



Optimal and robust design of nonlinear structures under uncertain loads using statistical optimization method

Q. Yu¹, T. Kashiwamura², M. Shiratori¹, T. Yamada¹

¹*Department of Mechanical Engineering and Material Science,
Yokohama National University, 79-5 Hodogaya-ku, Yokohama-shi,
Kanagawa-ken, 240-8501 Japan*

*Email: qiang@swan.me.ynu.ac.jp, masaki@swan.me.ynu.ac.jp,
tyamada@swan.me.ynu.ac.jp*

²*NHK Spring Co., LTD, 3-10, Fukuura, Kanazawa-ku, Yokohama,
236 Japan*

Abstract

The Statistical Design Support System^{[1]-[4]} is based upon the design of experiments (orthogonal array design) and FEM analysis. The orthogonal array is used to define a set of finite element analyses (FEA), and by the analysis of variance of the FEM results, the correlation between the objective functions and the design factors can be expressed as a set of chebyshev's orthogonal functions. And then, the other design processes are treated using these approximate functions (estimation expressions). In this study, the authors proposed a basic concept to evaluate the dispersion due to an uncertain variable (uncontrollable variable), and a method of optimal and robust design of nonlinear structures by SDSS, and gave an example of the optimal and robust design for an artificial ankle joint. It was concluded that the SDSS can be used as an optimum design tool taking into consideration its robustness.

1 Introduction

Generally, two kinds of dispersion should be taken into consideration for the optimization design to maintain its robustness. One is due to the manufacturing

98 Computer Aided Optimum Design of Structures

or assembling errors of the controllable design variables. This kind of dispersion can be evaluated with a first-order second-moment method^[5], and is easy to be treated in the process of the optimization. On the other hand, the second kind of dispersion is due to the uncertainty of the loads or the factors that deal with the way the user handles or uses the product. Here the uncertain factors are referred to uncontrollable variables. Because the uncertainty of the uncontrollable variables cannot be controlled by the engineering designers, and their dispersion should be treated uniformly over whole of the available design space, the dispersion cannot accurately be expressed with the first-order second-moment method.

G. Taguchi introduced The Taguchi Method^[6] on quality improvement. This method is essentially a principle that emphasizes the proper choice of levels of controllable variables in a process for the design of a product. The principle of choice of levels focuses to a great extent on variability around a prechosen target for the design. It is assumed that the majority of variability around the target is caused by the presence of the uncontrollable variables. As a result, the term of robust design (select the levels of the controllable variables) can be applied so as to achieve robustness to inevitable changes in the uncontrollable variables. The Taguchi Method revolves around the use of orthogonal designs where an orthogonal array (inner array) involving controllable variables is crossed with an orthogonal array (outer array) for the uncontrollable variables. However, the crossed array results in a considerable increase in the number of the FEM analysis.

This study deals with an approach for the evaluation of dispersion due to an uncertain factor (uncontrollable variables) using SDSS to reduce the run number of FEM analysis, a new method for the optimum and robust design of nonlinear structures.

2. The Robust Design by Statistical Optimization Method

2.1 Basic Concept

The basic idea about the dispersion evaluation of this method is the similar to the Taguchi Method. However, only one orthogonal array is used in the SDSS to reduce the analytic number, and not only objective functions and but also dispersion due to the uncertainty are quantitatively represented with estimation expressions. The basic concept and flow of the present method is shown in Fig. 1. At the first step, the uncontrollable variable is treated as a controllable variable, so the effectivity of the uncontrollable variable can be evaluated as the same as the other controllable design variables with only one orthogonal array to reduce the run number. Based upon the results of the analysis of variance, the estimation expressions of the objective functions are given taking into consideration the interaction between the uncontrollable variable and all of the controllable factors. And then, the study of the effectivity of the controllable design variables on the dispersion due to the uncontrollable variable is carried

