Multi-objective optimization in reliability based design

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Abstract

This paper discusses the development of a reliability-based multi-objective design tool for solving structural optimization problems. The developed design optimization tool has the capability to take into account the effects of variability on the proposed design through a user specified reliability design criterion. The nonlinear goal programming method used, provides for a design method that eliminates the difficulty of having to define an objective function and constraints, while at the same time has the capability of handling rank ordered design objectives or goals. In this work, multiple design criteria including structural weight, load induced stress, deflection, and mechanical reliability are considered.

For simulation purposes the design of a pressure vessel cover plate was undertaken as a testbed for the developed design tool. The formulation of this structural optimization problem into goal programming form is presented. The resulting optimization problem was solved using Powell's conjugate directions method. Some numerical test cases are included demonstrating the design tool's capabilities as it applies to this design problem.
1 Introduction

The concept of optimization is intrinsically tied to the engineering design process. The desire to develop and manufacture a product that is of superior performance and reliability than its predecessor is a major driving force in engineering design. As a result, design tools that allow the attainment of these goals in a timely and economical fashion have become essential in the design process. Over the past forty years, the development of numerical optimization techniques has been instrumental in this context.

Since the pioneering work by Schmit in 1960, the use of numerical optimization techniques in engineering design has gained widespread acceptance and popularity [1]. In the spirit of this work and that which has followed since [2] lies the motivation for the work in this paper. In particular, this work was conceived having as its primary objective to develop a reliability based multiobjective design tool. A tool that would have the capability to take into account the effects of variability on the proposed design, while at the same time would provide for a realistic design model capable of taking into account conflicting and multiple objectives. Multiple objectives of interest including structural weight, load induced stress and deflection, and mechanical reliability.

2 Boiler and Pressure Vessel Design Criteria

Pressure vessels are subject to a wide range of service and environmental loading conditions. As defined per the ASME Boiler and Pressure Vessel Code [9] they include:

- internal or external pressure
- weight of vessel and contents
- static reactions from attached equipment, lining, supports
- cyclic and dynamic reactions due to pressure or thermal variations
- impact loading due to fluid shock
- temperature gradients and differential thermal expansion
- Wind and seismic forces

Since each one of these loading conditions may constitute a possible mode of failure, the appropriate loading condition pertinent to the desired design must be identified. In the case of a pressure vessel cover plate, and hence in terms of this paper, an internal pressure loading condition was selected for simulation given its prevalence in real life applications. Associated with this loading condition, a set of core/primary design criteria were likewise selected. In particular, they included static linear strength, static linear stiffness, and
reliability. A brief discussion in reference to these design criteria is presented in the following.

2.1 Static Linear Strength

The concept of the static linear strength design criteria is based on the simple premise that the load induced stress at the critical location in the cover plate should be less than or equal to a maximum allowable/permissible stress. That is,

\[ \sigma \leq \sigma_{\text{max}} \] (1)

Normalizing yields,

\[ \frac{\sigma}{\sigma_{\text{max}}} - 1 \leq 0 \] (2)

It should be noted that the ASME Boiler and Pressure Vessel Code Section VIII Division 1 provides for a maximum allowable stress of one-quarter the ultimate tensile strength [3].

2.2 Static Linear Stiffness

Similarly, the static linear stiffness design criteria is based on the premise that the load induced deflection at the critical location in the cover plate should be less than or equal to a maximum allowable deflection. That is,

\[ z \leq z_{\text{max}} \] (3)

Normalizing yields,

\[ \frac{z}{z_{\text{max}}} - 1 \leq 0 \] (4)

The maximum allowable deflection is user defined and is selected based on engineering judgment and/or design specifications.

