Optimization of fiber arrangement with CAIO (computer aided internal optimization) and application to tensile samples

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Abstract

The understanding of the internal optimization of trees leads to the development of a method for the optimization of the internal arrangement of fibers in composite materials. The CAIO method is based on FEM and copies biological fiber orientation in order to minimize shear stresses within orthotropic or multilayer composite material structures. Tests on specimens of fiber-reinforced material proved that structures with optimized material directions bear higher loads than unidirectional ones. It is evident that the mechanism of failure changes from shear failure of the unidirectional specimen to an excess of the ultimate tensile strength of the optimized specimen. With the latest version of CAIO, it is possible to calculate fiber orientations in three-dimensional structures. The optimization procedure reduces the shear stresses as well as the principle stresses. CAIO also allows the consideration of local changes of stiffness for curvilinear fiber-reinforced materials.

1 Introduction

With the introduction of the term biomechanics into science, interest in natural constructions within classical mechanical engineering has increased steadily during the past years. It is the objective of this interdisciplinary scientific field to transfer structures and functions that can be observed in nature into practical applications. Some activities in the field of component design and optimization have been pursued by the Forschungszentrum Karlsruhe (Karlsruhe...
Research Center). Bones and trees play a major role as design teachers. Trees optimize their external shape such that stress concentrations are avoided. Using dozens of examples, this self-optimization of the tree shape was studied and the AXIOM OF CONSTANT STRESS was formulated. Wood and bones are structured in an lightweight optimum manner [Mattheck, 1998; Mattheck and Kubler, 1997].

Determination of a relationship between the internal architecture of trees and their strength leads to a better understanding of the tree as a mechanical component and to new design rules, e.g. for fiber-composite materials. Internal optimization of trees is currently being simulated using CAIO. It will be used for the production of technical fiber composites and is verified in first high-strength prototypes.

It is obvious from this internal self-optimization of trees that a very large optimization potential is hidden inside them. Individual mechanisms may well be copied and adapted by engineers. By the technical implementation of optimization strategies with CAO and SKO, which are currently applied in numerous fields of high-tech industry, applicability of natural strategies has already been demonstrated [Teschner, Mattheck, 1997]. While these methods are based on the reduction of reference stresses, such as the Mises stress, and applied above all to isotropic and quasi-isotropic materials, internal optimization by CAIO is particularly suited for orthotropic or anisotropic materials.

2 CAIO (computer aided internal optimization)

It is an advantage of fiber-composite materials that lightweight structures of high stiffness and strength can be achieved. Studies on natural load carriers have demonstrated that shear stresses, which are extremely dangerous for fiber-composite materials, are nearly eliminated by a force flow-tailored orientation of the fibers. Thus, the risk of splitting and formation of shear cracks, which may lead to failure of fiber composites, is averted.

After the example of nature, an FEM-based computer program called CAIO has been developed at the Forschungszentrum Karlsruhe. It allows to orientate maximum stiffness of the material (maximum Young’s modulus) along the flow of force and to considerate variable stiffness.

Based on several examples from nature, such as e.g. branch connections of trees and bone structures, comparative calculations were performed. As a result, a striking agreement between the fiber structures of nature and the computer simulations was found [Reuschel et. al, 1997, Reuschel and Mattheck, 1998].

When using CAIO, an orthotropic material with any orientation is assigned to the FE model to be optimized. With this orientation FE analysis is performed under operating load. The results of the FE stress analysis are read in by CAIO and the new orientation of the material axes is computed. The result of the CAIO computation is a local material orientation which is adapted to the force flow for each element. It serves as input data for the following FE analysis.
The actual version of CAIO is able to consider local variations of stiffness, dependent on the fiber density of the structure. Regions of low principle stresses are defined with the longitudinal Youngs-modulus of the matrix, regions of high principle stresses are defined with the longitudinal Youngs-modulus of the fibers. Between this states a compound formula is used. The application is realized via subroutines which are able to vary the local Youngs-modulus during the FE-analysis.