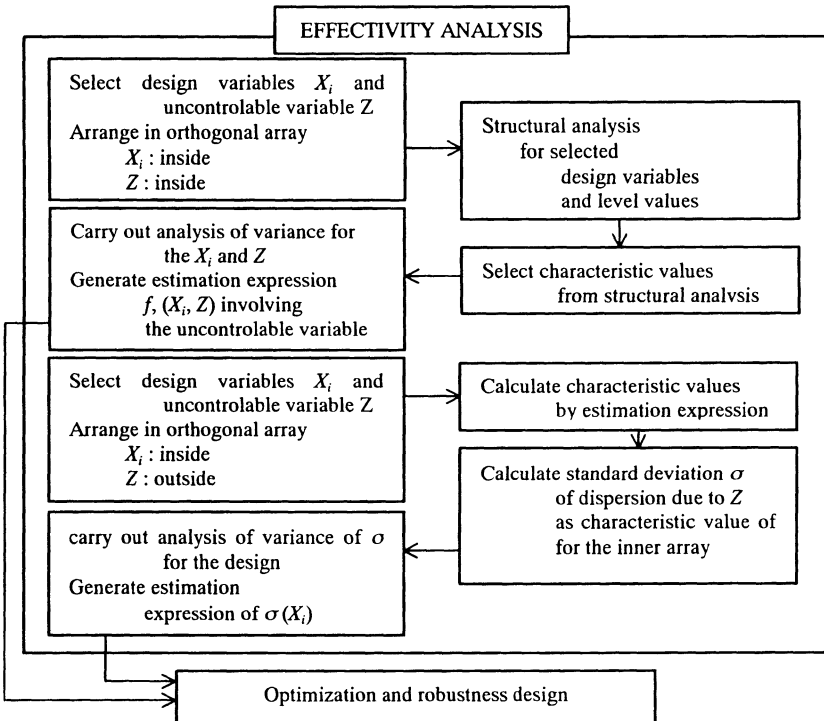


Fig. 1 Analysis flow of optimization and robustness design

out using the orthogonal array designs again, where an orthogonal array involving controllable variables is crossed with a set of levels of uncontrollable variables. Here, only every run was easily carried out using the pregenerated expressions. Following the above treatment, the correlation between the dispersion and the controllable design variables can be given with another estimation expression, when the dispersion due to the uncontrollable variable is assumed as an objective function (to be minimized).

Finally, based upon the expressions of the design objective functions and the dispersion, the optimum and robust design can be calculated with very simple optimization algorithms like SQP, and so on.

2.2 Effectivity Analysis Containing Uncontrollable Variable

The robust design places an emphasis on variance reduction, and the most important concept is to estimate the interaction between the uncontrollable variable and all of the controllable variables. Needless to say, if it is necessary, the interaction among the controllable variables should be considered, too.

The orthogonal arrays like L8, L16, L27, which can be used to estimate the interaction between a special variable (uncontrollable variable) and all of the other variables, are proper choices for this study. For example, three variables

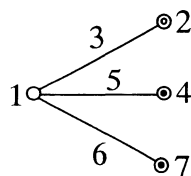


100 Computer Aided Optimum Design of Structures

(X_1 , X_2 , and X_3) are chosen as controllable design variables, and one factor (Z) is chosen as an uncontrollable variable. The design and uncontrollable variables are at three levels. Table 1 shows an example of L8, where uncontrollable factor Z is arranged at No.1 column, and design variables X_1 , X_2 , and X_3 are arranged at No. 2, No. 4, and No. 7, respectively. Therefore, the interactions between Z and X_1 , X_2 , X_3 , are observed as shown in Table 1 (b). After a 8-run analysis, 8 results of the objective functions can be given, and based upon the analysis of variance of the 8 observations, estimation expressions for the objective functions are generated involving the interaction terms of X_1Z , X_2Z , and X_3Z .

Table 1 (a) an orthogonal array L8(2^7)							
	Z	X_1	e	X_2	e	e	X_3
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

(b) a linear graph for L8



2.3 Dispersion Analysis for an Uncontrollable Variable

Because the uncertainty of the uncontrollable variable cannot be controlled by the engineering designers, its dispersion should be treated uniformly over the whole of the available design space. In this method, the dispersion of an uncontrollable variable is study by the array designs shown in table 2, where an orthogonal array involving controllable variables is crossed with a set of levels of the uncontrollable variable.

