Transactions on the Built Environment vol 28, © 1997 WIT Press, www.witpress.com, ISSN 1743-3509

# Statistical optimization method

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## Abstract

The authors have proposed a new practical optimal design method called the statistical optimization method, which consists of the following five steps : the effectivity analysis, reanalysis, evaluation of dispersion, the optimization and evaluation of structural reliability. The design of experiments, combined with a series of finite element analyses (FEA), is used to generate approximate evaluation functions for the controlling behavior depending on the changes in design variables of an object structure. The evaluation functions can also be used as direct tools for estimating the characteristic behavior of design structure. First-order second-moment method is employed to evaluate and generate approximate evaluation functions for the dispersion of the behavior. A mathematical programming etc. are employed to solve the optimization problem of the approximate evaluation functions of the behavior with dispersion. Finally, second-moment method is employed to evaluate the structural reliability. It is confirmed that the proposed method can be used for almost all kinds of the nonlinear problems including the impact behavior of structures, and that it can be carried out in much smaller number of FEA than the other existing methods.

## 1 Introduction

A variety of methods and systems for optimal design have been proposed. Most of these methods and systems incorporate structural and sensitivity analyses in their loops for optimum calculations to compute objective functions and constraints and, to evaluate the convergence properties. These systems have such difficulties as that their construction is complicated and their efficiency low. Under actual conditions computation for optimization of such nonlinear phenomena as collision is very difficult, because the sensitivity coefficient of design factors for characteristic value varies according to the time or load path.

Generally, structures have tolerance or dispersion in dimensions, material characteristics and so on, which will cause more or less dispersion in the characteristic values. Therefore, if one wants to design the structure with high accuracy, he has to consider about such dispersion as well. Evaluation of possible failure or lost function of structures is also important to design highly reliable and safe structures. Stochastic finite element method for the evaluation of structural reliability and dispersion in structural analyses are well-known. However the method has problems in that they require long calculation times in any nonlinear problems and that general purpose analytical software can't be employed. The conventional evaluation and optimization method have been processed individually and independently because of complicated analyses respectively. Therefore these methods reduce the efficiency of analyses.

This paper proposes a statistical optimization method, which is a practical, general purpose, highly efficient integrated design support system.

## 2 STATISTICAL OPTIMIZATION METHOD[1]

### 2.1 Flow of Statistical Optimization Method

As shown in the flow chart of Figure 1, the statistical optimization method is composed of five steps; the effectivity analysis, reanalysis, evaluation of dispersion, the optimization and evaluation of structural reliability. The details of these steps will be discussed in order below.

### 2.2 Effectivity Analysis

Combination of the design of experiments with the structural analyses enables us to quantitatively estimate the effectivity of design factors on the characteristic values with a less analytical labor.

#### 2.2.1 Design of Experiments[2]

The design of experiments with an orthogonal array has been widely used especially in the field of quality control as a method of systematic analyses of the effectivity of design factors. In the orthogonal array, the design factors are assigned to the columns, while the input data of structural analyses are assigned to the rows. The following shows some samples of the orthogonal arrays. $LA(B^C)$ is the description of the table, where "L" stands for orthogonal array, "A" for the number of rows, "B" for the number of levels, and "C" for the maximum number of design factors.

Representative samples : $L16(2^{15})$,$L18(2^1\times 3^7)$,$L27(3^{13})$,$L16(4^5)$

Multi-level samples : $L32(2^1\times 4^9)$,$L64(4^{21})$,$L50(2^1\times 5^{11})$

Multi-factor samples : $L36(2^{11}\times 3^{12})$,$L54(2^1\times 3^{25})$,$L81(3^{40})$

