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Probability of failure estimation of current reinforced structures using the Latin Hypercube Sampling

J. Delgado[1], M. de Azeredo[2], R. Delgado[2]
[1]*Viana do Castelo Polytechnic Institute, Portugal*
[2]*Departement of Civil Engineering, Faculty of Engineering*
Porto University, Porto

Abstract

The actual European design codes for reinforced concrete structures are based on partial factors of safety. In order to evaluate the safety margin associated to this type of structures design, a methodology of probability of failure estimation is proposed in the present paper. This methodology is supported by the well-known Latin Hypercube Sampling (LHS) simulation method for input variables random sampling, and by the Curve-Fitting (CF) technique for theoretical distribution functions adjustment to input and output variables. Limiting the number of basic variables to f_c - concrete strength and f_{sy} - steel yield stress, the main objective of this paper is the estimation of the probability of failure associated to Eurocode2 (EC2)[1] design rules and the analysis of LHS method performance. For this purpose two basic structures - a beam and a column - designed by EC2 code were studied and obtained results presented.

1 Introduction

International code procedures for reinforced concrete structures design are based on ultimate limit states and partial factors of safety. Ultimate limit states concept consists on establishing a prescribed actions combination and designing structural elements, assuming that design resistance of concrete and reinforcing steel are not exceeded under that actions combination. In this context, safety is guaranteed, increasing characteristic load values and decreasing characteristic materials resistance, through the introduction of partial factors of safety. Although its advantage in a practical point of view, the adoption of partial factors does not give any information about safety margins associated to structural design, and can not be used when non linear behavior has to be considered. With the present paper we intended to evaluate the safety margins associated to these EC2[1] design procedures by using a proposed methodology of probability of failure estimation. This methodology consists on associating a simulation method with a non-linear Finite Element Method (FEM) computational code, performing a reliability analysis of structural failure. Reliability analysis procedure starts with the theoretical distribution functions adjustment to input variables. By using a simulation method, input variables are sampled and a set of *n* structural simulations is obtained. Equal number of structural analysis is performed, with a non-linear FEM code, and output results, i.e. failure load on the present study, achieved. Statistical treatment of output results permit to adjust a theoretical distribution function, with CF technique, and in combination with load theoretical distribution function, an estimation of structural probability of failure is obtained. The LHS is the adopted simulation method and its main advantage is the small number of simulations required, when compared to a general random sampling method, like the Monte Carlo Method [2,3]. Below, in Section 2, the reliability assessment basis and methodology procedures are presented. Next, in Section 3, the LHS simulation method is explained and its performance for probability of failure estimation illustrated. In Section 4, by applying the proposed methodology to a beam and a column designed by EC2, probability of failure is evaluated and results obtained discussed. Finally the main conclusions are presented and some ideas about the proposed methodology potentialities for further investigations are given.

2 Reliability analysis

In the next subsections the probability of failure estimation concept, the variables probability distribution functions adopted and the structural output results treatment will be described.

2.1 Probability of failure estimation

A simplified version of the structural reliability problem consists on considering as input variables load S and resistance R. Through this simplification, probability of failure may be calculated as follows (1):

$$p_f = P[(R-S) \leq 0] \tag{1}$$

Considering S and R as Gaussean variables, their mean and standard deviation will be represented by: μ_S - load mean value; σ_S - load standard deviation; μ_R - resistance mean value; σ_R - resistance standard deviation.
Introducing the safety margin concept Z as (2):

$$Z = R - S \tag{2}$$

the mean value and standard deviation of Z become:

$$\mu_Z = \mu_R - \mu_S \tag{3}$$

$$\sigma_Z^2 = \sigma_R^2 + \sigma_S^2 \tag{4}$$

By substituting Z in (1) and considering the well-known standard Gaussean distribution Φ, the probability of failure follows into (5):

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$$p_f = P[(R-S)\leq 0] = P[Z \leq 0] = \Phi\left(\frac{0-\mu_z}{\sigma_z}\right) \tag{5}$$

A final aspect of probability of failure can be achieved in (7), once defined the safety index β (6).

$$\beta = \frac{\mu_z}{\sigma_z} \tag{6}$$

$$p_f = \Phi(-\beta) \tag{7}$$

Probability of failure and safety index are commonly used on safety margins evaluation, and a geometric interpretation of them is presented in Figure 1.

