The use of Voronoi diagram techniques in pocket machining toolpath planning

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Abstract

Since some practical areas of computer science, such as computer graphics, computer-aided design, robotics, and pattern recognition, give rise to geometrical problems, Voronoi diagrams have broad areas of application. Voronoi diagrams also have popular application in machining toolpath planning, using a contour-parallel strategy. When the contour-parallel strategy is applied to the machining toolpath planning, geometric degeneracies can occur and significantly require complicated and time-consuming computations of offset curves. The best way to deal with geometric degeneracies is with the bisector on each endpoint of the contour segment of the Voronoi diagrams. If islands are considered in the toolpath planning, it is essential to calculate the minimum passage width between islands, or between islands and contour segments. Furthermore, if the machining process is taken into account, it is also important to detect the location and thickness of thin walls in a pocket to avoid the machining damage on the working material. This paper presents the incremental algorithms based on the computations of offset curves to calculate the minimum passage width and to detect thin walls in pocket machining toolpath planning.

1 Introduction

Milling operations can be widely used for face finishing, edge finishing, general material removal, etc. When milling operations are considered, tool path generation becomes more important, especially for the NC pocket machining (Figure 1). Modern machine shops usually use sophisticated computer programs to generate instructions (including toolpaths) for NC machines to cut/mill 2½
dimensional parts. In order to generate accurate instructions for toolpaths, it is necessary to deal with the geometry of pockets and the size of cutting tools.

The Voronoi diagram technique has broad applications in industry to calculate offset curves, and thus it becomes more important in the area of machining toolpath planning in CAM for NC pocket machining. In order to determine the appropriate diameter of a cutting tool for pocket machining using a contour-parallel milling strategy, the maximum allowable diameter of the cutting tool needs to be determined, and thus the minimum passage width between islands, or between island and pocket contours should be pre-calculated. The determination of minimum passage width can be based on the calculation of offset curves of pocket and island contours. That is, the minimum passage width may be based on calculating increased/decreased (enlarged/shrunk) offset curves. This method can also apply to detect thin walls in prismatic parts. However, when the contour segments are enlarged/shrunk, geometric degeneracies can occur and require the performance of complicated and time-consuming computations of offset curves. Therefore, in order to solve the degeneracy problems, the method used to calculate offset curves becomes more important.

Based on the Voronoi diagram techniques, using the bisectors on every pair of consecutive contour segments is the best way to calculate offset curves. This is because the endpoints of a segment of an offset curve can be completed not only by constructing the elementary offset segment of a contour element, but also by intersecting the elementary offset segment with the bisectors bounding the Voronoi area that corresponds to the defining contour element. That is, the bisectors can be expressed as functions of their offset to the pocket boundary. Therefore, we can say that the best way to deal with geometric degeneracies is with the offset curves, and the best way to deal with offset curves is with the bisectors using Voronoi techniques.

In order to represent the contour profile using Voronoi diagrams, there are two main methods (algorithms) to approach the goal. They are the divide-and-conquer technique developed by Lee in 1982 [1] and a straightforward algorithm presented by C. Yao and J. Rokne in 1988 [2]. In [3, 4], Srinivasan et al. and Meshkat et al. developed an algorithm for constructing Voronoi diagrams based on simple multiply-connected polygonal domains, and this algorithm contains
simple polygonal islands in the pocket contour. Recently, Held [5] presents a fast incremental algorithm to represent the contour profile using Voronoi diagrams.

In modern industry, B-spline curves, known as Boundary-spline curves, have become the standard curve description in the field of CAD and graphics. In order to have better control flexibility over the curves and to fit the needs of modern industry, NURBS, known as Non-Uniform Rational B-Splines, should be applied to machining toolpath planning. This is because NURBS provide a unified mathematical basis for representing free-form entities, such as car bodies and ship hulls, and NURBS algorithms are fast and numerically stable [6]. However, representing closed contours that only consist of straight-line segments and circular arcs are the standard in constructing Voronoi diagrams, and there are no functions describing bisectors of NURBS using Voronoi techniques; therefore, a piecewise straight-line interpolation on the NURBS curves is used in this study. Furthermore, when a piecewise straight-line interpolation is taken into account, the distance between points on the contour becomes important. That is, it is important to know how close the points need to be specified on the contour. In [7], Wilson mentions that the distance between points on the contour depends on the amount of error that we are willing to tolerate in the interpolation algorithm, and he also gives equations to approximate the appropriate value of distance between two points on the contour.

