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Structural optimization methods

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ABSTRACT

This paper presents an overview of the basic techniques employed to perform shape and topology structural optimization. Sizing optimization and material selection are also briefly commented. The prominent methods employed to generate optimal topological designs are homogenization, bubble insertion, nature analogy and ground structure. Similarly, the main algorithms used to perform shape optimization fall in one of the following categories: boundary variation, nature analogy and hybrid, such as, a machine learning system interfaced to the shape optimization environment. The practical use of such methods in the design of structures is discussed. Special consideration will be given to the nature analogy software algorithms SKO (soft kill option) and CAO (computer aided optimization) both developed at the Karlsruhe Nuclear Research Centre. These are an integral part of the software package employed at EMPA for topology and shape optimization, respectively. In addition, it is planned to fabricate and test the structural components optimized with SKO and CAO in order to validate the algorithms.

1 INTRODUCTION

Today, there is a growing interest for the efficient use of materials. One approach being pursued to save on material is the application of optimization algorithms in the design of structural components or layouts. For example, the aerospace and automotive industries apply sizing and shape optimization to the design of structures and mechanical elements [1, 2]. Shape optimization is also employed in the design of electromagnetic, electrochemical and acoustic devices.

The aim in structural optimization is to generate, with minimal effort, optimal lightweight structures capable of safely carrying the imposed loads. This can be achieved by varying the following structural design variables:

- member size (e.g. cross section, thickness)
- shape of the structure (geometry)
- topology of the structure (pattern of members and joints or internal boundaries)
- material properties.

During the last decade there has been a revived interest for the search of better and more efficient sizing, shape and topology optimization. From a survey of the literature, it is possible to identify the different approaches that have evolved during this period. The basic methods employed to generate optimal structural topology designs may be categorized as follows:

- homogenization
- bubble insertion
- nature analogy
- ground structure.

Similarly, the main algorithms that are used to perform shape optimization of structures fall in one of the following groups:

- boundary variation
- nature analogy
- hybrid

Note that both the shape and topology optimization techniques complement one another, i.e., the shape algorithm has the task to fine tune the optimal topology design (e.g. reduce high stress concentrations).

Sizing optimization is a subset of shape optimization. Thus, both can be performed contemporarily. The effects of the size changes on the design can be determined separately with linear and nonlinear mathematical programming methods (MP) or with optimality criteria methods (OC) [3, 4, 5, 6]. Finite element codes in conjunction with software packages to control the design parameters can also be employed. Since, the sizing optimization problem is well established [7, 8, 9] and is not as complex as shape or topology optimization, it will not be considered further in this review.

Material selection presents a special problem when performing structural optimization, be it sizing, shape or topology. For example, conventional materials have to be selected from a finite set of variables (e.g. density, yield strength, modulus of elasticity). If these discrete variables were to be considered in the optimization procedure (e.g. MP and OC methods), the computation time and complexity would increase. Therefore, for a small number of conventional materials, it would be more efficient to perform the optimization separately for each material and to compare the results [4, 10]. On the other hand, the increasing application of high-performance composite materials in structural components has encouraged the use of material properties as design variables in shape and topology optimization [11, 12, 13, 14]. The inclusion of the composite material properties leads to the design of the materials themselves

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opening a new frontier in the application of such structural design tools. Moreover, for the advanced materials, such as sintered, composite or ceramic, special expressions for the description of the different failure criteria are required [15, 16]. Due to its complexity, material selection will not be discussed further.

Following this brief introduction, the aforementioned topology and shape optimization methods will be briefly treated. Particular attention will be devoted to the SKO and to the CAO algorithms.

2 TOPOLOGY OPTIMIZATION

Initial structural topologies are either defined a priori or dictated by design or manufacturing constraints. Instead, the goal in topology optimization is to remove or redistribute material in a rational iterative manner from within the given structural domain subject to loads and boundary conditions. This leads to an optimal preliminary topological design that can be shape optimized if necessary. The next step is to extract the optimal form that is embedded as a contour diagram of the material or stress distribution on the design domain (refer to Figure 1). This can be performed with the aid of an automatic image processor [17], that will be definitely required for three-dimensional optimization, or with manual techniques that yield good results for two-dimensional designs.