2.3 Reliability

The reliability design criteria is based on the theory of probability and statistics. In particular, reliability, as referred to in this paper, is defined as the probability that the strength exceeds the load induced stress at the critical
location in the cover plate [4]. Both strength and stress are observed to be random variables having a normal distribution according to the Central Limit Theorem of Statistics, and are characterized by a mean and a standard deviation [5]. That is,

\[ S \sim N(\mu_s, \sigma_s) \quad \text{and} \quad \sigma \sim N(\mu_\sigma, \sigma_\sigma) \]  

The so-called coupling equation relates reliability, through the standard normal variate \( z \), to the statistical parameters of the normally distributed strength and stress [4]. That is,

\[ z = \frac{\mu_s - \mu_\sigma}{\sqrt{\sigma_s^2 + \sigma_\sigma^2}} \]  

Based on the value of \( z \) in conjunction with the standard normal distribution curve, the associated reliability is determined as [4]:

\[ R = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{u^2}{2} \right) \, du \]  

Numerical computation of the observed reliability is a fairly simple task once the statistical parameters of the normally distributed strength and stress are known. Simpson’s Rule or trapezoidal approximation can be employed to undertake the necessary numerical integration.

Reliability, per its definition, is consequently a function of the strength and stress statistics. As a result, the reliability design criteria imposes the requirement that the observed reliability be equal to or greater than a target reliability. That is,

\[ R \geq R_0 \]  

where \( R_0 \) is the target and \( R(\sigma, S) \) is the observed reliability.

Normalizing and inverting the inequality yields,

\[ 1 - \frac{R}{R_0} \leq 0 \]
The target reliability is user defined and is selected based on engineering judgment and/or customer specifications.

3 Nonlinear Goal Programming Formulation

The standard form of the nonlinear constrained optimization goal programming model is formulated as follows [2]:

Minimize: \( z = \left\{ f_1(d^-, d^+) \right\} \)

Subject to:

\[
\begin{align*}
\text{achievement vector} \\
\text{design constraints (10)} \\
\text{nonnegativity requirement} \\
\text{side constraints}
\end{align*}
\]

To find: \( X = \{X_1, X_2, X_3, \ldots, X_n\} \) design variables

The vector \( z \) is referred to as the achievement vector. It is structured as an ordered set such that a preemptive priority structure is maintained [2]. That is, \( P_1 \) (most important goal) \( > \) \( P_2 \) \( >>> \) \( P_K \) (least important goal) [2]. The dimension of \( z \) represents the number of preemptive priority levels which is equal to or less than the number of objectives (design criteria), and the value of \( z \) will be equal to the zero vector if all the objectives meet their targets [2]. Lastly, it is important to note for clarity that in reference to Eq. (10), \( g_i(X) \) represent the design objectives (criteria) and \( b_i \) the aspired targets.

A variety of methods have been proposed/developed over the years to solve the resulting nonlinear optimization problems. One such approach, is Powell's conjugate directions method.
A reliability based nonlinear optimization design tool for solving the nonlinear goal programming (NLGP) problem of Eq. (10) was developed based on Powell's conjugate directions method. The developed NLGP design tool first minimizes, as nearly as possible, the objectives with the highest priority level. It then proceeds to satisfy the objectives of the next priority, as nearly as possible, without degrading the achievement of any objective in a higher priority level. This process is continued until all priority levels have been considered. At each priority level the search is terminated when the difference between the present and the previous achievement function value becomes sufficiently small. The value of the achievement vector \( z \) will be equal to the zero vector if all the objectives meet their preselected targets.

### 4 Test Cases

To demonstrate the capabilities of the developed NLGP design tool several numerical test cases, as applied to the design of a pressure vessel cover plate, were conducted based upon the preselected design criteria in conjunction with the following service conditions:

- a cover plate made of SA-515-70 grade carbon steel with its edges securely fixed (welded seal: SMAW)
- a maximum allowable stress per ASME Code [9]:
  \[ \sigma_{\text{max}} = \frac{S_{\text{ut}}}{4} = 120 \text{ MPa} \]
- a maximum allowable deflection: \( z_{\text{max}} = 0.1 \text{ mm} \)
- a target reliability: \( R_0 = 0.999 \)
- a target structural weight: \( W_0 = 3.50 \text{ kg} \)
- an inside diameter \( D = 200 \text{ mm} \)
- a maximum uniformly distributed internal design pressure: \( P_{\text{max}} = 4.2 \text{ MPa} \)

The finite element Software Package ALGOR was used in order to recover the stress and deflection response necessary for numerical optimization.