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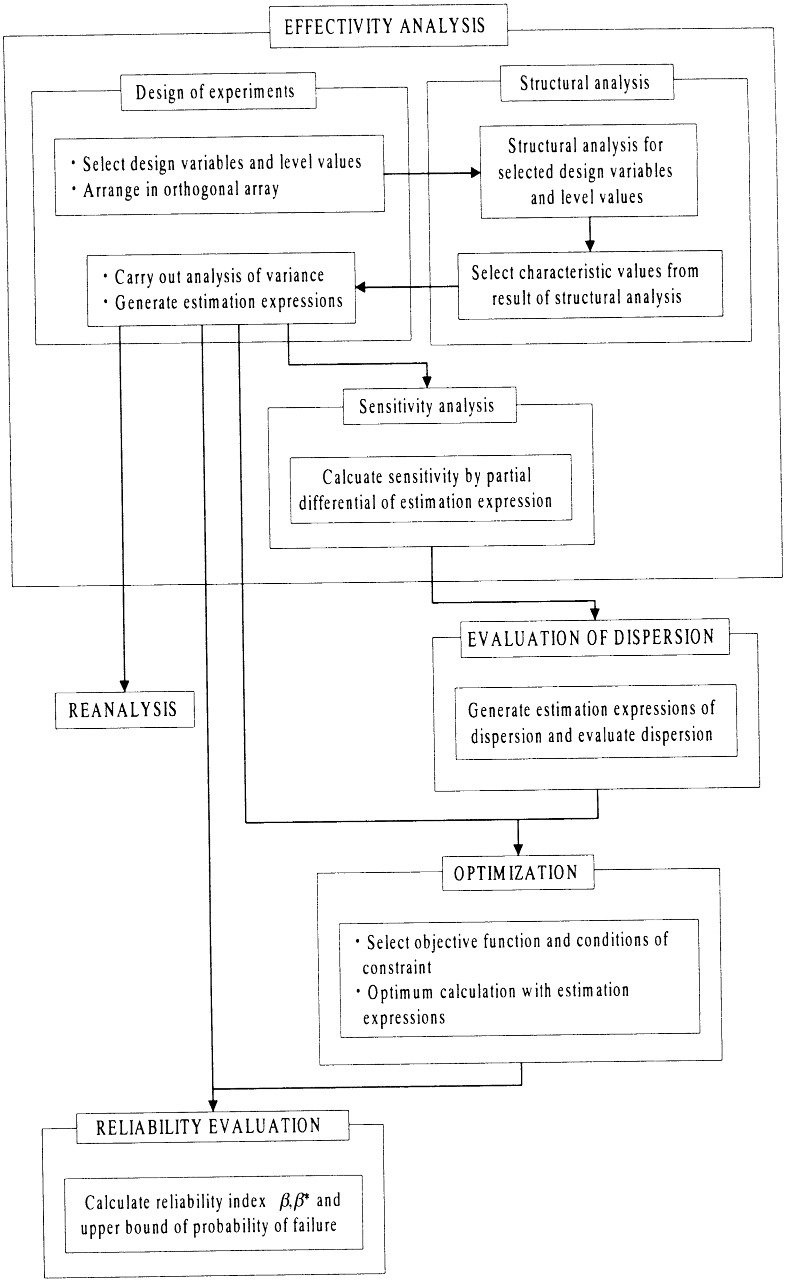


Figure 1 Analysis flow of statistical optimization method

The most appropriate orthogonal array should be selected depending upon the number of design factors, the settings of interaction, and the number of levels. Effectivity analysis can be carried out quite efficient by the orthogonal array. Utmost care should be exercised in setting the number of levels and the range of level values, which affect the accuracy of effectivity analysis, evaluation of dispersion and optimal solution. The number of levels should be determined by taking into account the variation characteristics (tendency) of design factors which will affect characteristic values. For example, if the estimated characteristics is an nth-degree function of design factors, the number of levels for the design factors should be chosen to be higher than $n+1$. The level value should be set within the range of actual design.

### 2.2.2 Structural Analysis

Structural analyses are conducted with the input data generated according to the orthogonal array. Based on the analytical results, the data of characteristic values are obtained for the analysis of variance. As shown in the flow of the statistical optimization method, structural analyses are independent of the other steps, which eliminates the necessity to consider its combination with optimum calculation. Therefore, this reduces the number of repeated structural analyses remarkably. Because of this advantage, the statistical optimization method can be applied to most of nonlinear problems such as impact problems, eigenvalue problems, and so on. Moreover, in this method, any commercial structural analysis software can be used to carry out the analysis of such nonlinear problems. Not only the direct results of structural analysis but also modified values such as maximum value, mean value and integrating value can be utilized as characteristic values. In this method, one set of structural analyses are conducted only once against the combined design factors.