Table 2 Level value of error variable

Level	Level 1	Level 2	***	Level n
Level value	Z_1	Z_2	***	Z_n

Here, only every run is easily carried out using the pregenerated expressions. For every run number of the inner array, n observations $w_{i1} \sim w_{in}$ are given, where "i" of the subscript of this characteristic value w_{ij} shows the run number of the orthogonal array, and "n" shows the level of the uncontrollable factor. Then the standard deviation σ of $w_{i1} \sim w_{in}$ is used for the index to evaluate the dispersion due to an uncontrollable factor, and σ can be expressed as the following equation. The effectivity of design variables on the standard deviation σ is evaluated conducting the analysis of variance, and the estimation expression of the standard deviation can be made with the Chebyshev orthogonal polynomial function to evaluate the dispersion due to the uncontrollable factor.

Table 3 An orthogonal array design crossed level of uncontrollable variable

		Design Variables							Error Variables	Characteristic value
		<i>e</i>	<i>X₁</i>	<i>e</i>	<i>X₂</i>	<i>e</i>	<i>e</i>	<i>X₃</i>	<i>Z</i>	
Data No.	1	1	1	1	1	1	1	1	<i>Z₁~Z_n</i>	<i>w₁₁~w_{1n}</i>
	2	1	1	1	2	2	2	2	<i>Z₁~Z_n</i>	<i>w₂₁~w_{2n}</i>
	3	1	2	2	1	1	2	2	<i>Z₁~Z_n</i>	<i>w₃₁~w_{3n}</i>
	4	1	2	2	2	1	1	1	<i>Z₁~Z_n</i>	<i>w₄₁~w_{4n}</i>

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^n (\bar{w}_i - w_{ij})^2}{n-1}} \quad \text{" } i \text{ " shows the data number.} \quad (1)$$

2.4 Optimum Calculation Taking into Consideration the Robustness

It is made an objective that an optimal calculation in consideration of the stability of the objective functions was made in consideration of the dispersion of the objective functions due the uncontrollable factor. If the value of the estimation expression of the standard deviation is taken into consideration, it is possible that the dispersion of the objective value due to uncontrollable factor is controlled, and an optimal design taking the robustness into consideration can be made. The establishment example of an optimization problem to maintain the general robust design is shown in the following form;

Objective function : $F \rightarrow \text{minimize}$

Behavior constraints : σ of characteristic value $Z \leq \text{Establishment value}$

Characteristic value $W \leq \text{Establishment value}$

Side constraints : Each design variables are within the level range.

3 Analysis of Artificial Human Ankle Joint

3.1 Optimal and Robust Design of Artificial Ankle Joint

Osteoarthritis is a common disease affecting human joints. Generally, an artificial operation of a joint replacement is performed on the patient whom experiences difficulty even in walking. However enough examination for the optimization study has not been conducted though the artificial foot joint being used for this operation had been designed using know-how and technology the orthopedist had gained from previous experiences. It is considered that various factors influence the stress condition of the bone around the artificial foot joint. The geometric factors of the artificial joint are needless to say, the condition of

102 Computer Aided Optimum Design of Structures

the lower limb joints such as the patient's Femorotibial Angle (FTA) influences stress conditions, too. Because FTA is the form which is characteristic of each patient, it is not a preknown factor and cannot be controlled by the engineering designers before the optimization design of the artificial joint. Therefore, the optimization must be done in consideration of the robustness when designing an artificial foot joint adapted to every patient as well.

3.2 Effectivity Analysis

3.2.1 Level Values of Variables

The variables and their values taken into consideration are shown in Figure 1^[7]. Five variables which contained the form of the components on the tibia side of the artificial foot joint and FTA were taken into consideration. The design variables are the geometric dimensions w , h , θ of the artificial ankle joint and the Young's modulus E of a shock absorption material to pour onto the surface of the artificial foot joint and the bone, and β (FTA) is the uncontrollable variable. The material properties in this analysis are shown in Table 4.