Alternating application of CAIO calculation and FE-analysis can be continued until a satisfactory reduction of shear stress is achieved. One iteration comprises a CAIO run and an FE analysis. All necessary steps for an optimization with CAIO are displayed in Fig. 1.

The mechanism of shear stress reduction can be easily be understood by means of the Mohr’s circle diagram for stress. For each element of the FE structure the stress state can be defined in Mohr’s circle. In the three-dimensional case, this stress state, the so-called stress tensor, consists of nine normal and shear stresses. By the reorientation of the material axes in the direction of the principle stresses, the shear stress fraction can be eliminated gradually. After this reorientation, the effective stresses are located on the abscissa of Mohr’s circle and are referred as principle stresses.

**Flow Chart**

**CAIO (Computer Aided Internal Optimisation)**

- **FE-analysis**
  - unidirectional orientation of the orthotropic axes

- **CAIO calculation**
  - orientation of the local orthotropic axes along the principal-stress-trajectories

- **FE-analysis**
  - newly arranged orientation of the local orthotropic axes in direction of the force flow
  - Variation of local stiffness depending on local principle stresses
    - **YES** principle stresses
    - **NO**

- **Aim of optimisation achieved**
  - **YES Sufficient reduction of stresses?**
  - **NO**

*Fig. 1: Flow Chart CAIO.*
The principle stresses are calculated from the normal and shear stresses of an element. In the three-dimensional case, the stress tensor is defined as following:

\[
\sigma_{ij} = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}
\]  

(1)

(1) can generally be expressed as \(t_i = \sigma_{ij} \cdot n_j\), with \(t_i\) vector of stress and \(n_j\) normalized direction vector. For the principle axes system \(t_i = \sigma \cdot n_i\) is valid. This leads to \(\sigma_{ij} \cdot n_j - \sigma n_j = 0\). With respect to \(n_i = \delta_{ij} n_j\) (\(\delta = 1\) for \(i = j\), \(\delta = 0\) for \(i \neq j\)) one obtain:

\[
\begin{align*}
(\sigma_{11} - \sigma_1)n_1 + \sigma_{12} \cdot n_2 + \sigma_{13} \cdot n_3 &= 0 \\
\sigma_{21} \cdot n_1 + (\sigma_{22} - \sigma_2)n_2 + \sigma_{23} \cdot n_3 &= 0 \\
\sigma_{31} \cdot n_1 + \sigma_{32} \cdot n_2 + (\sigma_{33} - \sigma_3)n_3 &= 0
\end{align*}
\]

(2)

\(\sigma_{ij}\) = normal or shear stresses,

\(\sigma_i\) = principle stresses (k = 1...3),

\(n_{ij}\) = normalized direction vector.

Additional there is the normalization condition for the direction vectors \(n_1, n_2, n_3\):

\[
n_1^2 + n_2^2 + n_3^2 = 1
\]

(3)

yield the normalized principle stress directions \(n_i\).

Based on the equations above, the normalized vectors for the orientation of the material along the principle stress trajectories are computed by CAIO for each element (or optionally for each node) of a structure. For this purpose, a vector-oriented approach was selected which allows to transfer the results to post-processors in a rather simple manner. The fibers of a 3D-structure can be presented as 3D volume rods.

For the evaluation and representation of the results, network quality is of decisive importance. The finer the network is, in regions of strong changes in orientation, the more precise the fiber lines can be calculated along the principle stress trajectories. In case of an insufficient discretisation, it may even be impossible to obtain reasonable results.

Representation of local orthotropic axes as trajectories is proved to be very useful. This type of representation is also used in the following figures here.
For the technical implementation, the patterns of the trajectories can be output as a group of coordinates with nearly any accuracy. For this, a special program was developed, which converts the points of integrated fiber lines into any ASCII format, e.g. for CNC controls. This method has been applied during the fabrication of matrices for the semi-automatic production of fiber composites.