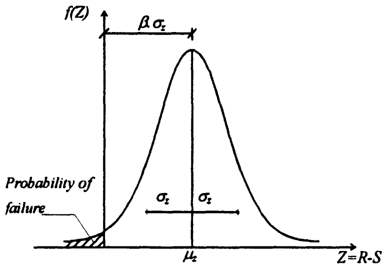

Figure 1: Safety margin geometric interpretation

2.2 Load theoretical distribution

Partial factors of safety for load ultimate limit state of ruin verification are in general equal to 1.5. An interpretation of these partial factors has been proposed by Calgaro [4]. According to this author the partial factor of safety γ_S, can be divided into two parts (8):

$$\gamma_S = \gamma_{md} \times \gamma_s \tag{8}$$

γ_{md}-model uncertainties factor equal to 1.125
γ_s- action uncertainties factor
This study refers that actions uncertainties can be described by its statistical parameters, while model uncertainties still remains as an explicit partial factor γ_{md}. Thus, a first statistical parameter to characterize a Gaussean distribution function to global load variable can be defined adopting as characteristic value the characteristic actions combination values once affected by the combination coefficient ψ and by the model uncertainties factor γ_{md} (9).

$$S_k = \gamma_{md}\left(G_k + Q_{k1} + \sum_{i>1} \psi_{0i} Q_{ki}\right) \tag{9}$$

The second parameter adopted is the variation coefficient (CV_S). Although a precise value for this coefficient could be calculated for each analyzed structural problem, the solution adopted is to calculate the probability of failure, or safety index, for a range of CV_S values between 0.2 and 0.5. This procedure gives us a larger field of information about structural safety margins and avoids the consideration of each individual action uncertainty.

2.3 Resistance theoretical distribution

Resistance theoretical distribution definition of a prescribed structure requires, as a first step, the application of the LHS method to selected basic variables, in order to obtain a set of structural simulations. Selected variables were those affected by partial factors of safety, according to EC2, concrete strength - f_c and reinforcing steel yield stress - f_s.
Increasing the combination actions, resulting from standard structural design, from zero till failure, by FEM code application to each simulation, a set of ruin configurations will be acquired. Each ruin configuration has a resistance or failure value associated, and statistical treatment of output results based on CF technique allows the adjustment of a theoretical distribution function to structural global resistance. In the present paper Gaussean distributions have been adjusted to output resistance results, being these adjustments validations verified by the *Modified Kolmogrov-Smirnov test: Lilliefors version (1967)* [6], statistical test.
LHS simulation method application to selected basic variables implies that their distribution functions are previously defined. Based on Calgaro [4] proposes, concrete strength - f_c and reinforcing steel yield stress - f_s theoretical distribution functions will be presented.

2.3.1 Concrete theoretical distribution definition

Partial factor of safety defined by EC2 [1] for concrete strength is γ_C =1,5. According to Calgaro [4] this factor can divided in two parts (10):

$$\gamma_C = \eta \times \gamma_c \tag{10}$$

η - factor converting differences between, concrete strength measurement in laboratory or in field, and equal to 1,15.
γ_s - concrete strength uncertainties factor.
Concrete strength uncertainties can be represented by its statistical parameters, while η remains as explicit partial factor of safety. Once EC2 limits concrete strength to 85% of its design value, this limitation has been introduced as an explicit factor as well. Assuming concrete strength as a Gaussean variable, two parameters are needed to define its distribution function. Considering a prescribed concrete class, the characteristic value f'_{Ck} may be obtained from the EC2 design characteristic value f_{Ck}, once divided by η and multiplied by 0.85, as shown in (11).

$$f'_{Ck} = \frac{0.85 \cdot f_{Ck}}{\eta} \tag{11}$$

The second parameter is the variation coefficient CV_{fc}, dependent on the characteristic and mean concrete strength values, but rising CV_{fc}=0,17 for current concrete classes.

2.3.2 Reinforcing steel theoretical distribution definition

Reinforced steel yield stress partial factor of safety, prescribed by EC2, is γ_s =1,15. Reliability analysis of this factor, made by Calgaro [4], shows that it is only related to reinforcing steel uncertainties, and that its coefficient of variation

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is in general equal to CV_{fS}=0,087. Therefore a Gaussean distribution representing reinforcing steel can be established by its variation coefficient CV_{fS} and the yield stress characteristic value prescribed by EC2[1].

3 Latin Hypercube Sampling

LHS is a well-known simulation technique used by several authors [2, 3, 5] in behaviour, failure and reliability analysis. The strategy used in this method is quite simple. Considering M basic variables X_j (X_j, $j=1,M$) of a structural problem, the cumulative distribution function of each variable $F(X_j)$ is divided into N non-overlapping intervals of equal probability, e.g., Figure 2, and each interval is represented for its centroid C_{ij} (C_{ij}, $i=1,N$; $j=1,M$).