Four test cases were run. In all four cases the maximum positive or negative deviation was limited to be less than 0.01.
Case I.

Minimum Weight Design with Reliability Constraints:

The minimum weight design problem with reliability constraints was solved using the developed NLGP design tool. In particular, the optimization problem focused on minimizing the structural weight of the preselected cover plate as a function of its thickness subject to reliability constraints. In Table 1, the predicted optimum minimum thickness along with the optimum weight and the expected reliability for the cover plate design are listed.

Case II.

Minimum Weight with Stress and Reliability Constraints:

In this case, the minimum weight design problem with reliability and stress constraints at different priority levels was solved using the developed NLGP design tool. Specifically, the optimization problem focused on minimizing the structural weight of the preselected cover plate as a function of its thickness with: (i) weight and stress constraints priority level 1; and (ii) reliability constraints priority level 2. In Table 1, the predicted optimum minimum thickness along with the optimum weight and the expected maximum principal stress and reliability for the cover plate design are listed.

Case III.

Minimum Weight with Deflection and Reliability Constraints,

In this case, the minimum weight design problem with deflection and reliability constraints at different priority levels was solved using the developed NLGP design tool. In particular, the optimization problem focused on minimizing the structural weight of the preselected cover plate as a function of its thickness with: (i) weight and deflection constraints priority level 1; and (ii) reliability constraints priority level 2. In Table 1, the predicted optimum minimum thickness along with the optimum weight and the expected maximum deflection and reliability for the cover plate design are listed.
Case IV.

Min. Weight with Stress, Deflection & Reliability Constraints:

Lastly, the minimum weight design problem with stress, deflection, and reliability constraints at different priority levels was solved using the developed NLGP design tool. Specifically, the optimization problem focused on minimizing the structural weight of the preselected cover plate as a function of its thickness with: (i) weight and reliability constraints priority level 1; and (ii) stress and deflection constraints priority level 2. In Table 1, the predicted optimum minimum thickness along with the optimum weight and reliability for the cover plate design are listed.

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Thickness (mm)</th>
<th>Weight W(h), Kg</th>
<th>Reliability R(h)</th>
</tr>
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<tbody>
<tr>
<td>I</td>
<td>14.20</td>
<td>3.51</td>
<td>0.9999</td>
</tr>
<tr>
<td>II</td>
<td>15.47</td>
<td>3.82</td>
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<tr>
<td>III</td>
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</tr>
<tr>
<td>IV</td>
<td>15.47</td>
<td>3.82</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

5 Conclusion

A reliability based nonlinear optimization method for solving structural optimization problems was developed. This method that affords the design engineer the ability to take into account the effects of variability on the proposed design, while at the same time provides for a realistic design model that takes into account conflicting and multiple objectives. The developed nonlinear goal programming (NLGP) design tool was used to solve several numerical test cases, as applied to the design of a pressure vessel cover plate. These test cases ranged from a minimum weight design with reliability constraints to a minimum weight design with stress, reliability, and deflection constraints at different priority levels. The developed NLGP design tool was able to: (i) take into account the effects of variability on the proposed design; and (ii) yield an optimum design within ASME Code specifications. However, the solution of the preselected numerical test cases also afforded the opportunity to demonstrate the most compelling attributes of the developed
goal programming method, namely: (i) its flexible problem formulation that eliminates the difficulty of having to define an objective function and constraints; and (ii) its ability to yield an optimum design suited to a user specified rank ordered preemptive priority system.

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6 References