2 Incremental Algorithms

This paper develops two incremental algorithms for calculating minimum passage width and detecting thin walls in the prismatic parts. Minimum passage width can help determine the maximum diameter of a cutting tool, and the detection of thin walls can prevent the machining process from damages on the working materials. In order to find the minimum passage width of a pocket with islands, there are three basic concepts that need to be discussed. They are: determine the location of a point inside/outside the contour, recognize reflex points, and generate monotonous areas.

2.1 Determine the location of a point inside/outside the contour

We can decide whether a point is inside/outside the contour by using the sign of the cross product of the vector $\vec{AB}$ and vector $\vec{AC}$ (Figure 2). Vector $\vec{AB}$ is a vector from point $A$ to the next clockwise point $B$ (point $A$ and $B$ are any two consecutive vertices on the contour) and vector $\vec{AC}$ is a vector from point $A$ to the point $C$ (point $C$ is inside/outside the contour). That is, $P = \vec{AB} \times \vec{AC}$. By using the right-hand rule, if the sign of $P$ is positive, point $C$ is outside the contour; otherwise, point $C$ is inside the contours.
2.2 Recognize reflex points

A vertex in which the internal angle between the segments incident upon the vertex is larger than 180° is called a reflex point. The extended segment lines, lines 1 and 2, shown in Figure 3, have an intersection at reflex point R. Line 1 and line 2 divide the area into 4 subareas (subarea A, B, C, and D). If a point is in subarea A, the point is outside lines 1 and 2. If a point is in subarea B, then the point is outside line 1 but inside line 2. If a point is in subarea C, then the point is inside line 1 but outside line 2. If a point is in subarea D, then the point is inside line 1 and line 2. Given this, once the reflex point is recognized, and if a point is located in area B, C, or D, the point should be considered as inside the contour.

Figure 2: Determine the location of point C

Figure 3: Four subareas divided by line 1 and line 2

2.3 Generate monotonous areas

The concept of "monotonous areas" was originally developed by Martin Held [8] for dealing with the automatic generation of tool paths for pocket machining. Generally speaking, a simple-connected area is called a monotonous area if its boundary contour can continuously and uniformly be shrunk to a point without splitting any offset of the boundary contours into two or more separate connected sets (shown in Figure 4). In order to generate monotonous areas, it is important to find where the bottlenecks take place. If a pair of non-consecutive contour elements define a common bisector and there is one point on the bisector such that 'walking away' from the point for any arbitrary small amount will not decrease the minimal distance to the contour, these elements will form a bottleneck [8]. Figure 5 shows three subareas (subarea A, B, and C) and two bottlenecks in the contour.

Figure 4: Monotonous areas are shrunk to three points.

Figure 5: Monotonous areas and bottlenecks.
Monotonous areas play an important role in the incremental algorithm for constructing Voronoi diagrams. Basically, the purpose of using monotonous areas is to simplify complicated cases of pocket machining. In addition, because generating monotonous areas can help locate the positions where bottlenecks take place, it will eventually find the minimum passage width when the pocket contains no islands.

2.4 The algorithm for calculating the minimum passage width

In order to find the minimum passage width of a pocket with or without islands, there are 7 steps that should be followed.

Step 1. Compute bisectors of every pair of consecutive contour segments.

Step 2. Simplify the length and number of intersections for each bisector using Cones of Influence [8].

Step 3. Check the existence of islands in the pocket. If there are no islands contained in the pocket, go to Step 4; otherwise, go to Step 5.

Step 4. Determine the monotonous areas and find the middle point of every bottleneck. Choose the smallest diameter (2r) as the minimum passage width and go to Step 7.

Step 5. Shrink the contour segments from zero to the distance that first meets a point on any island.

Step 6. Enlarge any island contour from zero to the distance that first meets any other point on an island.

Step 7. Determine the minimum passage width.

In Step 1, the calculations of bisectors, shown in Figure 6, are based on the formulas in [8].

