The MINLP optimization of Tunnel intake bulkheads at Sultartangi Hydroelectric Project Iceland

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Abstract

The paper presents optimization of sliding hydraulic steel gates Tunnel intake bulkheads, erected in Sultartangi Hydroelectric Project, Iceland. The gates have been optimised by the Mixed-Integer Non-Linear Programming optimization approach (MINLP). The MINLP optimization approach to structural synthesis is performed through three steps: i.e. the generation of a mechanical superstructure, the modelling of an MINLP model formulation and the solution of the defined MINLP problem. As the discrete/continuous optimization problem of the gates is non-convex and highly non-linear, the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm has been used for the optimization. The accompanied Linked Multilevel Hierarchical (LMH) strategy accelerated the convergence of the mentioned algorithm. The obtained optimal result yields about 30 per cent of savings in investment costs if compared to the design obtained by the classical method.
1 Introduction

This paper presents the optimization of sliding hydraulic steel gate structures, named Tunnel intake bulkheads, which were erected in spring 1999 in Iceland inside the Sultartangi Hydroelectric Project. The gates were optimized by the Mixed-Integer Non-Linear Programming approach (MINLP).

The MINLP approach handles with continuous and discrete variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (dimensions, stresses, strains, weights, costs, etc.), discrete variables are used to express discrete decisions, i.e. usually the existence or non-existence of structural elements inside the defined structure. Different materials, standard dimensions and rounded continuous dimensions may also be defined as discrete alternatives. Since continuous and discrete optimizations are carried out simultaneously, the MINLP approach also finds optimal continuous parameters, structural topology, material, standard and rounded dimensions simultaneously.

Since some structural elements are added or removed from the structure within the optimization process, so that they on the basis of a defined algorithm form various structural alternatives automatically, the paper concentrates on the discussion of structural synthesis. The MINLP discrete/continuous optimization problems of structural synthesis are in most cases comprehensive, non-convex and highly non-linear. The MINLP optimization approach is proposed to be performed through three steps. The first one includes the generation of a mechanical superstructure of different topology, material, standard and rounded dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The optimization of Tunnel intake bulkheads includes a very large number of discrete alternatives, particularly alternatives for rounding of different continuous dimensions like width of the gate, heights of the main gate elements, widths of flanges, heights of webs as well as intermediate distances between vertical and horizontal girders. Rounding of continuous dimensions has essentially increased the comprehension of the optimization problem, because a large number of rounded alternatives have been subjected to each continuous dimension due to the fact that each dimension may be defined inside the wide interval between its lower and upper bound.

The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm has been used for the optimization as well as the Linked Multilevel Hierarchical strategy (LMH), which accelerates the convergence of the mentioned algorithm. Since the number of discrete alternatives and defined binary 0-1 variables have been too high for normal solution of the MINLP, we have developed a new procedure which automatically reduces the number of binary variables for rounded dimension alternatives on a reasonable level.
2 A mechanical superstructure

The MINLP optimization approach to structural synthesis requires the generation of an MINLP mechanical superstructure composed of various topology and design alternatives that are all candidates for a feasible and optimal solution. While topology alternatives represent different selections and interconnections of corresponding structural elements, design alternatives include different materials, standard and rounded dimensions.

The superstructure is typically described by means of unit representation: i.e. structural elements and their interconnection nodes. Each potential topology alternative is represented by a special number and a configuration of selected structural elements and their interconnections; each structural element may in addition have different material, standard and rounded dimension alternatives.

Therefore, the main goal is to find within the given superstructure a feasible structure that is optimal with respect to topology, material, standard and rounded dimensions as well as all defined continuous parameters.

The usage of a high number of units and hence a high number of design alternatives in the superstructure usually leads to a better optimal result, but on the other hand, it can substantially increase the comprehension of the combinatorial problem that may become difficult to solve or even remain insoluble. By including heuristics and experience in the problems of synthesis, we may in many cases reduce the complexity of the optimization problem, particularly by using a less more reasonable number of units. However, the synthesis of large-scale problems always requires a high number of alternative units to be combined.

