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Kaunas dam overtopping probability estimation

E. Uspuras[1] & J. Simaityte[2]
[1]*Laboratory of Nuclear Installation Safety, Lithuanian Energy Institute, Lithuania.*
[2]*Department of Mathematics and Statistics, Vytautas Magnus University, Lithuania.*

Abstract

The dam of the Kaunas hydropower station in Lithuania is an important system from the safety point of view because only several kilometres downstream the dam Kaunas city is situated with population over 450 thousands.

The paper presents the Kaunas dam risk analysis aspects, where the main part takes overtopping probability estimation. The purpose of this study was the critical water level modelling by the dam and to estimate overtopping probabilities by several scenarios - including spillway failure probabilities.

The statistical data – maximum annual flow data of 76 years– water input to the Kaunas reservoir from The Nemunas River were collected and analysed. Also there were analysed water level and reservoir capacity data near the dam.

The mathematical model of Kaunas dam which let perform prognoses of the critical reservoir capacity and at the same time the critical water level, as it is known as dangerous for dam and may cause the dam failure. The maximum flow data were assumed to be independent and identically distributed. The extreme value distribution helped to estimate critical water levels probabilities. The every day flow and water release data thorough the dam spillways let to calculate everyday reservoir water volume and to know water level during the flood period. As a result, there were estimated three critical level probabilities under three gate operation scenarios.

Estimated outlet failure probabilities and water level probabilities in each scenario yields the overflow probability (when water level reaches the top of the dam). As the Kaunas dam height is 48 meters, such water level leads to overtopping and according international literature almost 50% leads to dam

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failure. The water level probability at 48 meters was calculated. Sensitivity analysis is performed, too.

The results of sensitivity analysis showed if the outlet failure probability went to zero, the dam overtopping probability is not sensitive to this decreasing. In that case the bigger attention should be given to analyse other components of dam safety and the strict measures, not only from hydrological system, are necessary to ensure stability of the dam body and its capability to resist extreme flooding.

1 Introduction

This paper presents the Kaunas dam, by the Kaunas hydropower station (KHPS), overtopping probability estimation, which should be performed in order to ensure safety level of the dam. Thorough risk assessment of the Kaunas dam has never been performed.

The said dam is an important system from the safety point of view. Yang [2] showed that dam failure could have serious consequences to the second largest city of Lithuania – Kaunas and other downstream towns.

Construction of the Kaunas dam began in 1955. KHPS has 4 turbines of 100.8 MW total nominal power. There are three spillways operated by motors or by crane. Kaunas dam is an earth filled structure. Its altitude is 48.0 meters and size varies between 21.0 and 51.0 meters. Water to Kaunas dam reservoir is supplied from a 45800 km^2 area. The reservoir nominal water level altitude is 44.0 meters high. Volume of the reservoir at this level is 462 million m^3 and occupies 63.5 km^2. Average flooding dates are from 6 March to 9 May with maximum flow on 24-30 March.

In order to evaluate the overtopping probability, the flow data was analysed – water input to the Kaunas reservoir from the Nemunas River. By analysing the data the seasonal prevalence was realised– the annual maximum flood being in the spring, so the maximum floods were selected. To calculate everyday reservoir water volume during the flood period we use everyday flow data and water release through the dam spillways.

2 Overtopping Probability Estimation

2.1 Data analysis

Annual flow data from the past 76 years was collected and analysed. It was collected from two water-measuring stations: Birstonas and Nemaniunai. The stations are located on the Nemunas River, upstream the Kaunas dam water reservoir.

Birstonas water-measuring station is nearer Nemaniunai station to the Kaunas reservoir. However the Birstonas station everyday flow data was available from 1920-1956, the other data were obtained from the Nemaniunai station from 1957-1996, under the assumptions that both water-measuring stations are located in the same location. Because of war in 1944 and 1945 the data of these two years was not measured. The everyday flow data is shown in Appendix 1.