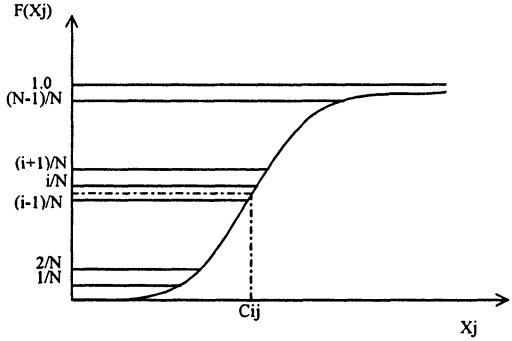

Figure 2: Division of the probability distribution function.

The LHS technique consists on a random simulation process of each variable centroids, providing N simulations (equal to the number of intervals of the probability distribution functions). The main criterion of this random simulation implies that each interval is used only once during the process of random simulation. Thus centroids are obtained as random permutations of integers $1,2 \ldots N$. The use of simulation techniques in structural analysis requires a number of structural analysis equal to the number of shots. A high number of shots may become an excessively time-consuming, particularly in non-linear reinforced concrete structures applications. The main advantage of this method is the small number of shots needed, when compared to other simulation techniques. According to several authors tens to hundreds shots are enough to achieve good results. The regularity of probability intervals of distribution functions is the basis of the good performance of the method [2].

In order to discuss the applicability of LHS, this method has been applied 10 times to study load failure values of a beam (B1) described in Section 4 (see Figure 3). Selecting as input variables, f_c - concrete strength and f_{sy} - steel yield stress, a number of shots equal to 30 have been adopted for each LHS application. An illustration of the random simulation obtaiend with LHS can be observed further in Section 4 for the studied examples. Statistical treatment of failure load output results, obtained from the 10 LHS applications, allowed the

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calculation of its mean, characteristic, and standard deviation values, which are presented in Table 1.

LHS application	Mean or nominal value	Characteristic value	Standard deviation	LHS application	Mean or nominal value	characteristic value	Standard deviation
1	147.23	133.00	8.65	6	147.33	130.42	10.28
2	147.30	131.62	9.53	7	147.13	132.77	8.72
3	147.37	131.86	9.43	8	147.18	132.53	8.90
4	147.28	130.79	9.32	9	147.42	130.00	10.59
5	147.46	131.96	10.13	10	147.41	131.22	9.84

Table 1: Statistical parameters of beam failure load distribution

Although using only 30 shots, the observation of Table 1 illustrate that results of each LHS application are very close to each other. Failure load mean value is near by the same for the 10 applications. Analysing the characteristic value, a reference value corresponding to the lower limit of 5% of its distribution function and influenced by the distribution dispersion, we concluded that the major difference between its maximum and the minimum is smaller than 2.3%. According to these results the applicability of LHS as random simulation method for the proposed reliability analysis is confirmed, once its main advantage of requiring a small number of shots did not influenced the output results distribution.

4 Examples

The proposed methodology to evaluate structural safety margins has been applied to a beam (B1) and a column (P1) of current reinforced concrete. These examples were designed according to partial factors of safety European code rules. The adopted materials were concrete class C20/25 and reinforcing steel class S400. The description of the examples, the methodology application and obtained results will be presented simultaneously and from now on quoted as B1 and P1. Structures geometry, elements width and height, actions, tensile reinforcements according to structural design and adopted mesh for FEM structural analysis will be presented in Figure 3.

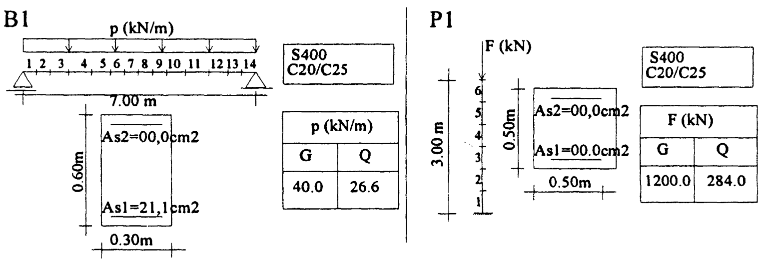

Figure 3: B1 and P1 definition

In correspondence with EC2 [1] material mechanical properties for concrete class C20/C25 and steel class S400 are presented in Table 2. Elasticity modulus

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has not been considered as a random variable and, consequently, only their nominal values are presented.