Since the selection or rejection of each unit implies a discrete decision, the higher the number of structural elements candidating inside the superstructure is, the more comprehensive the discrete optimization that has to be performed will be. Fortunately, not all elements need be subjected to discrete decisions. Thereby, two basic types of structural elements are proposed to essentially reduce the complexity of the discrete optimization:

- *alternative structural elements* which may be selected or rejected and
- *fixed structural elements* which always exist in the optimization.

While fixed structural elements are in general proposed to define border and some unavoidable intermediate structural elements, alternative elements almost always represent the intermediate structural elements of the superstructure. All possible combinations between selected alternative elements and fixed structural elements represent a multitude of topology/structure alternatives within the defined superstructure. A full representation of defined structural elements represents a maximal possible topology/structure. In the opposite case, when all alternative elements disappear, the fixed structural elements define the smallest structure and the smallest topology. An optimal topology/structure usually lies between both mentioned extremes.
3 MINLP model formulation for mechanical superstructures

Non-linear, non-convex, discrete/continuous optimization problem of structural synthesis can be formulated as an MINLP model formulation for mechanical superstructures (MINLP-MS) in the following form:

Economic objective function:
\[
\min \quad z = c^T y + f(x)
\]

s.t.:
Structural analysis constraints:
\[
\begin{align*}
& h(x) = 0 \\
& g(x) \leq 0 \\
& A(x) \leq a
\end{align*}
\]

Logical constraints:
- pure integer logical constraints:
\[
Ey \leq e
\]  
(MINLP-MS)
- interconnection logical constraints:
\[
Dy^e + R(x) \leq r
\]
- logical constraints for continuous variables:
\[
Ky^e + L(d^{cn}) \leq k
\]
- logical constraints for standard dimensions:
\[
P(y^e, y^{st}) + S(d^{st}) \leq s
\]
- logical constraints for rounded dimensions:
\[
V(y^e, y^r) + U(d^r) \leq v
\]

Variables:
\[
\begin{align*}
& x \in X = \{x \in \mathbb{R}^n; \ x^{LO} \leq x \leq x^{UP}\} \\
& y \in Y = \{y | y \in \{0,1\}\}
\end{align*}
\]

The above model formulation is given in the condensed form. More extended and comprehensive MINLP model formulation may be found elsewhere, e.g. in References^5.

In the MINLP model formulation for mechanical superstructures are included continuous variables \(x=\{d, p\}\) and discrete binary variables \(y=\{y^e, y^{st}, y^r\}\). Continuous variables are partitioned into design variables \(d=\{d^{cn}, d^{st}, d^r\}\) and into performance (non-design) variables \(p\), where subvectors \(d^{cn}, d^{st}\) and \(d^r\) stand for continuous dimensions, standard dimensions
and rounding continuous dimensions respectively. Performance variables represent all other non-design variables like cross-section characteristics of structural elements, load, strains, deflections, coefficients for stability analysis, stresses, economical parameters, etc. Binary variables $y^e$ are used to represent the potential existence of each structural element inside the superstructure: a structural element is selected if its assigned binary variable is 1, otherwise it is rejected. Subvectors of binary variables $y^{st}$ and $p^f$ denote potential selection for standard dimension alternatives and rounded continuous dimensions alternatives respectively.

The economical objective function $z$ involves fixed cost charges in the linear term $c^T y$ and dimension dependant costs in the non-linear term $f(x)$. Non-linear structural analysis constraints $h(x)=0$, $g(x) \leq 0$ and linear equality/inequality structural analysis constraints $A(x) \leq a$ represent the rigorous system of the design, loading, stress, deflection, stability, etc. constraints known from the structural analysis.