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The calculations are based on analysis of extreme values; so basically, from the whole data set the flood peaks measurements were needed.

2.2 Mathematical model of water level fluctuations

As Kaunas Dam reservoir nominal water level is 44.0 meters high the water level above mentioned bar becomes critical and that lead to the dam failure. The mathematical model let perform prognoses of the critical reservoir capacity and the critical water level near the dam.

2.2.1 Extreme value distribution fit

The hydrologic annual data analysed are assumed to be independent and identically distributed. This is achieved by selecting the annual maximum values of the data being analysed.

The study of extreme hydrologic event involves the selection of a sequence of the largest observations from sets of data (Te Chow, Maidment, Mays [3]). Hence, from the annual maximum peak flows were selected from the available flow data there. Since these observations are located in the extreme tail of the probability distribution of all observations from which they are drawn, so their probability distribution is different from that of the parent population. In this study the distribution of extreme values Type I is used (Embrechts, Klueppelberg, Mikosch [1]).

The Extreme Value Type I (EVI) probability distribution function is

$$F(x) = \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right], \qquad -\infty \le x \le \infty. \tag{1}$$

The parameters are defined as $\sigma = \frac{\sqrt{6}s}{\pi}$ and $\mu = \bar{x} - 0.5772\sigma$.

According our annual maximum flow data, the parameters μ and σ were estimated.

Comparing the theoretical and sample values of the relative frequency should test the goodness of fit of a probability distribution. In this study the χ^2 test is used. The fit of the extreme value distribution to the maximum flow data is accepted.

2.2.2 The linear dependencies of characteristics and distribution transformation

In order to calculate the water level by the dam during the flood period it is need to define the dependencies between flow data, reservoir capacity and water level. From the statistical data using approach of lest squares, the trend parameters were defined as following:

1) Let X be the maximum flow and Y – flood volume. Since we have a maximum flow dependence on total flood volume, $y = 2.0296x - 316.29$. The probabilistic behaviour of total flood volume can be described.

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Maximum flow dependence on total flood volume is shown in Figure 1.

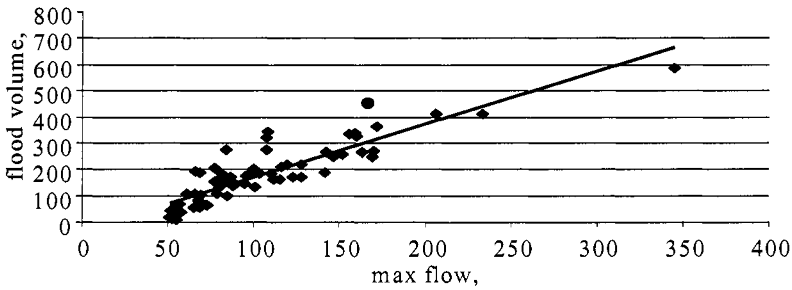

Figure 1: Maximum flow dependence on flood volume

2) The water level near the KHPS depends on reservoir volume. Let x – reservoir volume and y – water level near the dam, then we have dependence $y = 0.014x + 37.022$. Volume dependence on the elevation is sh

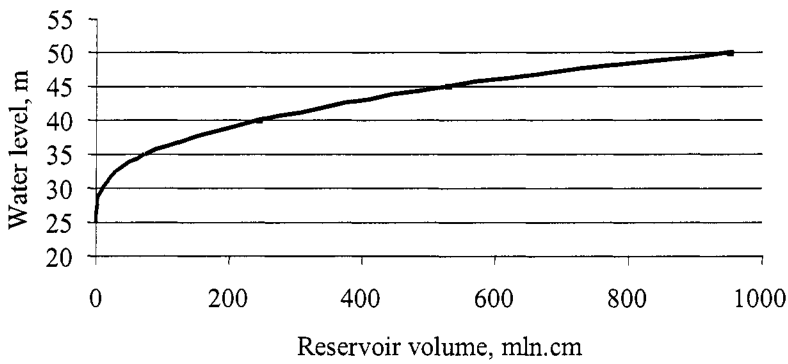

Figure 2: Water level dependence on reservoir volume

If X is a random variable with cdf $F_X(x)$, then function of X, say g(X), is also a random variable. Since Y is a function of X, we can describe the probabilistic behaviour of Y in terms of that of X, making a transformation from X to Y = g(X), When g is an increasing function, the transformed function has shape:

$$F_Y(y) = F_X\left(g^{-1}(y)\right). \tag{2}$$

Having the dependency between maximum flow and flood volume and making transformations the probabilistic distribution of flood volume was defined.

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2.2.3 Dam spillways probabilistic assessment

In order to calculate everyday reservoir water volume during the flood period we use everyday flow data and water release through the dam spillways. There are three spillways in KHPS, operated by motors or manually by crane.

Let's assume that the outlet failure is a discrete random variable X and has a *binomial distribution* if its pmf (probability mass function) is of the form

$$f_X(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0,1,2\ldots,n \ , \tag{3}$$

where n is a positive integer and $0 \le p \le 1$. In our case n=3 and probability of one outlet failure is assumed to be 1E-02 (Dan [5]). Failure probabilities to open the outlets are calculated and are shown in Table 1 below:

Table1: Outlets failure probabilities

p_0	p_1	p_2	p_3
1E-06	2.97E-04	2.94E-02	9.7E-01

p_0 - all outlets are failed, p_1 - 2 outlets are failed, p_2 -1 outlet is failed, p_3 - all outlets open.

2.2.4 Critical water level probability

Water release through the outlets depends on water level and we have one spillway discharge at full opening and the reservoir volume dependence on flood volume, so for the picked annual maximum flows we can calculate the everyday water level near the dam, for each scenario (with probabilities p_1, p_2, p_3). Also it is known the dependence between water release through the spillways and water level near the dam. The curve of this dependence and its approximation was found.

Modelling the water level fluctuations mathematical model it is need to have flood volume and reservoir capacity. The flood volume was calculated under the formula:

$$V_m = \sum_{k \in \{M_m\}} \theta\left(X(t_k) - v\right) X(t_k) \cdot T \ . \tag{4}$$

Where

V_m - flood volume, $m = 1920,\ldots,1996$;

T - day in seconds;

v - a flow bar, which means a flood if the flood flow is more than $v = 500\ \mathrm{m^3/s}$;

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t_k - day's number, $k = 1..27389$;

$\{X(t_k)\}$ - the average daily flow, $k = 1..27389$;

θ - Heaviside's function, $\theta(x) = \begin{cases} 1, \text{if } x \geq 0 \\ 0, \text{if } x < 0 \end{cases}$.

M_m - flow set, where $M_m = \{k : \theta(X(t_k) - v) = 1\}$, $m = 1920,...,1996$.

When the flood volume is realised it is possible to calculate reservoir capacity during the flood period and the water level near the dam at the same time. It is clear that water level near the dam depends on flood volume, as it impacts reservoir capacity, and on water quantity witch releases thorough the spillways. As it was mentioned above, the spillway failure probability also should be included in mathematical model. There are three scenarios $i = 1,2,3$, which mean one spillway operates, two spillways operate and three spillways operate respectively.

The dam high is assumed infinite and p_0 is not included in this calculation as if all outlets are failed the overtopping occurs anyway. The next step is to realise the maximum water level near the dam body during the flood period when dam high is assumed to be infinite. The maximum water level for each peak was modelled and picked up, and for each scenario the dependence was obtained.

The water level dependencies on flood volume for each scenario are shown in Figure 6.

