Transactions on the Built Environment vol 37 © 1999 WIT Press, www.witpress.com, ISSN 1743-3509

Genetic algorithm to optimal lay-up in thin composite panels

S. Rajasekaran, S.A.S. Srinivas
PSG College of Technology, Coimbatore,
Tamil Nadu, India
Email: psg/coimbatore@dartmail.dartnet.com

Abstract

The orientation of fibre direction in layers and number of layers of composites play a major role in determining the strength and stiffness. Thus the basic design problem is to determine the optimum stacking sequence of the composite laminate. Many methods are available at present for the design optimisation of structural systems. However, these methods are based on mathematical programming techniques involving gradient search and direct search. These methods assume the variables are continuous. In this paper a different search and optimisation algorithm, known as the Genetic algorithm of Goldberg [1], has been successfully applied to obtain optimal fibre orientation of multi-layer plates, which considers the angle of fibre orientation as a discrete variable. The principle of GA is applied to obtain optimal layers and the orientation of fibres for stiffened square plates for both the symmetrical and antisymmetrical orientation of fibres subjected to uniaxial, biaxial and shear loading. Plates composing two to nine layers were analysed for their critical buckling load.

1 Introduction

Most of the methods used for design optimisation assume that the design variables are continuous. In practice, in structural optimisation, almost all design variables are discrete. A simple Genetic algorithm proposed by Goldberg [1] is used to obtain the optimal fibre orientation of multi-layered composite plates. The simple Genetic algorithm is mainly based on three operators namely Reproduction, Cross over and Mutation. The Genetic algorithm combines

Darwinism principle of survival of the fittest and structural information exchange using randomised operators to evolve an efficient search mechanism avoiding local minimum. The working of simple genetic algorithm is explained by Goldberg [1].

Rajeev and Krishnamoorthy [3] have applied simple Genetic algorithm to the optimisation of two and three dimensional pinjointed members subjected to stress and deflection.

2 Genetic algorithm for optimal orientation of layered composite plates [4]

A simply supported square plate is optimised for its fibre orientation adopting genetic algorithm. The multilayered plate is analysed for buckling. Two to nine layered plate of constant thickness are considered separately. With reference to the middle plane symmetrical and antisymmetrical fibre orientations are adopted. Symmetrical orientation is possible only for all even and odd layers whereas an antisymmetrical orientation is possible only for even layered plate. The discrete variable is only the fibre orientation. A four-bit string is used to represent the variables and the equivalent angle for the decoded string as shown in Table 1. The objective is to find fibre orientation, which gives the maximum buckling load.

Table 1. Binary representation of angles

S.No	Binary Coding	Decoding	Fibre Orientation
1	0 0 0 0	0	0
2	0 0 0 1	1	10
3	0 0 1 0	2	20
4	0 0 1 1	3	30
5	0 1 0 0	4	45
6	0 1 0 1	5	60
7	0 1 1 0	6	75
8	0 1 1 1	7	80
9	1 0 0 0	8	90
10	1 0 0 1	9	-10
11	1 0 1 0	10	-20
12	1 1 0 0	11	-30
13	1 1 0 0	12	-45
14	1 1 0 1	13	-60
15	1 1 1 0	14	-75
16	1 1 1 1	15	-80

Transactions on the Built Environment vol 37 © 1999 WIT Press, www.witpress.com, ISSN 1743-3509

Binary coding system is used to represent the variable and substring of 4-bit length is used. A total length of (n/2) x 4 bit represent one solution for even layers and [{(n+1)/2} x4] bit for odd layers. If the layers are even and is symmetric lay-up the same orientation is repeated for the other half and if the layers are odd and is symmetric, the middle orientation is same and the remaining angles are repeated. In case of antisymmetric lay-up the angles are repeated with change in sign for the other half. The sample orientation is shown in Table 2. First depending upon the number of layers, the number of populations for one generation is determined and the populations are generated randomly for one half of the plate only. The binary strings are decoded and the corresponding angles for fibre orientations are taken from the Table 1. The plate is analysed and the buckling load for each population for the given loading is computed using the software FEAST-C [5]. The objective is to get the optimal fibre orientation to give the maximum buckling load. The fitness for each population is obtained by dividing the buckling load by the highest load in that generation. Hence the population having highest buckling load will have fitness of 1 and the population having least buckling load will have a lesser fitness. According to the fitness, the individuals get copies in the mating pool and these individuals are the best populations in this generation. The best populations are mated randomly and crossed at random lengths of the full string. The process is repeated till to get the maximum buckling load.