Linear pure integer logical constraints $Ey \leq e$ are proposed to describe relations between binary variables to avoid equal topology solutions. They also define bounds of the topology. Mixed linear interconnection logical constraints $Dy^e + R(x) \leq r$ restore interconnection relations between currently selected or existing structure elements (corresponding $y^e=1$) and cancel relations for currently disappearing or non-existing elements (corresponding $y^e=0$). Mixed linear logical constraints for continuous variables $Ky^e + L(x^{st}) \leq k$ are proposed to define continuous design variables for each existing structure element. The space is defined only when the corresponding structure element exists ($y^e=1$), otherwise it is empty. Mixed linear logical constraints for standard dimensions $P(y^{st}) + S(x^{st}) \leq s$ define standard design variables $d^{st}$. Each standard dimension $d^{st}$ is determined (the simplified definition) as a scalar product between its vector of $i$ ($i \in I$), standard dimension alternative constants $q = \{q_1, q_2, q_3, ..., q_i\}$ and its vector of $i$ binary variables $y^{st} = \{y^{st}_1, y^{st}_2, y^{st}_3, ..., y^{st}_i\}$. Only one discrete value can be selected for each standard dimension, since the sum of $i$ binary variables $y^{st}_i$ has to be equal 1:

$$d^{st} = \sum_{i \in I} q_i y^{st}_i$$

$$\sum_{i \in I} y^{st}_i = 1$$

Mixed linear logical constraints $V(y^e, y^f) + U(d^f) \leq v$ define rounded continuous dimensions $d^f$. These variables are determined in the similar way as for standard dimensions.
4 The OA/ER algorithm and LNH strategy

For the solution of non-linear and non-convex problems we used the Modified OA/ER algorithm\(^2,^3\), in which many modifications like deactivation of linearizations, decomposition and deactivation of the objective function linearization, use of the penalty function, use of the upper bound on the objective function to be minimized as well as a global convexity test and a validation of the outer approximations have been applied for the master problem.

The OA/ER algorithm consists of solving an alternative sequence of Non-linear Programming (NLP) and Mixed-Integer Linear Programming (MILP) master problem optimization subproblems. The former corresponds to continuous optimization of parameters for a mechanical structure with fixed topology, standard and rounded dimensions and yields an upper bound to the objective to be minimized. The latter involves a global approximation to the superstructure of alternatives in which new topology, standard and rounded dimensions are identified so that its lower bound does not exceed the current best upper bound. The search is terminated when the predicted lower bound exceeds the upper bound.

The optimal solution of complex non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to be obtained. Therefore, the LNH\(^4\) MINLP strategy has been used to accelerate the convergence of the OA/ER algorithm. We hierarchically decompose the original MINLP problem into subproblems which are then easier to solve than the original one. The MINLP optimization of discrete decisions is sequentially performed at different decision levels, starting from the highest (the most important) one. Decision levels are hierarchically classified as:

1. the level of discrete topology alternatives (the highest level),
2. the level of discrete standard dimension decisions (the middle level),
3. the level of rounded continuous dimension decisions (the lower level).

Higher levels give lower bounds to the original objective function to be minimized while lower levels give upper bounds. The MINLP subproblems are iterated about each level until there are no improvements in the NLP solution. Thus, we start with the discrete optimization of topology at the relaxed standard dimensions. When the optimal topology is reached, we proceed with simultaneous discrete topology and standard dimension optimization at the second level. Finally, after the optimal topology and standard dimensions are obtained, the MINLP is carried out once more for complete discrete decisions at the third level. Each higher lever accumulates a global linear approximation of the superstructure model representation to be used at its lower level, which can in this way be solved much more efficiently.

The synthesis of Tunnel Intake Bulkheads was defined as a very large optimization problem with more than \(10^{15}\) discrete alternatives of different
topologies/structures, standard and rounded dimensions. The optimization model contains 2983 binary 0-1 variables of alternatives. Most of them are subjected to rounded dimensions. Since this number of 0-1 variables is too high for a normal solution of the MINLP, we developed a new procedure, which automatically reduces binary variables for rounded dimension alternatives into a reasonable number. In the optimization at the third level are included only those 0-1 variables which determine rounded dimension alternatives close to continuous dimensions, obtained at previous MINLP optimization iterations.