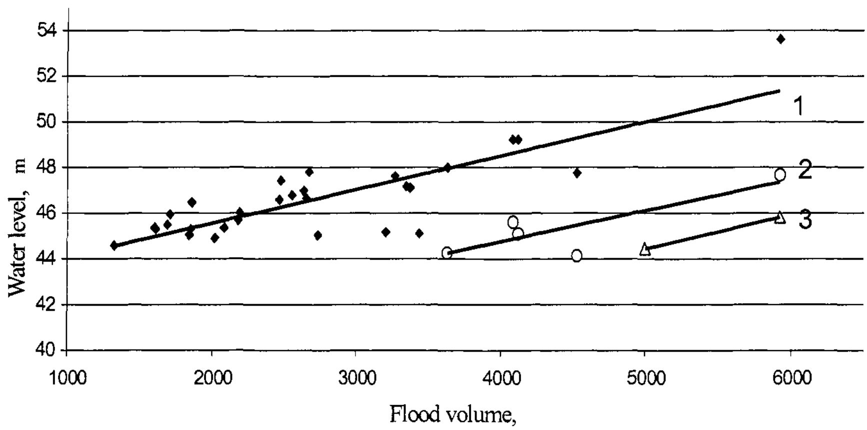

Figure 3: Water level dependence on flood volume in cases: 1 spillway operates, 2 spillways operate, 3 spillways operate.

The last step is to estimate the critical water level overtopping estimation. Let u is water level near the dam. If the water level exceeds u the overtopping

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occurs. Moreover, the dependencies between water level and flood volume are fixed and the flood volume empirical distribution is known it is possible to calculate overtopping probabilities under each scenario for any water level near the dam body. The general water level u overtopping probability can be calculated under the following formula:

$$P\{h > u\} = \sum_i \left(P\{x > V_m^i(u)\} \cdot P\left(A^{(i)}\right) \right), \; i = 1,2,3, \tag{5}$$

where

$$P\{x > V_m^i(u)\} = \int_{V_m^i(u)}^{\infty} f_i(x)dx \tag{6}$$

is flood volume V_m exceeding probability under each scenario i;
$P\left(A^{(i)}\right)$ - the spillways failure probability under each scenario.

2.3 Results and sensitivity analysis

The highest Kaunas Dam lift (the temporary water level designed to reduce spring flood flow) is 45.6 meters and the highest dam altitude is 48 meters, so it is important to estimate water level possibility to reach mentioned heights. There were picked up four critical dam heights and calculated water level overrun probabilities under three scenarios. The most interesting probability in this case is for water level to reach 48.0 meters height - overtopping, and according international literature almost 50% leads to dam failure. Taking into account calculated dependencies and extreme values distribution (EVI) the overflow probability with any given water level could be evaluated.

Table 2: Kaunas dam critical water level overrun probabilities

	Critical water level overrun probabilities under three scenarios			
Level, m	1 spillway operates	2 spillways operate	3 spillways operate	General overtopping probability
48.0	4,8E-02	1,5E-03	4,5E-04	**4,9E-04**
47.0	1,0E-01	3,9E-03	1,0E-03	1,2E-03
46.0	2,0E-01	9,7E-01	2,4E-03	2,7E-03
45.6	3,0E-01	1,4E-01	3,7E-03	4,1E-03

These probability estimations depend on the outlet failure probability, where one outlet failure probability was assumed 1E-02. This estimation was determined from documented mechanical failure rates from the literature (Dan [5]).

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In our study this estimation was assumed analysing overtopping probability sensitivity under different spillways failure probabilities. The analysis results are presented in Table 3.

Table 3: Overtopping probability sensitivity analysis results

One spillway failure probability	General overtopping probability	Primary probability and modified probability ratio
2,00E-03	4,57E-04	1,08
1,00E-03	4,53E-04	1,09
1,00E-02	**4,90E-04**	**1**
5,00E-02	9,30E-04	1,88

The probability sensitivity analysis showed that if the spillway failure probability go to the zero, the dam overtopping probability is not sensitive to this decreasing. Hence, we can assume that the given failure probability is the most optimal and evaluated quite logical.