Table 2. Sample fibre orientation for different layers

Layers	N/2	Sample Decoded Angle	Symmetric Layup	Antisymmetric Layup
2	1	10	10 10	10 -10
3	2	30 60	30 60 30	-
4	2	30 45	30 45 45 30	30 45 -45 -30
5	3	20 40 60	20 40 60 40 20	-
6	3	30 45 60	30 45 60 60 45 30	30 45 60 –60 –45 –30

2.1 Genetic algorithm: an introduction

In the last five years genetic algorithm have emerged as a practical robust optimisation and search method. First proposed by Holland [2] genetic algorithms are attractive classes of computational models that mimic natural evaluation to solve the problems in wide variety of domains. Pioneering work by Holland, Goldberg ,Dejong Grefensetette Davis [1], Muhlenbein (See Srinivas[4] and others fueling the spectacular growth of GAs. A genetic algorithm emulates biological evolutionary theory to solve optimisation problems. Genetic algorithm comprises a set of individual elements (the populations) and a set of biologically inspired operators defined out the population itself. According to evolutionary theory only the most suited element in a population is likely to survive and generate off spring, thus transmitting the biological heredity to new generation .In computing terms a GA maps a problem on to a set of (typically binary) strings each string representing a potential solution.

2.2 Power of genetic algorithm

Genetic Algorithm combines the Darwinian survival of the fittest procedure. Genetic algorithms are search procedures based on mechanics of natural genetics and natural selection. Genetic algorithm derives its power from the following genetic operators
1) Reproduction 2) Cross over 3) mutation 4) Inversion 5) Dominance 6) Deletion 7) Intra chromosomal duplication 8)Translocation 9) Segregation 10) Speciation 11) Migration 12) Sharing and 13) Mating.

In this paper a simple Genetic Algorithm with Reproduction , Cross over and mutation operators are used for the optimal lay-up of composite laminates. The reproduction operator emphasizes the survival of the fittest in genetic algorithm. Individual strings from a set of population are selected on the proportionate basis for reproduction according to their fitness. Fitness is defined as figure of merit, which is either maximized or minimized. In effective reproduction, individuals with higher fitness values have higher probability of being selected for mating and subsequent genetic action.

2. 3 Working of genetic algorithms

The buckling load of a composite structure varies with the orientation of fibres in the laminates. The basic design problem of composite structure is to find the optimum orientation for maximum buckling load. To find the orientation of fibres genetic algorithm is used .The orientation of fibres in layers is such that the laminates are either symmetric or antisymmetric of layers in the plate and the following scheme has been employed for the genetic algorithm. For symmetric orientation both odd and even number of layers are chosen and for antisymmetric orientation only even number of layers are chosen. In Case of even number of layers the first half of the layers about the middle surface are taken as design variable for GA and second half layer, if symmetric orientation the same layers as the first half are used and if antisymmetric the layers of first half with negative sign are used. Only one half of the layer orientations are used for all the operation of reproduction and cross over.

In this subsection, working of GA is explained with reference to a three layered symmetric orientation of thin composite square plate subjected to biaxial compressive load. The assumed data for square plate is 40 mm side and thickness is 0.8mm.

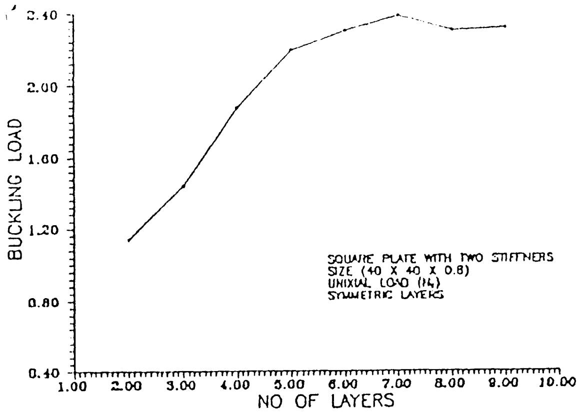

Figure 1. Buckling Load vs Number of Layers (Symmetric)

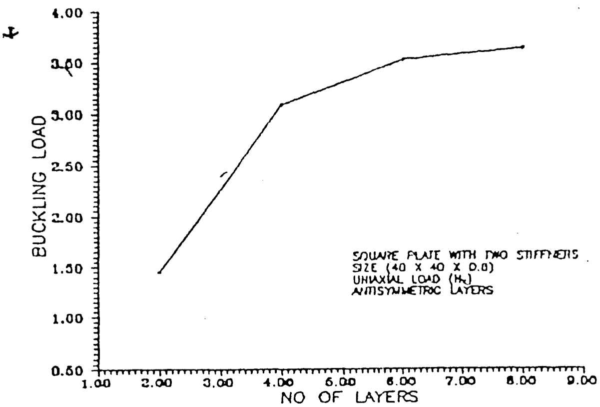

Figure 2. Buckling Load vs Number of Layers (Anti - Symmetric)

