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Size and shape optimization of long portal bridges

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Abstract

Most applications in optimum design of frame structures have been devoted to size optimization. In this paper a multispan bridge having a large number of elements is presented as a case for design optimization. First, a size optimization problem is carried out having a number of design variables varying from 4 to 118. Then size and shape optimization is formulated considering the span length as design variable for geometry optimization. Numerical results obtained show the advantages of this combined approach.

1 Optimum design of flexural systems

Optimum design of flexural systems composed by bar elements is a very well stablished topic on structural optimization. Plastic and elastic behaviour has been considered for many authors using classical mathematical programming or emerging techniques as genetic algorithms or neural networks. References [1-11] represent an overview of results in different problems and typologies including frames, grillages and beams.

While the number of applications is large, all of them are related with size optimization. The design problem is posed usually in terms of the cross section areas of the bars which are considered design variables. Other mechanical parameters, as inertia modulus I, or strength modulus W are linked to area A by using the following expressions

$$I = a_I A^{b_I} \qquad W = a_w A^{b_w} \tag{1}$$

Sometimes, design optimization is made in several steps by carrying out multilevel optimization techniques [12-13]. In that approach, the optimum values of the areas are first identified, then a second optimization problem is solved considering the internal dimensions of the cross section areas of the bar as design variables.

In this paper an application of size and shape optimization of portal bridges will be presented. The set of design variables will be composed by the values of cross section areas A defined as a vector $\underline{A}$, and the set of bars length $\underline{L}$ as indicated in Figure 1. Allowable stresses and vertical displacements will be adopted as constraints and the weight of material will be considered as objective function. Hence, minimization problem may be written as

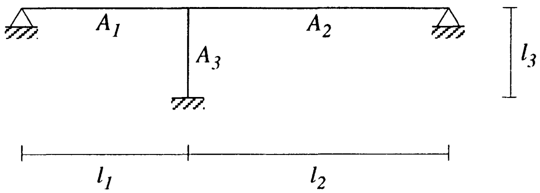

Figure 1. Size and shape of a portal bridge

$$min\ W_g = W_g\,(\underline{X}) \tag{2.a}$$

being $\underline{X}$ the vector of design variables

$$\underline{X} = \underline{X}\,(\underline{A},\ \underline{L}) \tag{2.b}$$

subjected to

$$\sigma_L \leq \sigma\,(X) \leq \sigma_U \tag{2.c}$$

$$u\,(X) \leq u_U \tag{2.d}$$

where

W_g weight of material
$\underline{A}$ vector of cross section areas
$\underline{L}$ vector of bars length
σ representative stress in a structural element
σ_L, σ_U lower and upper stress level of material
u representative vertical displacement of a structural element
u_U upper bound of vertical displacement

2 Sensitivity analysis approaches

Efficient numerical optimization algorithms require first order information of the objective function and the set of constraints of the design problem. Such kind of information is known as sensitivities. First order sensitivity of a structural response ψ with respect to a design variable x_i can be written as

$$\frac{d\psi}{dx_i} = \frac{\partial\psi}{\partial x_i} + \frac{\partial\psi}{\partial \underline{u}}\frac{d\underline{u}}{dx_i} \tag{3}$$

It is well know that expression (3) may be solved by two approaches.

In the direct differentiation method the following expression is finally obtained

$$\frac{d\psi}{dx_i} = \frac{\partial\psi}{\partial x_i} + \frac{\partial\psi}{\partial \underline{u}}\underline{K}^{-1}\left(\frac{d\underline{P}}{dx_i} - \frac{d\underline{K}}{dx_i}\underline{u}\right) \tag{4}$$

Alternatively, the adjoint approach leads to

$$\frac{d\psi}{dx_i} = \frac{\partial\psi}{\partial x_i} + \underline{\lambda}^T\left(\frac{d\underline{P}}{dx_i} - \frac{d\underline{K}}{dx_i}\underline{u}\right) \tag{5}$$

Both techniques require to obtain the following:

a) Derivatives of vector of loading $\frac{d\underline{P}}{dx_i}$: The main class of loading depending on the design variables is the self weight of the structure. This term is usually neglected in truss systems but it may be important in most flexural systems and it will be included in this paper.

b) Derivatives of stiffness matrix $\frac{d\underline{K}}{dx_i}$: Derivatives of the stiffness matrix with respect to design variables as cross section area or other parameter related to it, as inertia modulus, is quite straight forward. But major difficulties appear if coordinates of nodes are included in the set of design variables. In this case the derivatives usually are not evaluated explicitly, and instead that are carried out by finite differences procedures. In the piece of research presented in this paper the derivatives of the stiffness matrix was done in closed form. Optimization was carried out using ADS [14] software using sensitivities provided by the authors for both types of design variables: cross section areas and nodes coordinates.