In Step 2, the concept of the Cones of Influence is applied, and the result of this application is shown in Figure 7. The purpose of the Cones of Influence is to shorten the infinite bisectors and those unnecessary intersections. That is, it has the beneficial side effect of shortening bisectors, which results in fewer useless intersections between bisectors. Step 3 is used for checking the existence of islands. If there are no islands contained in the pocket, Step 4 should be followed for generating monotonous areas (Figure 8); otherwise, go to Step 5. In Step 5, it is important to make sure that when we “walk” clockwise on the increased offset lines, all the points on any island are on the right hand side; otherwise, there must be intersections between increased offsets lines and islands (Figure 9).
If there are $i$ islands ($i \geq 2$) in a pocket, the algorithm needs to repeat Step 6 $i$ times, and each time the final offset distance obtained from this step should be saved (Figure 10 and Figure 11). In Step 7, we compare the offset distances that we got from Step 5 and Step 6 and choose the shortest offset distance as the minimum passage width (Figure 12).

2.5 The algorithm for detecting thin walls in prismatic parts

The machining damage on the working material caused by inappropriate cutting speeds, spindle speeds, material feeds, ... etc. is fairly common in industry, especially when the contour-parallel milling operation is taken into account for the pocket machining. Therefore, it is important to prevent the machining damage occurring at thin walls by locating and pre-calculating the thickness of the thinnest wall in prismatic parts. The method of detecting thin walls in the prismatic parts is basically the same as the method of finding the minimum passage width between island and pocket contour walls. That is, simply
consider the interior of the part as an island and calculate the minimum passage width between the island and the outside contour. In order to detect the thickness and location of the thinnest wall, there are 4 steps that should be followed:

Step 1. Compute bisectors of every pair of consecutive contour segments.
Step 2. Simplify the length and number of intersections for each bisector using *Cones of Influence*.
Step 3. Shrink the outside contour segments from zero to the distance that first meets a point on any island.
Step 4. Determine the thickness of the thinnest wall (Figure 13).

![Figure 13: Find the thinnest wall between inside and outside contour walls](image)

3 Program Implementation

The computer program developed for Voronoi techniques is used for generating the minimum passage width when the shapes of pocket and island contours are not only convex hulls but also non-convex hulls. In addition, the program is used to handle the cases involving circular arcs and NURBS as pocket/island contour segments. The program also detects thin walls in prismatic parts and determines the minimum passage width when the pocket contains no islands or multiple islands.

The program uses a double-linked list for representing the contour. That is, every contour object has two pointers, which indicate the next object in counterclockwise and clockwise direction, respectively. In order to save the coordinate information, keeping objects’ starting points and normalized directions for straight lines or the objects’ centers, endpoints, and radii for circular arcs is sufficient. Figures 14-21 show the screen captures while the Voronoi program is executing.

The purpose of this Voronoi program is to construct a Voronoi Advisor. Once the minimum passage width and the thickness of the thinnest wall in the prismatic parts are obtained from the incremental algorithms by reading the input data from CAD (SAT/IGES files), the Voronoi Advisor will output these data (minimum passage width and thin wall thickness) to the Decision Module in CAM and help determine the tool size, speeds, feeds, ... etc. for generating machining tool paths. The flowchart of Voronoi Advisor and CAM can be referred to Figure 22.
Figure 14: Pocket contour
Figure 15: Simplified bisectors
Figure 16: Monotonous areas
Figure 17: Minimum passage width (no islands)
Figure 18: A pocket contains two islands
Figure 19: Minimum passage width
Figure 20: Inside and outside pocket contours
Figure 21: Thickness of the thinnest wall
4 Conclusion

Among the different techniques for metal removal, a milling operation is one of the most popular operations in the industry. When milling operations are considered, it is important to discuss pocket machining since it is one of the common operations in milling. If the method of contour-parallel milling used for the pocket machining is considered, Voronoi diagrams can be particularly useful in dealing with geometric degeneracies.

In particular, representing closed contours that only consist of straight-line segments and circular arcs are standard in constructing Voronoi diagrams. When expressing the bisectors as functions of their minimal distance to the defining contour segments, the computation of these intersections can be calculated and reduced to simple evaluations of bisector formulas. Thus, computing the toolpath consisting of several closed offset curves is not very difficult when the technique of Voronoi diagrams is applied.

From a practical point of view, the Voronoi diagram technique can simplify geometrical computations of deciding the machining toolpath. Other applications that could be considered include constructing offset curves and minimum passage width in 3-dimensional contour segments using Voronoi techniques.
References