Variable	Mean or nominal value	characteristic value	Standard deviation	Variation coefficient
Concrete strength, f_c	28 [Mpa]	20 [Mpa]	4.86 [Mpa]	0.174
Concrete elasticity modulus *Ec*	29 [GPa]	-	-	-
Steel yield stress, f_s	467 [Mpa]	400 [Mpa]	40.6 [Mpa]	0.087
Steel elasticity modulus *Es*	200 [Gpa]	-	-	-

Table 2: Material mechanical statistical parameters

4.1 Load and materials theoretical distributions

Load and material Gausean distributions will be defined by its characteristic and variation coefficient values. According to §2.2 the characteristic load value could be achieved by (9) and the variation coefficient CV_S has been considered with a range of values between 0.2 and 0.5. Under these considerations the load Gaussean distribution, for both examples B1 and P1, can be defined respectively by (12) and (13).

$$L_S^N(75.0(kN/m), CV_S, (0.2 to 0.5)) \tag{12}$$

$$L_S^N(2120(kN), CV_S, (0.2 to 0.5)) \tag{13}$$

Considerations made in §2.3 and material mechanical properties presented in Table 2 permit now the establishment of its Gaussean distributions (14) and (15).

$$L_{fc'}^N(14.78, 0.174) \tag{14}$$

$$L_{fs}^N(400, 0.087) \tag{15}$$

4.2 LHS simulation method application

As already referred the simulation process has been made admitting a number of shots equal to 30. Adopting an independent random simulation for each example, representative points of basic variables are presented in Figure 4.

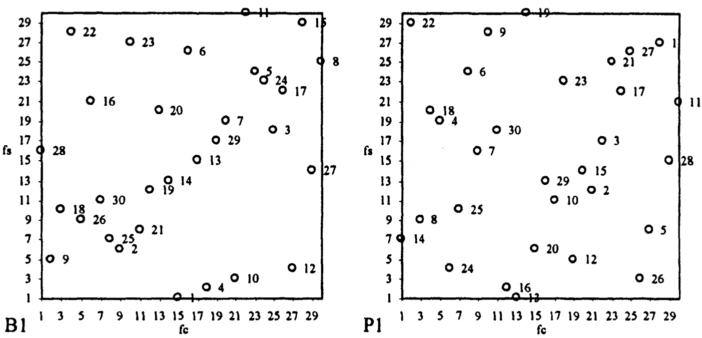

Figure 4: B1 and P1random simulation order

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4.3 Structural analysis. Resistance theoretical distribution

Supported by a computational non-linear FEM code, resistance values were obtained increasing load from 0 till ruin. This procedure has been applied to the 30 shots of each example, and the set of failure output results (resistances) are shown in Table 3. Statistical parameters of output resistance distributions are presented in Table 4.

Example B1				Example P1			
Simulation	R_i	Simulation	R_i	Simulation	R_i	Simulation	R_i
1	161.46	16	124.97	1	5142.5	16	4366.7
2	142.34	17	152.87	2	4664.4	17	6116.0
3	147.31	18	140.87	3	5995.3	18	3933.7
4	141.17	19	164.00	4	5372.5	19	4914.8
5	140.58	20	134.05	5	5790.8	20	4990.7
6	148.00	21	156.19	6	5218.0	21	4834.0
7	141.46	22	147.02	7	5530.4	22	4106.5
8	130.73	23	151.70	8	7099.5	23	4751.7
9	154.83	24	127.31	9	3698.4	24	5888.4
10	140.39	25	135.22	10	5610.9	25	4572.7
11	156.00	26	131.80	11	5696.5	26	4245.1
12	133.36	27	158.43	12	6255.8	27	6663.6
13	119.70	28	148.97	13	5292.2	28	3263.1
14	123.02	29	141.95	14	5064.8	29	5450.1
15	144.00	30	144.09	15	6428.6	30	4474.5

Table 3: R_i values for examples B1and P1

	λ_R resistance factor			
	Mean or nominal value	Characteristic value	Standard deviation	variation coefficient
Example B1	142.94	124.04	11.49	0.080
Example P1	5181.1	3703.9	898.0	0.173

Table 4: Statistical parameters of R

Gaussean distribution functions adjusted to resistance values have been defined by its characteristic value and the variation coefficient (16) and (17). The adjustment validation has been verified by *Kolmogrov-Smirnov* test [6].

$$L^N_{R(B1)}(124.04,0.080) \tag{16}$$

$$L^N_{R(P1)}(3703.9,0.173) \tag{17}$$

4.4 Probability of failure estimation

Along the above subsections, load and resistance distribution functions have been established and justified. According to §2.1it is now possible to estimate the probability of failure p_f (7) and the safety index β (6), associated to each example design. Remembering that load distribution function have been defined for a

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range of variation coefficient, probability of failure and safety index become dependent on this coefficient.
Results of p_f and β are presented in Figure 5 and Figure 6.