2.1 Homogenization Method

Bendsøe and Kikuchi [12] were the pioneers of the homogenization technique. Basically, this method defines a material composed of infinitely many unit cells and optimizes the medium porosity with an optimality criteria procedure. Each cell can consist of (i) a given material with one or more holes or (ii) a layered material. The mechanical properties of the medium can be described by macroscopic material properties that depend on the basic cell geometry and orientation. These macroscopic properties can be determined with the aid of the homogenization theory [11, 12, 18]. The theory invoked to calculate the macroscopic material properties and whether the material properties themselves are also optimized dictate the type of structures generated. Typical designs range from truss-like structures to cellular continua or composite systems [11, 13, 19]. Note that the optimization of material properties can be employed to control the frequency response characteristic of the structure. Moreover, the homogenization method can be employed to study complicated designs requiring thermoelastic and elastoplastic formulations. Due to its versatility there is great interest in this method as a tool for topology optimization [11, 12, 13, 19, 20, 21, 22, 23, 24].

2.2 Bubble Method

The bubble approach presented by Eschenauer et al. [15] positions iteratively new holes (bubbles) in a structure by means of a definite function and a hierarchically secondary shape optimization. Note that only one new hole at a time can be inserted in the continuum. Each simultaneous shape optimization of the holes and the specified variable boundaries of the structure yields a variety of possible topologies.

2.3 Ground Structure Approach

The pioneer of topology optimization of trusses in the form of grid-like continua was Michell [25]. Today, optimization of the size, shape and topology of grid-type structures (e.g. trusses, frames, shell-grids, cable nets) can be formulated with the ground structure method [4, 11, 26, 27]. This approach assumes an initial ground structure that contains many joints and connecting members, i.e., all possible non-overlapping connection between the nodes. Some authors choose any given set of nodal connections for the ground structure [11]. The ground structure method, which employs an optimality criteria procedure, removes from a highly connected discrete structure uneconomical members by permitting the areas to reach zero and resizes the remaining ones. This leads to an optimal substructure (truss-like structure) of the structural universe. The continuum-based optimality criteria (COC) and the discretized continuum-based optimality criteria (DCOC) methods [27, 28, 29, 30, 31], incorporated in the ground structure approach, lead to a discretized optimal layout solution. In addition COC and DCOC have been employed for generalized shape optimization (shape and topology) of cellular continua or composite systems [14, 27, 28, 29, 30, 31].

2.4 Nature Analogy Methods

Over the past years there has been an active involvement towards the development of topology optimization algorithms based on the simulation of natural phenomena or biological growth. Some of the most well-known methods are: material annealing [32]; neural networks [33]; genetic [34, 35]; adaptive bone mineralization (SKO - soft kill option) [36, 37]; functional adaption of bone [38, 39] and evolutionary fully-stressed (hard kill) [40, 41, 42, 43]. Each algorithm has its particular advantage and application. At EMPA, structural topology optimization is carried out with the SKO software tool and as pointed out in the introduction only this method will be discussed. Details of the other methods can be found in the respective references cited.

2.4.1 Adaptive Bone Mineralization (SKO - Soft Kill Option) Method

Even though this technique is not a conventional optimization method, it removes inefficient material and generates an optimal topology design in a manner similar to nature. SKO is based on the constant stress state of biological

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growth of load carriers [44, 45]. Load changes in the bone will cause it to modify its shape, adaptive remodelling and mineralization mechanisms [36]. As a consequence, the high stresses lead to a high degree of mineralization as compared to the lower stresses. In other words the areas subjected to higher loads are stiffer than those exposed to lower loads resulting in a variation of Young's modulus in the load carrier. SKO simulates this biological growth mechanism that weakens the structure at the unloaded areas and reinforces those subjected to high loads. The procedure involves the following steps [36]:

I. The structure is discretized into a fine mesh and the stresses are calculated for the specified load case, boundary conditions and a constant modulus of elasticity.
II. Then the modulus of elasticity is expressed as a function of stress and the new stress distribution is calculated for the same load case and boundary conditions. This step is repeated until the specified criterion is satisfied.

Note that during each iteration the non-loaded elements are switched off and at the end of the optimization procedure these are killed (soft kill option) yielding a stress contour diagram plotted over the reference domain.