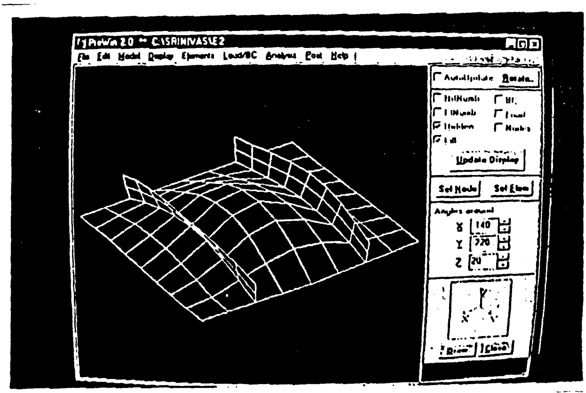

Figure 3. Buckled Shape of Stiffened Plate

$$E_L = 2500kN/mm^2; E_T = 100kN/mm^2; \gamma = 0.25; G_{LT} = 50kN/mm^2$$

In the example design variable for three layered plate is two. Since only half of the layers are above the middle layer including the middle are considered for the symmetric orientation of fibres. The orientation of fibres can be varied as discrete values and varies from + 90 to – 90.

A four bit binary string is used to code each variable in which case a variable can take 16 discrete values (Since angle varies from + 90 to – 90,it is divided as –80,-75,-60,-45,-30,-20,-10,0,10,20,30,45,60,75,80,90, sixteen angles to select a four bit binary string). Here eight bit-concatenated strings are adopted to represent two design variables.

The number of population depends on the importance of the problem and the complexity involved. The number should be even to facilitate mating. In this example number of populations are limited to eight for the purpose of illustration. The string representing individuals in the population is generated randomly as shown in column 2 of Table 3.

The third column, first value shows the design variable corresponding to layer 1.This is obtained by decoding the first substring of length of four of column 2 and getting the corresponding angle from Table 1.

For example, the substring corresponding to layer 1 from the first string is 1000.The decimal equivalent of the binary number is 8. The fibre orientation represents from 1 to 16. Now the angle for the first variable is 9^{th} from the list which is 90 degrees the count starts from 0 not from 1.Similarly, other strings are also decoded and the corresponding angles from the list are obtained. After knowing the angles, they are repeated for the second half, which is the mirror image of first half, as it is symmetric. For the above angles in fibres the Buckling load is calculated using Feast-C for each population. Column 4 shows the buckling load for each population.

2.3.1 Reproduction

Having obtained the fitness values for all the population, the next step is to generate the population for the next generation, which are the offspring's of the current generation. The genetic operators reproduction and cross over are used to generate population for the next generation .The reproduction operator selects the best individual from the current population and places them in mating pool. The reproduction process is carried out in the following manner .The population, which gives the highest buckling load, gets two copies and the population, which gives the lowest buckling load dies off. The other populations are getting single copy in the mating pool. This process of reproduction confirms the Darwinian principle of survival of the fittest. The mating pool is shown in column 6 of Table 3.

Table 3. Details of Computation

Population No.	Population	Angles (Decoded)	Buckling Load(KN)	Count
1	1000 1001	90/-10/90	0.345625	1
2	1010 1100	-20/-45/-20	0.362014	1
3	0100 1001	45/-10/45	0.358740	1
4	1100 0101	-45/60/-45	0.410290	2
5	0001 0010	10/20/10	0.309203	1
6	0110 0101	70/60/70	0.323253	1
7	1000 0111	90/80/90	0.273597	0
8	1110 1111	-70/-80/-70	0.313600	1

Cont...

Mating Pool	Mate	CS1	CS2	Population After Cross over
1000 1001	7	1	4	1100 1001
1010 1100	6	1	4	1110 1100
0100 1001	8	1	3	0110 1001
1100 0101	5	2	4	1101 0101
0001 0010	4	2	4	0000 0010
0110 0101	2	1	4	0010 0101
1100 0101	1	1	4	1000 0101
1110 1111	3	1	3	1100 1111

The operator which is responsible for search in genetic space, cross over is carried now. The cross over rate is selected, as 1.That is all the populations in the mating pool are to be mated. Before selecting the cross sites.the individuals in the mating pool are matched. Since there are eight individuals there are four matching pairs. The pairs are selected randomly along the length of the string for each pair. Column 10 shows the population after cross over, which is the population set for Generation 2. Now the same process is repeated for Generation 2 and so on.

It can be observed that the average buckling load in second iteration is more than the previous generation. It clearly shows the improvement among the set of population .As proceeds with more generations, there may not be much improvement among the set of populations and the best individuals with only slight deviation from the fitness of the best individual may progress .The populations get filled by more fit individuals with only slight deviation from the fitness of the best individual so far found and the average fitness comes very close to the fitness of the best individual. Number of generations is left to the personal interest .If the satisfied result is obtained, iteration can be stopped when there is no significant improvement in the performance from generation to generation. In the present study, the convergence criterion used is according to Rajeev and Krishnamoorthy[3], a fixed percentage of 80% to 85% of the population matrix becomes the same when compared to the previous generation, the iteration has been stopped.