### 2.2.3 Analysis of Variance and Estimation Expression

As a part of the effectivity analysis, it is necessary to conduct the analysis of variance and to generate estimation expressions. The detailed analyses of variance are carried out by an evaluation method which decompose the effectivity of design factors on characteristic values into the orthogonal components. This method of analysis enables us to express the effectivity of design factors by concrete characteristics as primary effects and secondary ones. However, it should be noted that we can extract only the qualitative nature through the evaluation of the effectivity which uses the F-value and contribution rate obtained from the analysis of variance.

The estimation of characteristic values are represented by a multivariate polynomial expression, based upon the design factors with the degrees regarded as effective by the analysis of variance. This method can generate simple estimation expressions. Estimation expression for each design factor is given by an orthogonal polynomial of equation (1), expressed by a Chebyshev's orthogonal function[2]. [equation (1) is applicable only to the levels with an equal interval.] An estimation expression containing different design factors can be composed by adding identical terms other than "$b_0$" in the estimation expression of each design factor. The highest degree of each design factor in the estimation expression is equal to $a-1$, where $a$ is the number of levels.

$$Y = b_0 + b_1(A-\bar{A}) + b_2\left\{(A-\bar{A})^2 - (a^2-1)\frac{h^2}{12}\right\} + \cdots + b_n\xi_n(A) + \cdots \quad (1)$$

$$\xi_0(A) = 1 \qquad (n=0)$$

$$\xi_1(A) = A - \bar{A} \qquad (n=1)$$

$$\xi_n(A) = \xi_{n-1}(A)\xi_1(A) - \frac{(n-1)^2\left\{a^2-(n-1)^2\right\}h^2}{4\left\{4(n-1)^2-1\right\}}\xi_{n-2}(A) \qquad (n = 2,3,\cdots)$$

In the formula above, "$A$" stands for a variable, "$\bar{A}$" for a mean level value, "$a$" for the number of levels and "$h$" for the interval between levels. Coefficient "$b_o$" and "$b_i$" can be expressed by equation (2), based on the orthogonal relations of estimation expressions.

$$b_o = \text{average of all analytical values} \qquad (2)$$

$$b_i = \sum_{v=1}^{a}\xi_i(A_v)y_v \Big/ \sum_{v=1}^{a}\xi_i^2(A_v) \qquad (v = 1,2,\cdots,a)$$

In this equation, "$A_v$" stands for the level value of Variable (factor) $A$, and "$y_v$" for the mean analytical value of each level.

### 2.2.4 Sensitivity Analysis

Since the estimation expressions are simple explicit equations, partial differential of the estimation expression ($Y$) about a design factor ($f_i$) can produce the sensitivity as shown in equations (3). This sensitivity ($S_i$) is a primary differential coefficient. In this equations, "$m$" stands for the total number of design factors.

$$S_i = \frac{\partial Y}{\partial f_i} \qquad (i = 1,2,\cdots,m) \qquad (3)$$

This sensitivity represents the variation of a characteristic value against the unitary variation of a design factor. And the sensitivity indicate the quantitative effectivity of the design factor on the characteristic value. The sensitivity ($S_i$) changes along with the design factor ($f_i$). Substitution of a definite value of the design factor ($f_i$) for the equation ($\partial Y/\partial f_i$) can produce the sensitivity at any value of the design factor. In addition, since the estimation expression is the function which continuously represents a phenomenon, the sensitivity obtained by partial differential of the estimation expression can be considered to express a quantitative effectivity. This is free from the choice of the preset range of level value as discussed in the analysis of variance. Therefore, to determine the effectivity precedence of design factors, employment of sensitivity gives more precise results than the analysis of variance.