Due to its simplicity, SKO can be used with any commercial FEM programmes, without enormous mathematical formulations to express Young's modulus as a function of stress. Baumgartner et al. [36] have demonstrated that this method can be applied without special optimization algorithms with Abaqus by exploiting the capability of the code to define Young's modulus as a function of temperature. Instead the three-point loaded beam, shown in Figure 1, was discretized and post processed with Patran while the Nastran solver was invoked in the SKO optimization procedure to perform only the FE calculations.

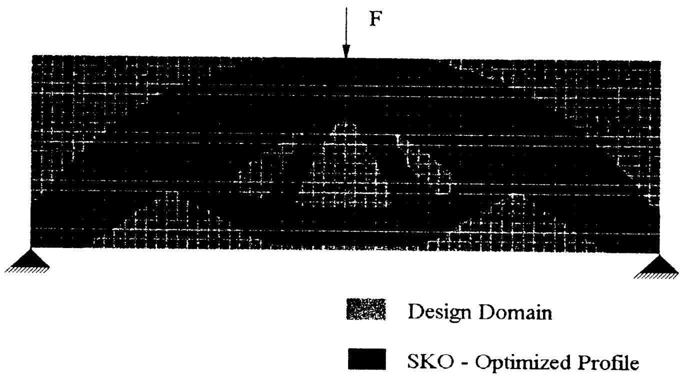

Figure 1: Von Mises stress contours plotted on the design domain.

3 SHAPE OPTIMIZATION

As mentioned in the introduction, shape or geometric optimization can be employed to fine tune the optimized topology or simply the shape of a proposed structural design, for instance, by reducing localized stress peaks. The additional

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design variables introduced by shape optimization, as compared to sizing optimization, allow for boundary movement.

3.1 Boundary Variation Method

The aim of the boundary variation method is to minimize some objective function such as weight that in turn is a function of the design variables. In this method the boundary of the structure is represented by a suitable curve (e.g. parametric cubic splines [46]). Thus the coordinates of the important points defining the splines can be the design variables to be optimized. At first, the structure is calculated for the specified load case, boundary conditions and material properties with a FEM programme. Afterwards, the effects resulting from small changes in the design variables on parameters such as weight, stresses and displacements are evaluated. These variations (sensitivities) are used in a mathematical programming technique [3, 4, 5, 6] to optimize the design variables in order to determine a new geometry. This iterative process is repeated until the convergence criteria for the optimization algorithm is satisfied.

3.2 Nature Analogy Methods

In the past, intense efforts have also been dedicated to the search of shape optimization methods that simulate natural phenomena or biological growth. A few of the most noteworthy methods are: material annealing [47]; neural networks [33]; genetic [48, 49]; functional adaption of bone [38] and biological growth, namely, the mechanism of adaptive growth (CAO - computer aided shape optimization) [36, 37, 50, 51] and the method proposed by Chen and Tsai [52]. Due to its inherent simplicity and the ease with which it can be coupled to existing FEM programmes, CAO was selected by EMPA to perform structural shape optimization. Therefore, only this method will be reviewed. For information concerning the other techniques the reader is directed to the respective references reported.

3.2.1 Adaptive Growth (CAO - Computer Aided Optimization) Method

During the process of biological growth, load carriers try to adapt to a state of constant stress at their surfaces [44, 45]. For instance, in trees only the outermost growth ring is active and is able to react to external loads so as to avoid localized stress peaks (notch stresses). The aim of the adaptive method (CAO), developed at the Karlsruhe Nuclear Research Centre by Mattheck [50], is to simulate a constant stress state on the surface of structures. The procedure involves the following steps [50, 51]:

I. Prepare the finite element model of the design domain to be optimized for the specified load case and boundary conditions. Beneath the surface there should be a thin layer of finite elements (soft surface) that have the same depth. The soft surface is to have a very low modulus of elasticity as compared to the rest of the elements (hard surface) in the structure only during swelling. Note that the thin layer has the same function as the outermost growth ring in trees.

II. Determine the von Mises stress distribution in the whole structure. The modulus of elasticity in the soft and hard surfaces should be the same. According to Mattheck [50], the von Mises stress criterion yields good growth prediction results.