3 Structural optimization of long portal bridges

The example included in this paper is a long bridge composed by twenty four spans of 93.75 m of length and forty and three spans 250 m long each one. Because the symmetry only half of the bridge is shown in Figure 2.

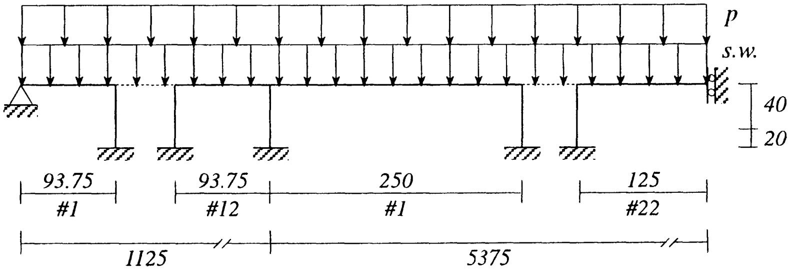

Figure 2. Geometry and loads considered

The design is similar to the Confederation bridge recently built in Canada [15-16]. The actual bridge was designed in concrete but in this paper steel and composite cross sections were considered as indicated in Figure 3.

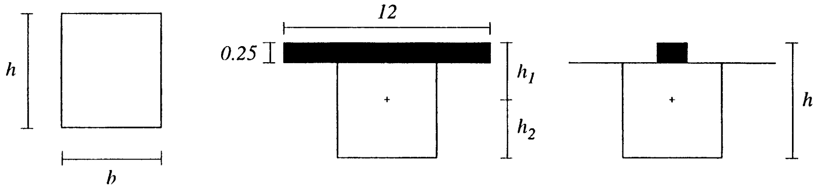

Figure 3. Cross sections considered

Defining I as inertia modulus and W_1, W_2, as the strength moduli corresponding to depth h_1 *and* h_2, respectively the following relationship were set up

$$I = a_I\, A^{bI} \qquad W_1 = a_{w1}\, A^{bw1} \qquad W_2 = a_{w2}\, A^{bw2} \tag{6}$$

Numerical values of each parameter in (6) depended on the cross section areas as follows.

—Columns:

For $A \geq 11000$ cm^2

$$a_I = 2.27 \cdot 10^{-20} \quad b_I = 6.94 \quad a_{w_1} = a_{w_2} = 1.42 \cdot 10^{-9} \quad b_{w_1} = b_{w_2} = 3.72 \tag{7.a}$$

For $A \leq 11000$ cm^2

$$a_I = 5.36 \cdot 10^{-42} \quad b_I = 12.37 \quad a_{w_1} = a_{w_2} = 9.43 \cdot 10^{-21} \quad b_{w_1} = b_{w_2} = 6.52 \tag{7.b}$$

—Bridge deck:

For $A \geq 17000$ cm^2

$$\begin{gathered} a_I = 7.53 \cdot 10^{-13} \quad b_I = 4.94 \quad a_{w_1} = 4.32 \cdot 10^{-6} \\ a_{w_2} = 3.74 \cdot 10^{-6} \quad b_{w_1} = 2.81 \quad b_{w_2} = 2.82 \end{gathered} \tag{7.c}$$

For $A \leq 17000$ cm^2

$$\begin{gathered} a_I = 2.99 \cdot 10^{-13} \quad b_I = 5.21 \quad a_{w_1} = 4.23 \cdot 10^{-4} \\ a_{w_2} = 1.73 \cdot 10^{-7} \quad b_{w_1} = 2.40 \quad b_{w_2} = 3.18 \end{gathered} \tag{7.d}$$

The set of loads considered were self weight of the structure and a distributed load p = 9.36 t/m, equivalent to a service load of 0.4 t/m^2 on bridge deck.

3.1 Size optimization

In size optimization only cross section areas were considered as design variables. Three cases were examined.

1) Four cross section areas: In this design the shorter spans and columns had constant area A_1, A_4, respectively and the longer spans had two different area size for the deck, namely A_2, A_3, as indicated in Figure 4a.