## 2.3 Re-analysis

The estimation expression for characteristic value obtained in the effectivity analysis can be used for the re-analysis. A designer has only to substitute a set of concrete values of the design factors into the estimation equation, then he has can obtain the corresponding characteristic value. Use of this estimation expression allows the designer to know easily any variation of the characteristic value accompanied by the change of design. The re-analysis method is highly efficient and practical. The authors believe that the use of the simplified re-analysis method will contribute largely to the enhancement of the efficiency in design work.

## 2.4 Evaluation of Dispersion

In this paragraph, the authors will shows the evaluation method of the characteristic values which are dispersed along with design factors. First-order second-moment method is employed for estimation and evaluation of the dispersion[3]. The response ($U$) of characteristic values can be expressed by the functional equation (4), in which the design factors ($f_1, f_2, ..., f_m$) are treated as random variables. This equation is equivalent to the estimation expression of characteristic values which is expressed by Equation (1). The response ($U$) of characteristic values with the dispersion of design factors is expressed by equation (5), which uses the definite term ($\bar{U}$), the width of variation ($\Delta f_i$) of design factor ($f_i$), sensitivity ($\partial Y/\partial f_i$). Equation (6) expresses the expectation ($E[U]$) of the characteristic values at a definite value ($\bar{f}_i$). The variance ($Var[U]$) in this instance can be obtained from equation (7) by using "$Var[f_i]$" and "$Var[f_j]$" which represent the variance of design factor ($f_i$) and ($f_j$), respectively.

$$U = g(f_1, f_2, \cdots, f_m) \tag{4}$$

$$U = \bar{U} + \sum_{i=1}^{m} \left(\partial g/\partial f_i\right)_{\bar{f}} \Delta f_i \tag{5}$$

$$E[U] = g(\bar{f}_1, \bar{f}_2, \cdots, \bar{f}_m) \tag{6}$$

$$Var[U] = \sum_{i \neq j}^{m} \sum^{m} \left(\frac{\partial g}{\partial f_i}\right)_{\bar{f}} \left(\frac{\partial g}{\partial f_j}\right)_{\bar{f}} \rho_{ij} \sqrt{Var[f_i]}\sqrt{Var[f_j]} + \sum_{i=1}^{m} \left(\frac{\partial g}{\partial f_i}\right)_{\bar{f}}^{2} Var[f_i] \tag{7}$$

In the equations above, " $\bar{f}$ " is equivalent to $\{\bar{f}_1, \bar{f}_2, \cdots, \bar{f}_m\}$ and " $^-$ " stands for definite values, and "$m$" for the number of design factors. "$(\bullet)_{\bar{f}}$" means to evaluate the differential about the definite values $(\bar{f}_1, \bar{f}_2, \cdots, \bar{f}_m)$. Since the standard deviation ($\sigma$) is the square root of the variance ($Var$), the standard deviation ($\sigma_U$) which indicates the dispersion of characteristic values can be calculated as the square root of equation (7). Therefore, the estimation expression of the standard deviation ($\sigma_U$) is represented by an explicit equation. Incidentally, the correlation coefficient ($\rho_{ij}$) of autocorrelation is 1. By using the results, equation (8) produces the coefficient of variation ($C.O.V$) which is often used as an index of relative dispersion against expectation.

$$C.O.V = \sigma_U / E[U] \tag{8}$$

## 2.5 Optimum Calculation

General problems of optimization can be formulated by equation (9):

Constraints: $g_i(x) \geq 0 \quad (i = 1,2,\cdots,k)$

$h_j(x) = 0 \quad (j = k+1, k+2, \cdots, l)$

Under the constraints,

Objective function: $f(x) \rightarrow$ minimum (9)

These $g_i(x)$, $h_i(x)$ and $f(x)$ in equation (9) can be represented by the estimation expressions for expectation and dispersions of the characteristic values. Since this method enables us to use any functions ( =estimation expressions) obtained from different analyses(for example, intensity analysis and eigenvalue analysis), at the same time, an optimum calculation, where plural phenomena is taken into account, is possible. Therefore, this method facilitates to set problems for optimization.

(1) General Example

Function: Mass $\rightarrow$ Minimum

Characteristic value $A$ (expectation + standard deviation * n) $\leqq$ Set value

Characteristic value $B$ (expectation) = set value

Where n in the constraints is a coefficient that represents the allowance as set by standards, codes, regulations, and sometimes, by experience.