2.4 Conclusions

This study analysed the critical overtopping probability estimation for Kaunas hydropower station dam. Basically, there were described simplified probability problems that arise in the theory of storage – we have a random continuous input to the Kaunas reservoir and a rule of release, which depends on spillway failure probability. Also there are several constraints which limit the accuracy of estimations, such as lack of detailed information about dam spillways construction, lack of hydrologic data, incomplete hydrologic analysis.

Though in this analysis were included not much particular KHES features the preliminary probabilistic safety analysis showed that results are close to the international dam failure statistical results (Graham [4]).

The sensitivity analysis reveals that spillways failure probability's reduction do not improve the general overtopping probability. That is dam safety improvement actions, not only to the hydrological system but also to other components and objects of the Kaunas Dam are necessary to insure the stability of the dam body.

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Appendix 1

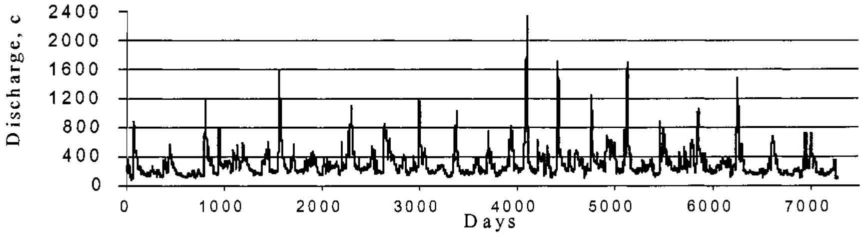

Figure 1: 1920-1939 Birstonas station everyday flow

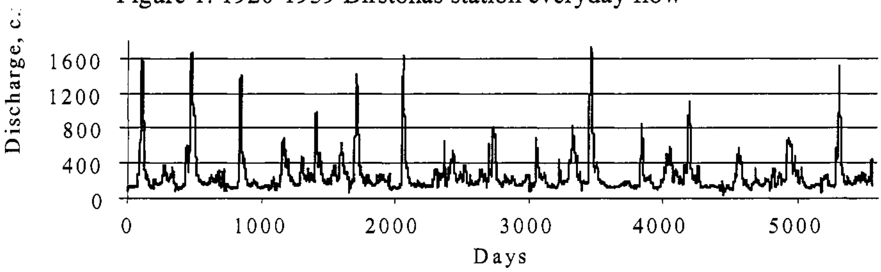

Figure 2: 1940-1956 Birstonas station everyday flow

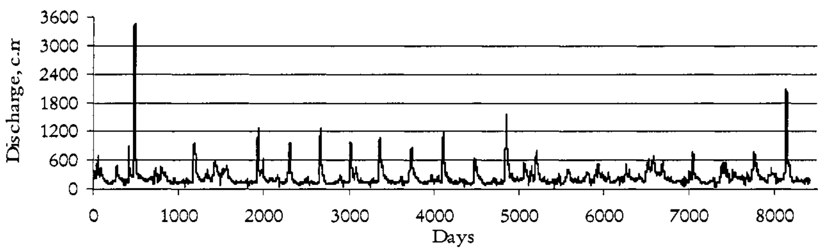

Figure 3: 1957-1979 Nemaniunai station everyday flow

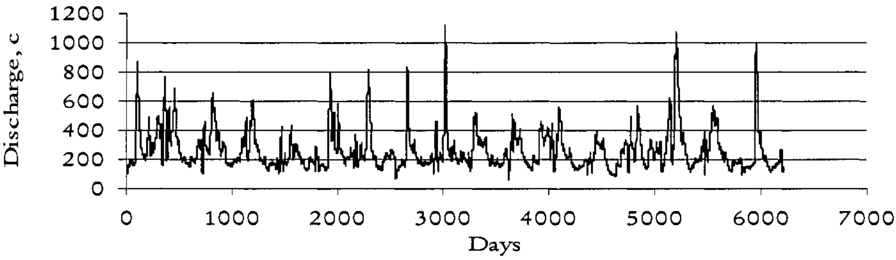

Figure 4: 1980-1996 Nemaniunai station everyday flow