5 MINLP optimization of the Tunnel intake bulkheads

An example of the synthesis of Tunnel intake bulkheads is presented here. These gates were designed, constructed and erected by the Slovenian company Metalna ECCE from Maribor for Sultartangi Hydroelectric Project inside the contract consortium Sulzer Hydro - Metalna ECCE - ESB International. The Sultartangi Hydroelectric Project is located in the lower highlands of southern Iceland on the river Pjorsa some 80 km upstream from the coast. The tunnel intake is regulated by two identical sliding hydraulic steel gates, named Tunnel intake bulkheads, each consisting of four vertical main elements.

Each gate is 14 m high, 6.65 m wide, made from steel St 52-3N (Fe 510) and loaded by hydrostatic pressure force of 16606 kN and earthquake force of 27243 kN. The design horizontal earthquake acceleration is 0.7g! Dynamic pressure on gates is calculated with the Westergaard formula\(^6\). Structural gate topology, defined in the contract documents, was 4-4-4-4/8 (4 horizontal girders for each of the four main gate elements and 8 vertical girders for the entire gate). The optimization of the Tunnel Intake Bulkheads has been made according to the technical design criteria defined in Reference\(^1\) and DIN 19704\(^7\).

The synthesis of the gates has been performed by the MINLP optimization approach. The gate superstructure has been generated in which all possible structures are embedded by topology variation between 3 to 6 horizontal girders for each main gate element and 5 to 9 vertical girders for the entire gate. In the addition, 14 standard dimension alternatives have been proposed for all structure elements including standard thicknesses of sheet-iron plates and 2958 different rounded dimension alternatives have been subjected to the continuous dimensions. In this way, each continuous dimension have been rounded on the obtained optimal whole number (in millimetres) inside its defined bounds.

The optimization model of the sliding gate structure GATOP\(^8\) (GATe OPtimization) has been developed according to the proposed MINLP-MS model formulation for mechanical superstructures. As an interface for mathematical modelling and data inputs/outputs GAMS\(^9\) (General Algebraic Modelling System), a high level language, has been used. The self manufacturing costs of material, sheet-iron cutting, welding and anti corrosion
Figure 1: Tunnel intake bulkheads, vertical section, optimal structure
resistant painting have been accounted for in the economical type of the objective function, subjected to the given design, material, stress, deflection and stability constraints.

As the model is non-convex and highly non-linear, the Modified OA/ER algorithm accompanied with the LMH strategy has been used for the optimization. Structural synthesis of the gate was carried out by our MINLP computer package TOP\textsuperscript{10} (Topology Optimization Program). MINOS\textsuperscript{11} has been used to solve the NLP subproblems and OSL\textsuperscript{12} to solve the MILP master problems.

The optimal solution yields about 30 per cent of savings and optimal topology 4-5-4-4/5 (4 horizontal girders for the lower, 5 girders for the second, 4 girders for the third, 4 horizontal girders for the upper main gate element and 5 vertical girders for the entire gate), see Figure 1.

6 Conclusions

The paper presents the MINLP optimization of the Tunnel intake bulkheads, erected in Iceland. The modified OA/ER algorithm together with the LMH strategy has been used.

The obtained optimal result yields about 30 per cent of savings in investment costs if compared to the design obtained by the classical method. Beside the optimal self manufacturing costs (material, welding, sheet-iron cutting and anti-corrosion resistant painting), an optimal gate topology with the optimal number of girders and plate elements, optimal standard dimensions (all necessary standard thickness of sheet-iron plates) and optimal rounded continuous dimensions (global geometry, all intermediate distances between structural elements and their cross-section sizes) have been obtained.

References


7. DIN 19704, Stahlwasserbauten, Berechnungsgrundlagen.


12. OSL, Optimization Subroutine Library, From IBM, Release 2.