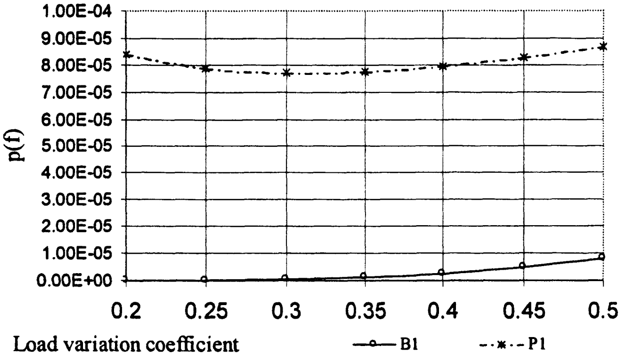

Figure 5: Probability of failure p_f

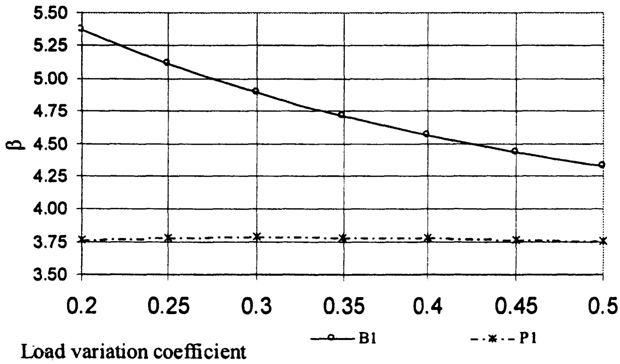

Figure 6: Safety index β

Once we intended to evaluate probability of failure associated to partial factors of safety design, two quite simple isostatic structures have been chosen, in order to introduce the minimum number of variables in this analysis. B1 corresponds to an isostatic beam under a distributed load, subjected to bending moments, and which ruin occurs by reinforcing steel tension failure. P1 is a simple column under axial load, without reinforcing steel, which ruin occurs by concrete strength failure. Thus, for each example, probability of failure is highly influenced by different materials resistance and consequently results achieved are close related with partial factor of safety for reinforcing steel in B1 example and with partial factor of safety for concrete strength in P1 example. Safety margin, associated to ultimate limit state of ruin structural design, is commonly refereed (e.g. Calgaro [4]) as corresponding to a probability of failure between 10^{-04} and 10^{-05}, which corresponds to β values between 3.75 and 4.25. The analysis of Figure 5 and Figure 6 shows that B1 has a very low probability of

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failure never exceeding 10^{-05}, while P1, though inside the accepted interval, has a probability of failure near by the limit value of 10^{-04}. Different level of uncertainties associated to each materials property is the main reason for these results, and leads to the conclusion that the use of different partial factors of safety for steel and concrete are not enough to achieve similar safety margins. As concrete strength has a high uncertainty associated, load variation coefficient has a very low influence on P1 probability of failure. In opposite, despite its high safety margin, B1 is highly influenced by load variation coefficient. Finally a reference to the fact that adopting a range of values for load variation coefficients gave us a larger field of knowledge about structural failure and did not limite conclusions.

5 Conclusions

With the present paper we intended to give a contribution to evaluate probability of failure associated to ultimate limit state of ruin design. The LHS simulation method supported the reliability analysis and its good performance has been tested and confirmed, namely when non linear behaviour has to be considered. The adoption of Guassean theoretical distributions for input variables and output results adjustment resulted reasonable, once confirmed by a statistical test. Applying the proposed methodology of probability of failure estimation to two simple examples, it could be concluded that structural design of structures, which failure is conditioned by steel strength, has a high safety margin associated. Otherwise structures which failure is conditioned by concrete strength have a higher probability of failure but lower than the commonly acceptable limit value of 10^{-04}. Civil engineers always want to achieve safety structures and to evaluate safety margins. The proposed methodology, since it is easy to be used, intends to be a useful contribute to a new structural reliability design concept, in which each structure safety margin could be estimated, without any restriction about considering non linear material or geometric behavior.

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