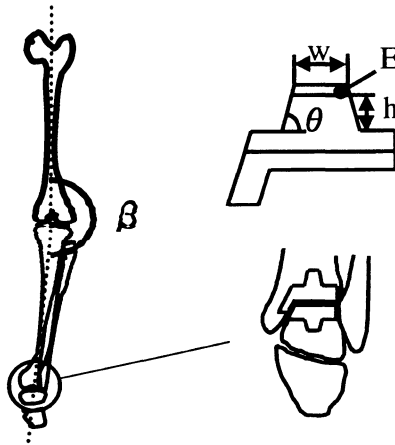


Fig. 2 Human foot model in this analysis and their variables

Table 4 Material properties

	Young's Modulus[MPa]	Poisson Ratio
Bone	1.8×10^4	0.3
Cartilage	14	0.4
Ceramics	3.5×10^5	0.3
HDP	5.0×10^3	0.3

3.2.2 Effectivity Analysis

At first, all of the variables are at three levels. An orthogonal array L27 was used for the array designs involving the evaluation of interaction between β and all of the design variables.

Table 5 Level value

	Level 1	Level 2	Level 3
β [deg]	176	180	184
w [mm]	7.8	11.7	15.6
h [mm]	4.57	6.81	9.08
θ [deg]	60	75	90
E [MPa]	0.72e+04	1.26e+04	1.8e+04

A 27-run analysis was carried out by FEM simulations, and the analytical code was ABAQUS5.7. The maximum stress “ s ” which occurred on the interface between the artificial foot joint and the bone was calculated as the objective function, and its estimation expression was made as;

$$\begin{aligned}
 s = & -1347.37 + 14.38\beta - 3.85e-02\beta^2 + 190.89w - 8.69w^2 - 133.5h + 7.13h^2 \\
 & + 23.41\theta - 0.15\theta^2 - 7.57e-03E + 1.07e-06E^2 - 2.1w\beta + 5.8e-03w\beta^2 \\
 & + 9.58e-02w^2\beta - 2.64e-04w^2\beta^2 + 1.45h\beta - 3.93e-03h\beta^2 - 7.68e-02h^2\beta \\
 & + 2.06e-04h^2\beta^2 - 0.24\theta\beta + 6.19e-04\theta\beta^2 + 1.52e-04\theta^2\beta - 3.89e-06\theta^2\beta^2 \\
 & + 7.43e-05E\beta - 1.75e-07E\beta^2 - 1.15e-08E^2\beta + 3.05e-11E^2\beta^2 \quad [\text{MPa}]
 \end{aligned}$$

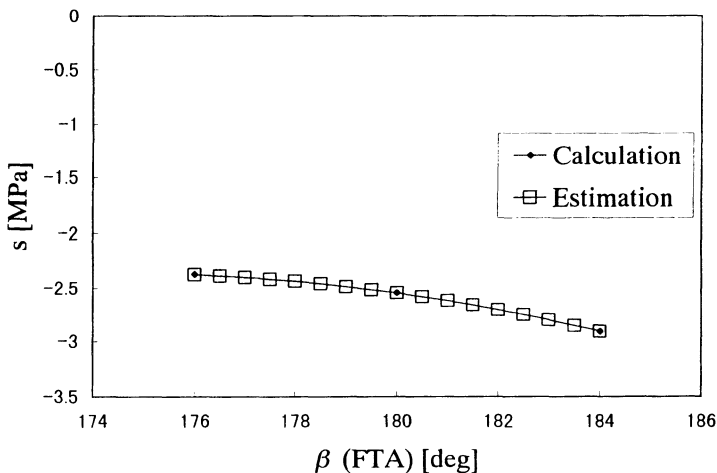


Fig. 3 The comparison between the FEA and estimation expression

104 Computer Aided Optimum Design of Structures

The comparison between the result of FEA and estimation expression within the level range was done to confirm precision of estimation expression (Fig. 3). Furthermore, L27 crossed 5 levels of uncontrollable variable β and was used to evaluate the standard deviation σ_s for the dispersion of the maximum stress s . All 27×5 maximum stresses were calculated by using the estimation expression of the maximum stress made previously. Then, the standard deviation σ_s was given from equation (1).