(2) Example Intended for Robust Design

Standard deviation of characteristic value $\rightarrow$ Minimum

Set value $\geqq$ Expectation of characteristic value $\geqq$ Set value

Mass $\leqq$ Set value

The computational method to solve the optimization problems may be selected from mathematical programming, genetic algorithm, and others.

### 2.5.1 Optimum Calculation of Continuous Variables

As the tool of the optimum calculation for continuous variables the authors use the mathematical programming that can be applied most generally and universally for mathematical optimization. Among many mathematical programming methods, they have adopted the successive (or sequential) quadratic programming (SQP) method. This method has been recognized the most efficient against nonlinear optimization problems with constraints that are very common in the practical problems. In the SQP method, successively approximated quadratic programming problems are generated as partial problems of equation(9) and the processes are repeated one after another to get the solution. Equation(10) shows an approximate expression when a point $x^{(k)}$ is given at the $k$-th iteration. In this approximate expression, the objective function takes a form of quadratic approximation, while the constraints are expressed in a form of the linear approximation.

$$\text{Objective function:} \quad \nabla f(x^{(k)})d + \frac{1}{2}d^T B^{(k)} d \rightarrow \text{minimum}$$

$$\text{Constraints:} \quad g_i(x^{(k)}) + \nabla g_i(x^{(k)})^T d \geq 0$$

$$h_j(x^{(k)}) + \nabla h_j(x^{(k)})^T d = 0 \qquad (10)$$

$B$: Matrix in which Hessian Matrix $(\nabla^2 L(x^{(k)}, \lambda^{(k)}))$ of Lagrangian ($L$) was
approximated.
$d$: Vector ($d$) (in the search direction from $x^{(k)}$)

Estimation expressions can be used as the objective functions or the constraints. With these estimation expressions given in explicit forms, the above equations can be solved easily, and we can get optimal solutions efficiently.

### 2.5.2 Optimum Calculation for Discrete Variables

For the optimization problems where the design factors are composed of only discrete values because of the requirements such as standards, codes and marketability of products, we can adopt the round robin computation method where all the combinations of design factors with discrete values are calculated. These satisfied conditions can be chosen out of the calculation results. Discrete values need not be equidistant. The round robin method is very time consuming in the general optimization algorithm because the number of combination is extraordinarily large. However, in this case, we can expect very short computing time because we use the simple estimation equations, instead of the direct FEM analyses, to calculate the characteristic values.

## 2.6 Evaluation of Structural Reliability

The authors will describe how to evaluate the structural reliability by the present method. The second moment method is applied to evaluate the reliability with expectation values and variance leaving the distribution form unknown [4][5]. The reliability index, which is a scale corresponding to the probability of failure, can be obtained by this method. Equation (11) represents the performance function that defines the limit conditions of the characteristic value, where $Z > 0$ stands for the safe side and $Z \leqq 0$ for failure region. The performance function can be generated as an explicit equation by using the estimation expression (1) of the characteristic value and the limit value for design.

### 2.6.1 Evaluation by FOSM Method

Equation (12) gives a reliability index $\beta$ of the first-order second moment (FOSM) method. With the distribution form of $Z$ left unknown, the reliability index $\beta$ indicates how far the expectation $E[Z]$ is away from the critical point ($Z = 0$) with the standard deviation $\sigma_z$ as a scale. The standard deviation $\sigma_z$ can be obtained as a square root of the variance $Var[Z]$ as shown in the equation (13) Since the performance functional equation is explicit, the variance $Var[Z]$ can be obtained in the similar fashion to the equation (7).

$$Z = H(f_1, f_2, \cdots, f_m) \tag{11}$$

$$\beta = \frac{E[Z]}{\sigma_Z} \tag{12}$$

$$\begin{aligned} Var[Z] = &\sum_{i \neq j}^{m} \sum^{m} \left( \frac{\partial H}{\partial f_i} \right)_{\bar{f}} \left( \frac{\partial H}{\partial f_j} \right)_{\bar{f}} \rho_{ij} \sqrt{Var[f_i]} \sqrt{Var[f_j]} \\ &+ \sum_{i=1}^{m} \left( \frac{\partial H}{\partial f_i} \right)_{\bar{f}}^2 Var[f_i] \end{aligned} \tag{13}$$

However, the FOSM method based upon the reliability $\beta$ is lack of universality because it depends on the representation of a numerical formula even if the performance function remains physically the same. Further the performance functions are nonlinear in most cases. In these cases there are the lack of universality in the FOSM method.

