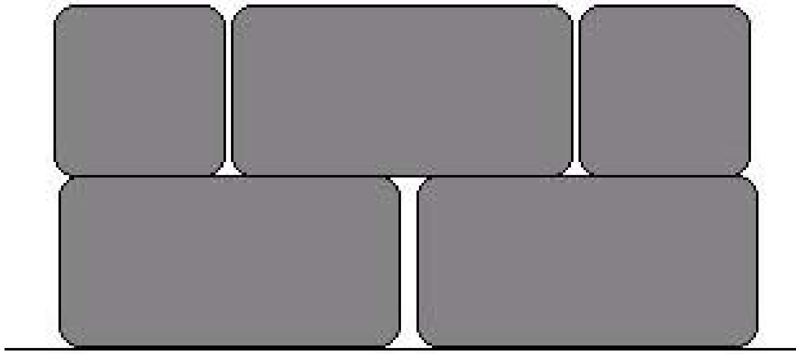

Figure 5: Condition of the model with variable friction after Miyagi input motion.

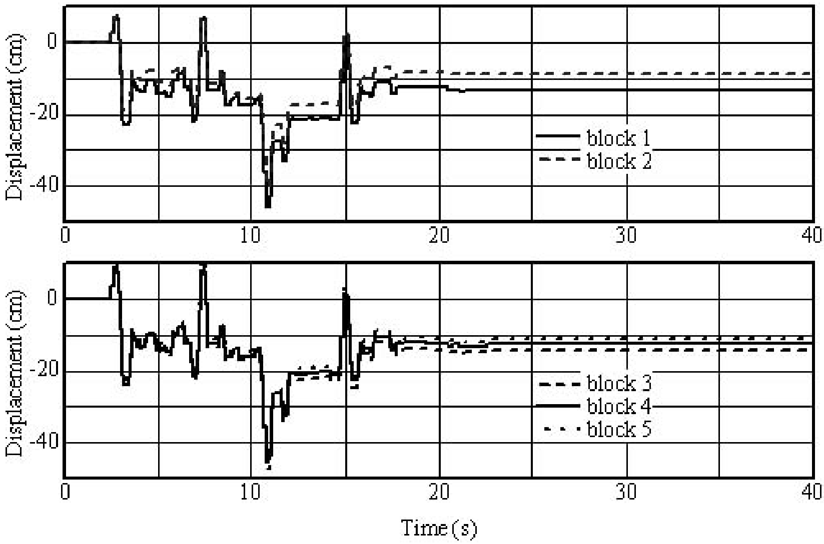

Figure 6: Displacement response: model with variable friction, Miyagi input motion.

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In Figure 6, the displacement response of each block, when the wall is subjected to Miyagi earthquake motion amplified by 5, is presented. Since the friction coefficient is assumed to be variable, each block has different displacement pattern, particularly after 10 seconds of motion. In the case of uniform coefficient of friction, only small differences between displacements of upper and lower blocks was observed, and no differences on displacements of adjacent blocks was detected.

4 Conclusions

Multiple rigid body model was employed to analyze the dynamic behavior of stone masonry Inca walls. The model takes into account the friction between stone blocks and the action of impact forces between adjacent blocks.

Considering uniform friction and variable friction among the blocks, sinusoidal wave with different amplitudes and frequencies was used as input motion. It was observed that the maximum displacement response become larger for low frequencies while for high frequencies this maximum decreases. Therefore it is believed that actual earthquake motions with low frequency content may produce more severe damage to this type of structures than earthquakes with high frequency content.

Comparing the responses for the cases of uniform friction and variable friction is was observed that the last one gives larger maximum displacement response. This is due to the action of impact forces that acts when relative displacement appears between adjacent blocks originated by the differences in friction forces. In the case of uniform friction impact forces does not appear since all adjacent blocks move on together. Then it can be concluded that uniform or constant friction is preferable to decrease or avoid impact forces and to ensure a behavior of the wall as a whole.

These findings are part of the ongoing collaborative research initiative at Akita Prefectural University and more sophisticated and detailed investigations are currently underway.

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