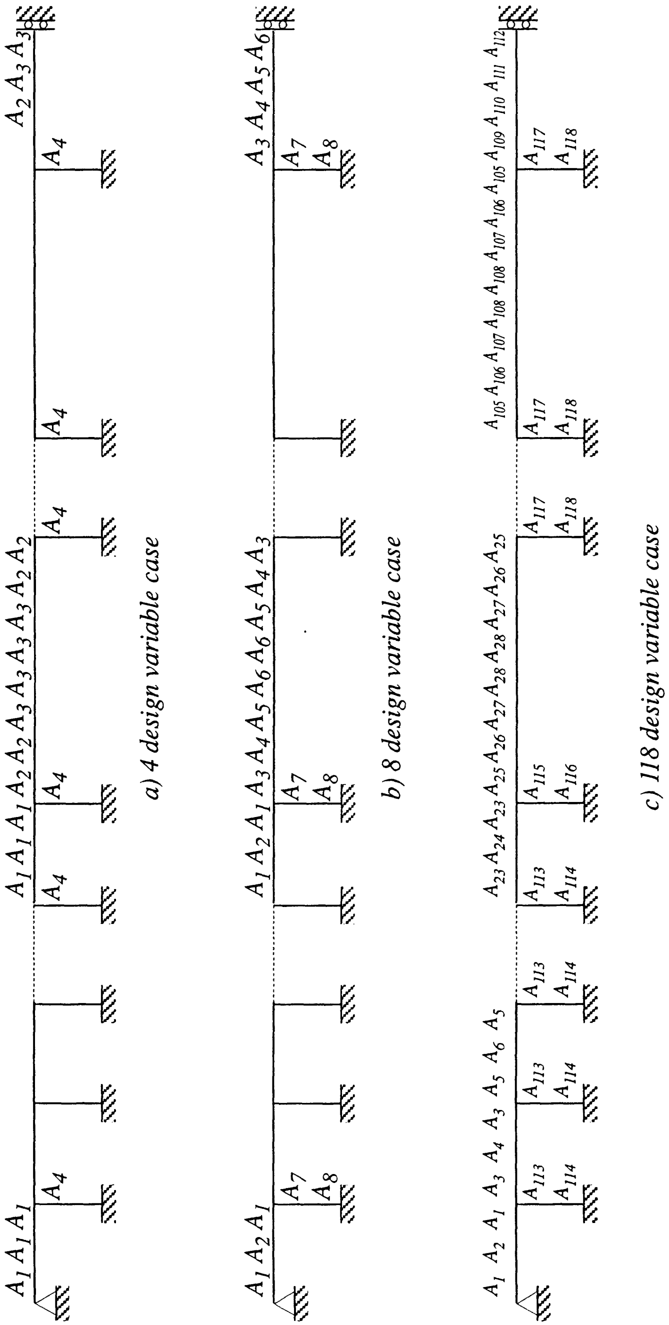

Figure 4. Design variables in size optimization

2) Eight cross section areas: In this case the longer spans had four cross section sizes, A_3, A_4, A_5, A_6, the shorter spans had two different areas A_1, A_2, and also columns had two areas A_7, A_8.
3) A hundred and eighteen areas: The distribution of area variables is indicated in Figure 4c.

For each design case two different problems were considered depending on the class of constraints included:

a) Stress constraints: Allowable stress level, $\sigma_U = -\sigma_L = 2600$ Kp/cm^2.
b) Displacement constraint: Maximum vertical displacement $l/250$, being l any span length.

The optimum values of the objective function are presented in Figure 5.

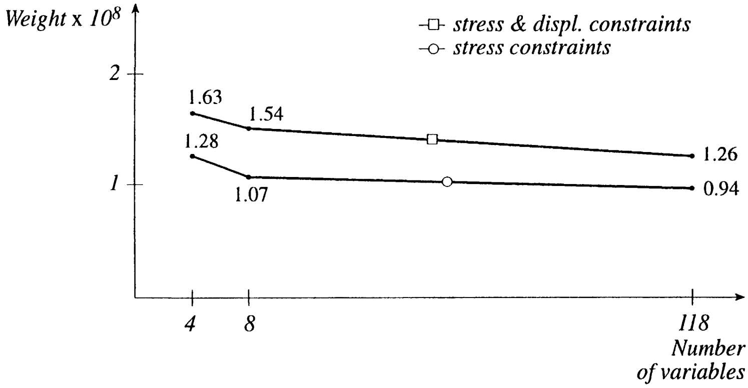

Figure 5. Evolution of objective function values

Optimum values of the objective function and the set of design values for each case appear in Tables 1 to 3.