### 2.6.2 Evaluation by AFOSM Method

Advanced first-order second moment (AFOSM) method [5] allows one to have a universal reliability index independent of the form or expression of the performance function. The reliability index $\beta^*$ in the AFOSM is defined as the shortest distance from the origin to the point on the curve $h(Y) = 0$ lying on the limit state surface of equation (15) in the space of normalized $Y_i$ as shown in the equation (14). The end point of this shortest distance on the surface is called design point ($Y_i^*$). The new variable $Y_i$ has been so normalized that the mean $\mu_{Yi} = 0$ and standard deviation $\sigma_{Y1} = 1$. Where $\mu_{fi}$ and $\sigma_{fi}$ are the mean values and standard deviation of the random variable $f_i$.

$$Y_i = \frac{(f_i - \mu_{f_i})}{\sigma_{f_i}} \tag{14}$$

$$Z = H(f_1, f_2, \cdots, f_m) = h(Y_1, Y_2, \cdots, Y_m) \tag{15}$$

$\beta^*$ and $Y_i^*$, when the performance function is nonlinear, are obtained by the iteration method, and they are applied to the nonlinear optimization problem as shown in the equation (16). The SQP of the mathematical programming method is adopted for the calculation of these values. It is assumed that there is not correlation among the variables $f_1, f_2, ..., f_m$.

$$\begin{aligned} &\text{Objective function: } \sqrt{Y_1^2 + Y_2^2 + \cdots + Y_m^2} \rightarrow \text{Minimum} = \beta^* \\ &\text{Equality constraints: } h(Y_1, Y_2, ..., Y_m) = 0 \end{aligned} \tag{16}$$

Note that $\beta$ is equal to $\beta^*$ when the performance function is linear.

The partial safety factor $\gamma_{xi}$, which is the ratio of design point to the mean value of random variable $\mu_{fi}$, is given by equation (17). The $f_i^*$ showing the design point is $Y_i^*$ as inverse-transformed into the original variable by the equation (14)

$$\gamma_{xi} = f_i^* / \mu_{fi} \qquad (i = 1, 2, ..., m) \tag{17}$$

### 2.6.3 Upper Bound of Probability of Failure

If the distribution form of $Z$ remains unknown, the probability of failure $P_f$ cannot be obtained from only the expectation value and variance. However the upper bound of the probability of failure that shows the value on the safe side can be obtained by this method. In the present method we can use the equation (18)[6][7] that gives an upper bound, which is rather low.

$$P_f < \frac{\sigma_Z^2}{\sigma_Z^2 + E[Z]^2} \tag{18}$$

## 3 Application

It is important that cars should secure the safety of its passengers when they crash into each other. The behavior of automobile seats at a collision raises complicated problems where dynamic behaviors with large deformations and material nonlinearities are involved. In this section, the authors solved quantitatively the effectivity and sensitivity of the components of seat frame on the collision response from the estimation expressions. Using the results of these calculations, authors carried out evaluation of dispersion and the optimum calculation for minimum cost of materials of seat frame.

## 3.1 Analysis of Effectivity

### 3.1.1 Design of Experiments

Eight design factors that consist of the thickness and yield stress of four steel members that compose the seat frame were considered in the analysis. On assumption that the characteristic value would show nonlinear relations, we employed 3 levels as the number of levels of each design factor . The orthogonal array L27 was used. Each level of thickness is t1.2,t1.4,t1.6 and each level of yield stress is 196MPa,245MPa,294MPa, respectively.