3 Application of CAIO

Previous workers built also prototypes with drilled in holes and compared it with moduled in holes. They used a steel punch to bypass the continuous fiber around the present circular hole and could find a increase of strength of the factor 1.3-1.7 [Lin and Lee, 1992; Lin and Yang, 1993]. The arrangement of fibers, calculated in this work, is not to compare with the one of the former workers. They neglected transversal contraction effects by pushing a steel punch between the non impregnated fibers. After the impregnation of the fibers with resin the specimen have other material properties. This was not taken in consideration.

A plate with two holes under pure tension loading was calculated with the FEM-program ABAQUS [HKS, 1998] and optimized with CAIO. The Results of the second stress analysis with optimized local orthotropic directions is shown in Fig. 2. The data of calculated fiber directions was transformed in CNC-code and used by a CNC-milling machine. Because no well proved automatically manufacturing technics was available a half automatic stencil technic was chosen. Channels were milled out of an aluminium plate (Fig. 3a) by a numerical milling machine in which the fibers were laid in (Fig. 3b). In a first test the relative strength of two kinds of prototypes were measured. The first type of specimen was a plate with a drilled in hole, the second one was manufactures with an optimized arrangement of fibers. The fiber content of all prototypes was constant with glass fibers as fiber material and resin as the matrix.

Fig. 2: Arrangement of material of a tension-loaded plate with two holes.
Fig. 3: a)Stencil for manufacturing prototypes, milled out of an aluminium plate, b) resin impregnated fibers taken out of the stencil.

Tensile tests of the specimen were made with an Instron-1185. Shear cracks in the early state of tension test, localized beside the hole, characterized the damage process of the non-optimized prototypes (Fig. 4). The optimized specimen failed spontaneous in less than 1/25 second, caused by a overload breakage, how video-recordings showed (Fig. 4). The strength of the optimized samples was two times as high as of unidirectional (drilled in) prototypes.
4 Optimization of three dimensional structures

Three dimensional structures can be optimized with CAIO as well as shell constructions or two dimensional samples. As an nontrivial example a cantilever beam with different kinds of load can serve. The geometry and boundary conditions are displayed in Fig. 5. Below in Fig. 6 are the corresponding optimal fiber arrangements, displayed in the cross-section of the cantilever. All fibers are orientated along the principle stress trajectories. In case a), where an axial pressure is attached at the end of the beam the trajectories are nearly parallel at the right end the lines are merging in one point (Optimization for minimum principle stresses). The force flows through a narrow channel out of the structure (left lower corner in Fig. 5a). Case b) displays a bending dominated part of the cantilever beam where the force is applicated and a shear stress dominated part with principle stress trajectories in a 45° helix where the cantilever is restrained to the ground.

These and other examples can help the engineer to take the force flow into consideration. In addition to this optimized fiber structures can be developed with CAIO. Cooperations about the optimization of two- and three dimensional structures with different partners of the aircraft industry are currently running.
Fig. 5: Optimization of three dimensional cantilever beam: a) pressure in negative x-direction b) pressure in negative y-direction.

5 Summary

Technical use of CAIO is possible due to the extensions of the method presented. To make use of the entire potential of technical fiber-composite materials, the fibers of technical components have to be oriented along the force flow. As the fabrication of optimized fiber structures today is still associated with manual work (manual lamination), if the fiber arrangement is taken into account at all, the manufacturing costs are extremely high. For highly loaded technical components, the optimization of fiber orientation with CAIO and primarily its combination with CNC allows to automate the fabrication processes with a simultaneous stress optimization and, hence, to reduce the production costs. The concept of a mechanically optimized "ecological design" which is actually used by the industry by CAO (Computer Aided Optimization) and SKO (Soft Kill Option) has been extended by CAIO. This includes the local variation of stiffness, dependent on the fiber content, represented by the law of mixture, depending on the principle stresses. Tensile tests show a significant increase of strength of optimized structures. It is also possible to perform three dimensional calculations of optimum fiber arrangement with CAIO.
References