Table 1. Four size variables case

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	W (Kg)*10^8
Stress	9182	27234	18660	14818	1.28
Stress & Displ	9231	35620	24022	18536	1.63

Table 2. Eight size variables case

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	A_5 (cm^2)
Stress	10580	7950	25390	21314	17114
Stress & Displ	9142	9296	36577	31937	26575
Constraints	A_6 (cm^2)	A_7 (cm^2)	A_8 (cm^2)	W (Kg)*10^8	
Stress	11414	13714	9516	1.07	
Stress & Displ	18799	18800	12401	1.54	

Table 3. A hundred and eighteen size variables case

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	A_5 (cm^2)	A_6 (cm^2)
Stress	9065	8350	8460	7950	7950	7950
Stress & Displ	8488	9249	8488	8048	7982	7950
Constraints	A_7 (cm^2)	A_8 (cm^2)	A_9 (cm^2)	A_{10} (cm^2)	A_{11} (cm^2)	A_{12} (cm^2)
Stress	7950	7950	7950	7950	7950	7950
Stress & Displ	7982	7950	7970	7950	7970	7950
Constraints	A_{13} (cm^2)	A_{14} (cm^2)	A_{15} (cm^2)	A_{16} (cm^2)	A_{17} (cm^2)	A_{18} (cm^2)
Stress	7950	7950	7950	7950	7950	7950
Stress & Displ	7970	7950	7970	7950	7985	7950
Constraints	A_{19} (cm^2)	A_{20} (cm^2)	A_{21} (cm^2)	A_{22} (cm^2)	A_{23} (cm^2)	A_{24} (cm^2)
Stress	7987	7950	7950	7950	12397	8600
Stress & Displ	7985	7950	8045	7950	12246	7950
Constraints	A_{25} (cm^2)	A_{26} (cm^2)	A_{27} (cm^2)	A_{28} (cm^2)	A_{29} (cm^2)	A_{30} (cm^2)
Stress	21912	17776	15686	12038	22827	18919
Stress & Displ	33025	28198	23088	18719	33096	28689
Constraints	A_{31} (cm^2)	A_{32} (cm^2)	A_{33} (cm^2)	A_{34} (cm^2)	A_{35} (cm^2)	A_{36} (cm^2)
Stress	14727	8247	22531	18418	14219	8685
Stress & Displ	23534	15692	32730	28016	22604	15138
Constraints	A_{37} (cm^2)	A_{38} (cm^2)	A_{39} (cm^2)	A_{40} (cm^2)	A_{41} (cm^2)	A_{42} (cm^2)
Stress	22579	18499	14291	8597	22584	18515
Stress & Displ	32806	28194	22855	15025	32846	28245
Constraints	A_{43} (cm^2)	A_{44} (cm^2)	A_{45} (cm^2)	A_{46} (cm^2)	A_{47} (cm^2)	A_{48} (cm^2)
Stress	14303	8539	23708	19750	14270	9772
Stress & Displ	22914	15088	32639	27906	22529	15291
Constraints	A_{49} (cm^2)	A_{50} (cm^2)	A_{51} (cm^2)	A_{52} (cm^2)	A_{53} (cm^2)	A_{54} (cm^2)
Stress	22746	18800	14654	8418	22228	17886
Stress & Displ	33010	28652	23508	15171	31887	26418
Constraints	A_{55} (cm^2)	A_{56} (cm^2)	A_{57} (cm^2)	A_{58} (cm^2)	A_{59} (cm^2)	A_{60} (cm^2)
Stress	13789	9314	25042	22165	18615	9423
Stress & Displ	20656	16193	39044	35940	31644	23665
Constraints	A_{61} (cm^2)	A_{62} (cm^2)	A_{63} (cm^2)	A_{64} (cm^2)	A_{65} (cm^2)	A_{66} (cm^2)
Stress	25390	21314	17114	11415	22218	17943
Stress & Displ	41846	35929	29236	19391	32810	28122
Constraints	A_{67} (cm^2)	A_{68} (cm^2)	A_{69} (cm^2)	A_{70} (cm^2)	A_{71} (cm^2)	A_{72} (cm^2)
Stress	14725	10876	22714	18748	14577	8370
Stress & Displ	22712	15226	32801	28184	22843	15073
Constraints	A_{73} (cm^2)	A_{74} (cm^2)	A_{75} (cm^2)	A_{76} (cm^2)	A_{77} (cm^2)	A_{78} (cm^2)
Stress	22561	18465	14261	8643	22589	18518
Stress & Displ	32765	28116	22747	15008	32777	28142
Constraints	A_{79} (cm^2)	A_{80} (cm^2)	A_{81} (cm^2)	A_{82} (cm^2)	A_{83} (cm^2)	A_{84} (cm^2)
Stress	14315	8586	22584	18508	14306	8600
Stress & Displ	22784	15000	32770	28127	22763	14994
Constraints	A_{85} (cm^2)	A_{86} (cm^2)	A_{87} (cm^2)	A_{88} (cm^2)	A_{89} (cm^2)	A_{90} (cm^2)
Stress	22586	18510	14309	8600	22586	18510
Stress & Displ	32773	28134	22773	14995	32771	28131