2.3.2 Cross over

Cross over operator is applied to the mating pool with a hope that it would create a better string .In this paper strings are selected from the mating pool at random and some portion of the strings are exchanged between the strings. The following types of cross over operators are available in genetic algorithm.1) Single site cross over 2) Two point cross over 3) Multipoint cross over 4) Uniform gross over and 5) Two dimensional cross over. In this paper two point cross over is adopted. In the mating between second and sixth individuals are taking place in the mating pool with cross site 1 as 1 and the cross site 2 as 4 and the mating between first and seven individuals with cross site 1 as 1 and cross site 2 as 4 then the populations after cross over is taken for the next generation.

Thus population obtained after crossing will form new population set for next generation. The process is repeated until to get optimal design with out violation of constraints or with a little violation.

2.3.3 Mutation

After cross over, strings are subjected to mutation. Mutation of a bit involves flipping it, changing 0 to 1 vice versa. Just as PC controls the probability of cross over, another parameter PM (the mutation rate), gives the probability that a bit will be flipped. The bits of string are independently mutated, i.e., the mutation of a bit does not affect the probability of mutation of other bits. The simple genetic algorithm treats the mutation as a genetic operator with the role of restoring lost genetic material .For example, suppose all the strings in population have converged to 0 at a given position and optimal solution has a 1 at that position ,the cross over cannot regenerate a 1 at that position while a mutation could.Thus mutation is simply an insurance policy against irreversible loss of genetic material. The mutation operator introduces new genetic structure in the population by randomly modifying some of its building blocks,helping the search algorithm escape from local minima's traps .Since the modification is not really to the previous genetic structure representing other sections of the search space.

3 Numerical results and discussion

Example 1: Buckling of Thin Composite Square Plate with two-blade stiffener subjected to uni axial and bi-axial and shear loadings.

Transactions on the Built Environment vol 37 © 1999 WIT Press, www.witpress.com, ISSN 1743-3509

Size of Square Plate is 40x40x0.8 mm with stiffeners of each blade 4mm height and .8mm thickness of carbon/epoxy. The boundary conditions assume the plate is simply supported.

CROSS OVER OPERATOR
(Cross over rate: 1)

In this case, cross over operator is performed on the total population. The optimum layers and orientations are found for both symmetric and antisymmetric orientation of fibres in the layers.
The optimum number of layers for symmetric and antisymmetric orientation of fibres for different types of loading is shown in Table.4.

Table 4. Optimal layup for the square with stiffeners

Loading	No of layers	Sym/ Antisym	Fibre Orientation	Buckling load
Uniaxial	9	Sym	10,-10,80,-75,80,-75,80,-10,10	3.830480
	8	Antisym	10,10,-45,75,-75,45,-10,-10	3.630610
Biaxial	7	Sym	-10,60,-80,-75,-80,60,-10	2.386210
	6	Antisym	-30,30,80,-80,-30,30	2.251770
Shear	6	Sym	-45,10,75,75,10,-45	8.222950
	8	Antisym	30,-45,-75,90,-90,75,45,-30	6.884290

4 Conclusions

In this paper, the optimum fibre orientations and the optimum number of layers for thin composite plates with stiffeners under different types of loading is arrived for the maximum buckling load criteria.

(1) It is found that by varying the orientation of fibres, there is a notable change in the buckling load.
(2) It is found that as the number of layers increases, the buckling load also increases.
(3) The maximum buckling load for a square plate with stiffeners for both symmetric and antisymmetric orientation of fibres has been found out for various loading.

5 Acknowledgements

The authors thank the management and the principal Dr.P.Radhakrishnan of PSG College of Technology for giving necessary facilities to the authors to carry out the work reported in this paper.

References

1.Goldberg D.E, (1989) *Genetic Algorithm in Search Optimization and Machine Learning*, Addison – Wesley Publishing Company Inc., Reading Massachusetts.

2. Holand,J.H., (1975) *Adaptation on Natural and Artificial System,* University of Michigan Press, Ann Arbor, Michigan, USA.

3.Rajeeve S. and Krishnamoorthy C. S, (1992) Discrete optimization of Structure using Genetic Algorithms, *Journal of Structural Engg.*Vol 118,No.5 pp 1233-1250.

4. Srinivas,S.A.S (1997) Genetic Algorithm to optimal lay-up in thin composite panels, *ME Thesis, Bharathiar University , Coimbatore.*

5.Vikram Sarabai Space Centre (VSSC), Trivandrum,(1995) *FEAST-C User Manual* , SEG,SDS Group, ISRO, Trivandrum.