III. The soft surface is allowed to swell (adaptive growth), for example, according to a simple swelling law for volumetric strain rate:

$$\dot{\varepsilon}_V = k(\sigma_M - \sigma_{reference})$$

where k is an artificial constant, σ_M the von Mises stress and $\sigma_{reference}$ the reference stress field (e.g. the average or far-field Mises stress). If the FEM code does not possess this feature, the swelling can be carried out by heating up the surface layer. The temperature distribution is set to a relation of the difference between the von Mises stress values, determined in the previous step, and the reference stress. Moreover, the coefficient of thermal expansion in the soft layer should be non-zero while the hard layer (stiffer material) must have a value of zero.

IV. The incremental displacement resulting from swelling or thermal expansion is added to the nodal points of the undeformed mesh and the elements in the thin layer are corrected to the original depth.

V. With the updated mesh as input, the iterations from step (II) are repeated until the specified criterion has been fulfilled (e.g. the peak stresses have disappeared).

VI. A final calculation can be carried out to assess the stress reduction achieved through the application of shape optimization in comparison with the original design. Of course, the modulus of elasticity in the thin layer should be the same as that for the hard layer.

Shape optimization with CAO has been successfully applied to two and three-dimensional problems [37, 50, 51]. The high peak stress in a fillet design, under tension, was reduced with CAO together with Patran and the Nastran solver without difficulty (refer to Figure 2 for the mesh results). In the near future, it is planned to subject the optimized component to fatigue tests in order to determine the effectiveness of CAO to increase the fatigue strength and endurance limit.

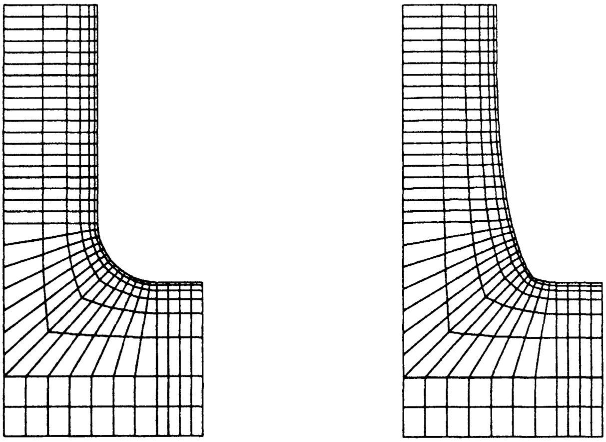

Figure 2: Mesh results of the fillet design before and after optimization.

3.3 Hybrid Method

There are a large number of hybrid techniques (e.g. artificial intelligence coupled to a genetic algorithm [1]) that could be assembled and applied for shape optimization. The method, that will be briefly commented, consists of a machine learning technique [53, 54] interfaced to an environment such as the structural shape optimization domain. Samuel [55] was the first to publish work on this fascinating area of machine learning. According to Richards [54], the learning classifier system developed by Holland and Reitman [56] is well suited to perform structural optimization. The classifier system is a machine learning system that learns to classify messages from the environment into general sets (classifiers) with the aid of a genetic algorithm. Once the machine learning technique has been selected, it is coupled to the shape optimization environment (boundary representation, analysis, optimization algorithm, termination criterion and final design). The optimization process is controlled via the set of rules communicating between the machine learning operator and the shape optimization environment. The main drawback of such a hybrid system is the large number of calculations that need to be performed. On the other hand, the immense information stored can be employed as a data bank in the search for optimized shapes from complex designs. Even though this method is in the infant stage, it holds promising prospects for future applications as demonstrated by Richards [54] with the SPHINcsX software package.

4 CONCLUSION

An attempt has been made to present the basic methods that have been developed to optimize structures with respect to size, shape, topology and material properties. Each technique has its particular application, function and

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advantage. For instance, SKO can be employed to generate a preliminary optimal topology design of a structure or component manufactured from traditional materials. On the other hand, for composite structures, where material properties can also be optimized, the homogenization method can be employed. Then the optimized topology can be fine tuned with a simple method like CAO. The selection of the optimization techniques is strongly dictated by the complexity of the design requirements imposed upon the component or structure layout. Up to now general economical packages integrating efficient and reliable optimization methods with CAD or CAE have not been devised. Principally, this is due to the fact that shape and topology optimization are at its beginning. However, the perfection of promising shape, topology and material properties optimization algorithms, as well as the continual evolution of new methods, will contribute to make this a reality. Only then can such design tools make a substantial contribution to a management programme aimed at controlling the depletion rate of the raw material resources.

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