Table 3. A hundred and eighteen size variables case (cont.)

Constraints	A_{91} (cm^2)	A_{92} (cm^2)	A_{93} (cm^2)	A_{94} (cm^2)	A_{95} (cm^2)	A_{96} (cm^2)
Stress	14310	8601	22586	18511	14310	8603
Stress & Displ	22768	14994	32772	28133	22770	14994
Constraints	A_{97} (cm^2)	A_{98} (cm^2)	A_{99} (cm^2)	A_{100} (cm^2)	A_{101} (cm^2)	A_{102} (cm^2)
Stress	22586	18511	14311	8603	22586	18511
Stress & Displ	32772	28132	22769	14994	32772	28132
Constraints	A_{103} (cm^2)	A_{104} (cm^2)	A_{105} (cm^2)	A_{106} (cm^2)	A_{107} (cm^2)	A_{108} (cm^2)
Stress	14311	8604	22586	18511	14311	8604
Stress & Displ	22770	14994	32772	28132	22769	14994
Constraints	A_{109} (cm^2)	A_{110} (cm^2)	A_{111} (cm^2)	A_{112} (cm^2)	A_{113} (cm^2)	A_{114} (cm^2)
Stress	23988	19912	15712	10009	9516	9516
Stress & Displ	34674	30034	24672	16897	6515	6574
Constraints	A_{115} (cm^2)	A_{116} (cm^2)	A_{117} (cm^2)	A_{118} (cm^2)	$W*10^8$	
Stress	12381	9516	12517	9516	0.94	
Stress & Displ	15911	11179	11451	7478	1.26	

3.2 Size and shape optimization

When shape optimization was considered in the problem the length of spans were included as design variables. In this way variables L_1 to L_{12} represented length of shorter spans and L_{13} to L_{34} corresponded to longer spans. The total length of each class of spans was considered a parameter. Therefore the following relationship arised.

$$\sum_{i=1}^{12} L_i = 1125 \ m \qquad \sum_{i=13}^{34} L_i = 5375 \ m \tag{8}$$

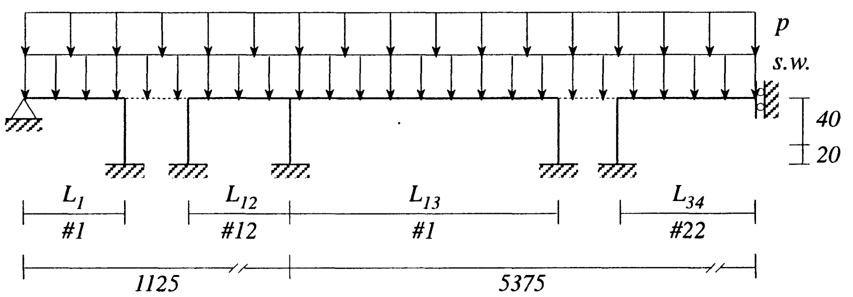

Figure 6. Definition of shape design variables

Design variables representing cross section areas were those aforementioned. Hence, the following cases were worked out.

Table 4. Number of design variables

Span length	*Cross section areas*	*Total*
34	4	38
34	8	42
34	118	152

Again, an optimization problem considering only stress constraints and another problem including also vertical displacement constraints were carried out. In both cases the constraint bounds were those mentioned in the size optimization problem.

Evolution of the objective function with regards to the number of design variables is presented in Figure 7.