This standard deviation σ_s shows the dispersion of the characteristic value which occurs due to the change in the whole territory of FTA.

Table 6 Level value of error variable

	Level 1	Level 2	Level 3	Level 4	Level 5
β [deg]	176	178	180	182	184

The analysis of variance for the standard deviation σ_s was made to examine how the design variables influence the standard deviation σ_s .

Table 7 The maximum stress "s" and standard deviation

Date No.	The maximum stress "s"					Standard deviation
	Level1	Level2	Level3	Level4	Level5	
1	-2.15	-2.26	-2.36	-2.46	-2.55	0.16
2	-2.3	-2.42	-2.54	-2.65	-2.76	0.18
3	-3.07	-3.23	-3.37	-3.48	-3.57	0.2
4	-2.04	-2.36	-2.59	-2.73	-2.79	0.31
5	-2.44	-2.55	-2.65	-2.74	-2.83	0.15
6	-2.46	-2.55	-2.64	-2.72	-2.81	0.14
7	-2.09	-2.18	-2.25	-2.31	-2.31	0.11

A calculation example of σ_s is shown in the following.

$$\begin{aligned}
 \sigma_{si} &= \sqrt{\frac{\sum_{j=1}^n (w_i - w_{ij})^2}{n-1}} \\
 &= \sqrt{\frac{[-2.356 - (-2.15)]^2 + \dots + [-2.356 - (-2.46)]^2 + [-2.356 - (-2.55)]^2}{4}} \\
 &= 0.16
 \end{aligned}$$

The estimation expression of the standard deviation σ_s was made by using the order element of the design variables judged predominant by the analysis of variance as;

$$\alpha_s = -1.81 + 5.63e-02w - 2.61e-03w^2 - 0.12h + 8.53e-03h^2 \\ + 6.16e-02\theta - 4.05e-04\theta^2 - 3.83e-05E + 1.62e-09E^2$$

3.3 Optimization Calculation

The optimum and robust design for the artificial joint can be formulated as a single objective optimization problem in the following form;

Objective function : $s=f(w, h, \theta, E) \rightarrow \text{minimize}$

Design variables : w, h, θ , and E

Behavior constraints : $\alpha_s \leq 0.01, \beta = 182$

Side constraints: Each design variables are within the level range

A Sequential Quadratic Programming was used for the optimization calculation, and the optimized results are;

$w=15.6$ [mm] , $h=4.54$ [mm] , $\theta=60$ [deg] , $E=10800$ [MPa]

4 Conclusion

The authors proposed an optimum and robust design method taking into consideration the robustness of nonlinear structures by the Statistical Design Support System. By the present approach, the dispersion of objective functions due to the uncertainty of an uncontrollable factor can be evaluated quantitatively and easily. Furthermore, because the robust design can be treated in the same way as the optimum design, an optimum and robust design can be carried out with the same system and at the same time. The optimization results of the application show that the SDSS could be applied to the various optimum and robust problems very practically and efficiently.

5 References

1. Kashiwamura, T., Shiratori, M., Yu Q., and Kouda, I., Transactions of the Japan Society of Mechanical Engineers , 62-601, A(1996),2180
2. Kashiwamura, Takao, M., MShiratori, M., Yu Q., and Osamu, M., Transactions of the Japan Society of Mechanical Engineers , 62-603, A(1996),2422
3. Kashiwamura, T., Shiratori, M., Yu Q., Transactions of the Japan Society of Mechanical Engineers , 63-607, A(1997),624
4. Kashiwamura, T., Shiratori, M., Yu Q., Transactions of the Japan Society of Mechanical Engineers , 63-610, A(1997),1948
5. Ang, A.H-S. and Tang, W.H., "Probability Concepts in Engineering Planning and Design", Vol. 1,1975, John Wiley & Sons
6. Taguchi, G., Design of Experiments (3rd edition), 1976, Maruzen
7. J. E. Anderson., Grant's Atlas of Anatomy (3rd edition), 1991, Igaku-Shoin