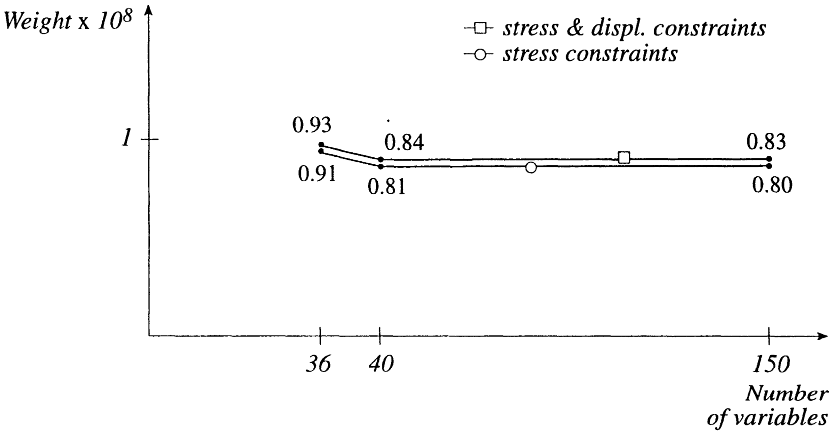

Figure 7. Evolution of objective function values

Values of objective function, cross section areas and span length appear for each case in Tables 5 to 7.

Table 5a. Size and shape optimization (38 variables case)

Constraints	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	L_5 (m)	L_6 (m)
Stress	80.77	98.76	98.77	98.76	98.77	98.77
Stress & Displ	80.76	98.73	98.67	98.66	98.63	98.59
Constraints	L_7 (m)	L_8 (m)	L_9 (m)	L_{10} (m)	L_{11} (m)	L_{12} (m)
Stress	98.76	98.77	98.47	96.64	95.10	62.66
Stress & Displ	98.57	98.52	98.54	98.23	96.31	60.70

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Table 5a. Size and shape optimization (38 variables case) (cont.)

Constraints	L_{13} (m)	L_{14} (m)	L_{15} (m)	L_{16} (m)	L_{17} (m)	L_{18} (m)
Stress	174.45	255.49	274.77	270.67	273.94	254.21
Stress & Displ	199.26	254.52	261.81	262.44	261.71	248.84
Constraints	L_{19} (m)	L_{20} (m)	L_{21} (m)	L_{22} (m)	L_{23} (m)	L_{24} (m)
Stress	273.25	273.66	250.69	154.38	205.75	256.55
Stress & Displ	265.27	261.04	241.89	154.03	212.02	257.94
Constraints	L_{25} (m)	L_{26} (m)	L_{27} (m)	L_{28} (m)	L_{29} (m)	L_{30} (m)
Stress	270.49	271.90	272.65	266.66	251.15	249.00
Stress & Displ	262.11	262.54	262.59	262.60	262.61	262.61
Constraints	L_{31} (m)	L_{32} (m)	L_{33} (m)	L_{34} (m)		
Stress	250.52	249.74	250.13	125.00		
Stress & Displ	262.62	262.62	262.62	131.30		

Table 5b. Size and shape optimization (38 size variables case)

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	W (Kg)*10^8
Stress	8279	21717	10784	9516	0.91
Stress & Displ	8263	21944	10766	10701	0.93

Table 6a. Size and shape optimization (42 size variables case)

Constraints	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	L_5 (m)	L_6 (m)
Stress	79.96	98.73	98.69	98.65	98.34	98.58
Stress & Displ	79.99	98.78	98.75	98.72	98.68	98.65
Constraints	L_7 (m)	L_8 (m)	L_9 (m)	L_{10} (m)	L_{11} (m)	L_{12} (m)
Stress	98.54	98.49	98.52	98.24	96.10	61.89
Stress & Displ	98.62	98.58	98.61	98.36	98.58	61.28
Constraints	L_{13} (m)	L_{14} (m)	L_{15} (m)	L_{16} (m)	L_{17} (m)	L_{18} (m)
Stress	194.91	253.08	265.91	263.85	264.39	243.19
Stress & Displ	199.30	254.73	263.37	264.32	263.67	250.83
Constraints	L_{19} (m)	L_{20} (m)	L_{21} (m)	L_{22} (m)	L_{23} (m)	L_{24} (m)
Stress	265.54	266.87	240.87	149.56	207.44	244.73
Stress & Displ	267.06	261.75	237.39	135.54	207.19	257.29
Constraints	L_{25} (m)	L_{26} (m)	L_{27} (m)	L_{28} (m)	L_{29} (m)	L_{30} (m)
Stress	259.00	264.14	265.15	265.52	265.77	266.01
Stress & Displ	263.66	264.45	264.55	264.58	264.59	264.60
Constraints	L_{31} (m)	L_{32} (m)	L_{33} (m)	L_{34} (m)		
Stress	266.19	266.38	266.47	133.33		
Stress & Displ	264.61	264.60	264.61	132.31		

Table 6b. Size and shape optimization (42 size variables case)

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	A_5 (cm^2)
Stress	8348	7950	20860	14087	10347
Stress & Displ	8355	7950	22064	13910	9997
Constraints	A_6 (cm^2)	A_7 (cm^2)	A_8 (cm^2)	W (Kg)*10^8	
Stress	9158	10700	10700	0.81	
Stress & Displ	11375	10700	10700	0.84	

Table 7a. Size and shape optimization (152 variables case)

Constraints	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	L_5 (m)	L_6 (m)
Stress	80.20	98.56	98.68	98.59	98.60	98.57
Stress & Displ	80.94	98.65	99.59	97.80	98.41	96.19
Constraints	L_7 (m)	L_8 (m)	L_9 (m)	L_{10} (m)	L_{11} (m)	L_{12} (m)
Stress	98.56	98.41	98.93	96.42	95.82	63.64
Stress & Displ	96.37	93.71	93.45	93.14	92.44	84.32
Constraints	L_{13} (m)	L_{14} (m)	L_{15} (m)	L_{16} (m)	L_{17} (m)	L_{18} (m)
Stress	194.19	256.14	264.10	264.44	263.14	243.81
Stress & Displ	208.99	250.06	259.36	260.85	259.79	246.75
Constraints	L_{19} (m)	L_{20} (m)	L_{21} (m)	L_{22} (m)	L_{23} (m)	L_{24} (m)
Stress	263.33	261.15	243.24	152.35	206.60	246.28
Stress & Displ	261.80	258.44	246.79	165.69	204.34	248.35
Constraints	L_{25} (m)	L_{26} (m)	L_{27} (m)	L_{28} (m)	L_{29} (m)	L_{30} (m)
Stress	259.37	262.96	264.86	265.22	265.66	265.95
Stress & Displ	258.09	261.90	263.68	264.59	265.04	265.17
Constraints	L_{31} (m)	L_{32} (m)	L_{33} (m)	L_{34} (m)		
Stress	266.20	266.32	266.40	133.29		
Stress & Displ	264.92	264.40	264.04	131.96		

Table 7b. Size and shape optimization (152 variables case)

Constraints	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	A_5 (cm^2)	A_6 (cm^2)
Stress	8366	7950	8332	7950	8347	7950
Stress & Displ	8431	7951	8344	7950	8394	7950
Constraints	A_7 (cm^2)	A_8 (cm^2)	A_9 (cm^2)	A_{10} (cm^2)	A_{11} (cm^2)	A_{12} (cm^2)
Stress	8343	7950	8346	7950	8347	7950
Stress & Displ	8275	7950	8300	7950	8146	7950
Constraints	A_{13} (cm^2)	A_{14} (cm^2)	A_{15} (cm^2)	A_{16} (cm^2)	A_{17} (cm^2)	A_{18} (cm^2)
Stress	8347	7950	8349	7950	8345	7950
Stress & Displ	8138	7950	7950	7950	7950	7950
Constraints	A_{19} (cm^2)	A_{20} (cm^2)	A_{21} (cm^2)	A_{22} (cm^2)	A_{23} (cm^2)	A_{24} (cm^2)
Stress	8205	7950	8278	7950	8287	7950
Stress & Displ	7950	7950	7950	7950	10406	7950
Constraints	A_{25} (cm^2)	A_{26} (cm^2)	A_{27} (cm^2)	A_{28} (cm^2)	A_{29} (cm^2)	A_{30} (cm^2)
Stress	20451	13678	10394	8997	20619	14026
Stress & Displ	19411	12959	10371	10984	22280	13459

Table 7b. Size and shape optimization (152 variables case) (cont.)

Constraints	A_{31} (cm^2)	A_{32} (cm^2)	A_{33} (cm^2)	A_{34} (cm^2)	A_{35} (cm^2)	A_{36} (cm^2)
Stress	9795	9032	20702	13953	9828	9019
Stress & Displ	8716	11125	22671	13550	8562	11230
Constraints	A_{37} (cm^2)	A_{38} (cm^2)	A_{39} (cm^2)	A_{40} (cm^2)	A_{41} (cm^2)	A_{42} (cm^2)
Stress	20746	13988	9834	8911	20806	14042
Stress & Displ	22703	13527	8484	11219	22521	13520
Constraints	A_{43} (cm^2)	A_{44} (cm^2)	A_{45} (cm^2)	A_{46} (cm^2)	A_{47} (cm^2)	A_{48} (cm^2)
Stress	9893	8977	20875	14072	9877	9019
Stress & Displ	8509	11204	22039	13558	8634	11410
Constraints	A_{49} (cm^2)	A_{50} (cm^2)	A_{51} (cm^2)	A_{52} (cm^2)	A_{53} (cm^2)	A_{54} (cm^2)
Stress	20741	14040	9650	8886	20609	13922
Stress & Displ	22111	13659	8658	11253	22438	13499
Constraints	A_{55} (cm^2)	A_{56} (cm^2)	A_{57} (cm^2)	A_{58} (cm^2)	A_{59} (cm^2)	A_{60} (cm^2)
Stress	9845	8873	20392	14042	10049	8900
Stress & Displ	8530	11166	22847	13505	8875	11137
Constraints	A_{61} (cm^2)	A_{62} (cm^2)	A_{63} (cm^2)	A_{64} (cm^2)	A_{65} (cm^2)	A_{66} (cm^2)
Stress	20547	13780	10418	8794	20345	13718
Stress & Displ	21151	14208	11409	8935	19504	13116
Constraints	A_{67} (cm^2)	A_{68} (cm^2)	A_{69} (cm^2)	A_{70} (cm^2)	A_{71} (cm^2)	A_{72} (cm^2)
Stress	10308	8957	20334	13719	9816	8687
Stress & Displ	10551	10959	21785	13454	8723	11165
Constraints	A_{73} (cm^2)	A_{74} (cm^2)	A_{75} (cm^2)	A_{76} (cm^2)	A_{77} (cm^2)	A_{78} (cm^2)
Stress	20674	14048	9746	8772	20705	14020
Stress & Displ	22226	13614	8662	11302	22451	13683
Constraints	A_{79} (cm^2)	A_{80} (cm^2)	A_{81} (cm^2)	A_{82} (cm^2)	A_{83} (cm^2)	A_{84} (cm^2)
Stress	9646	8780	20785	14036	9713	9005
Stress & Displ	8651	11348	22586	13712	8645	11363
Constraints	A_{85} (cm^2)	A_{86} (cm^2)	A_{87} (cm^2)	A_{88} (cm^2)	A_{89} (cm^2)	A_{90} (cm^2)
Stress	20804	14040	9793	8961	20840	14045
Stress & Displ	22672	13721	8637	11362	22736	13720
8627	A_{91} (cm^2)	A_{92} (cm^2)	A_{93} (cm^2)	A_{94} (cm^2)	A_{95} (cm^2)	A_{96} (cm^2)
Stress	9850	8996	20861	14045	9896	9005
Stress & Displ	8627	11355	22792	13709	8611	11342
Constraints	A_{97} (cm^2)	A_{98} (cm^2)	A_{99} (cm^2)	A_{100} (cm^2)	A_{101} (cm^2)	A_{102} (cm^2)
Stress	20881	14044	9927	9025	20892	14034
Stress & Displ	22827	13682	8584	11320	22773	13654
Constraints	A_{103} (cm^2)	A_{104} (cm^2)	A_{105} (cm^2)	A_{106} (cm^2)	A_{107} (cm^2)	A_{108} (cm^2)
Stress	9954	9049	20879	14036	9939	9048
Stress & Displ	8562	11294	22776	13639	8554	11282
Constraints	A_{109} (cm^2)	A_{110} (cm^2)	A_{111} (cm^2)	A_{112} (cm^2)	A_{113} (cm^2)	A_{114} (cm^2)
Stress	20865	14040	10112	9076	10700	10700
Stress & Displ	22750	13633	8553	11279	10700	10700
Constraints	A_{115} (cm^2)	A_{116} (cm^2)	A_{117} (cm^2)	A_{118} (cm^2)	$W*10^8$	
Stress	10700	10700	10700	10700	0.80	
Stress & Displ	10700	10700	10700	10700	0.83	

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4 Conclusions

Some conclusions may be obtained from the results presented in the paper.

— Size optimization of long civil engineering structures produces a quite important reduction on structural weight without complicating the construction process.

— Optimum design of frame structures has been made frequently, but applications taking in account changes on geometry are scarce.

— Shape optimization of bridges produce even further advantages that those obtained by size optimization. Introducing spans length as design variables allows the material to be distributed along the bridge in a more efficient way.

— For the example posed, optimal design with size and shape design variables leads to almost the same solution, in terms of the structural weight, for stress constraints that for stress and displacement constraints.

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