### 3.1.2 Finite Element Analysis

The explicit finite element method software, LS-DYNA3D, was used for the analysis of collision. FEM model is shown in Figure 2. The weight in the model was set to add to the frame a load by inertia force due to input acceleration. The half waves of a sine wave (max. acceleration 300 $m/sec^2$, cycle 200 *ms*) were applied to all the restraint points.

### 3.1.3 Analysis of Variance

The maximum displacement of weight portion that obtained from FEA was selected as a typical characteristic value of the collision phenomenon. Through the variance analysis we can decompose the effectivity into first-order and second-order ones. Table 1 shows the results of the variance analysis. The Figures 1 and 2 in the column "FACTOR" means first-order and second-order components of design factors. And "**" and "*" in "F RATIO" means that these components have a significant difference at 1% and 5% risk rates, respectively. From this results it is clear that there is not significant difference in interaction of $t_C$ and $t_D$.

### 3.1.4 Estimation Expression

An estimation expression (19) for the maximum displacement $d_{max}$ was generated with the design factors and degrees regarded as significant by the variance analysis (Criterion used 5% risk rate in F-Table). The variables t and $\sigma$ in the expression represent the design factors of respective members.

$$\begin{aligned}
\text{Maximum displacement } d_{max} &= 61.24 - 22.04(t_A - 1.4) - 14.51(t_B - 1.4) \\
&-56.56(t_C - 1.4) + 98.96\left\{(t_C - 1.4)^2 - 0.02667\right\} - 33.62(t_D - 1.4) \\
&-0.1314(\sigma_A - 245) + 0.001715\left\{(\sigma_A - 245)^2 - 1601\right\} \\
&-0.03218(\sigma_B - 245) - 0.3144(\sigma_C - 245) \\
&+0.002074\left\{(\sigma_C - 245)^2 - 1601\right\} - 0.1830(\sigma_D - 245) \\
&\pm 4.296 \quad (mm)
\end{aligned} \tag{19}$$

### 3.1.5 Sensitivity Analysis

The differentiation of equation (19) about individual design factors can produce the sensitivity to the maximum displacement as shown below. Clear and quantitative effectivity can be obtained easily from this method.

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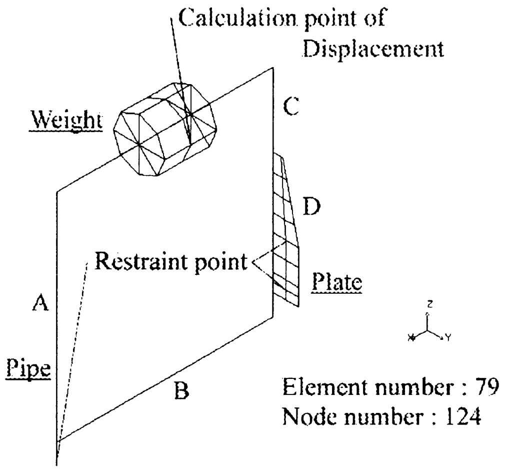


Figure 2 FEM model

Table 1 Analysis of variance for maximum displacement

| Factor | | Sum of Square | Degree of | Variance | F Ratio | | Contribution Ratio(%) |
|---|---|---|---|---|---|---|---|
| $\sigma_A$ | 1 | 7.46113E+02 | 1 | 7.46113E+02 | 8.901E+01 | ** | 6.94 |
| | 2 | 1.01770E+02 | 1 | 1.01770E+02 | 1.214E+01 | ** | 0.88 |
| $\sigma_B$ | 1 | 4.47710E+01 | 1 | 4.47710E+01 | 5.341E+00 | * | 0.34 |
| | 2 | 9.56765E+00 | 0 | 9.56765E+00 | 0.000E+00 | | 0.00 |
| $\sigma_C$ | 1 | 4.27279E+03 | 1 | 4.27279E+03 | 5.097E+02 | ** | 40.14 |
| | 2 | 1.48819E+02 | 1 | 1.48819E+02 | 1.775E+01 | ** | 1.32 |
| $\sigma_D$ | 1 | 1.44629E+03 | 1 | 1.44629E+03 | 1.725E+02 | ** | 13.54 |
| | 2 | 3.28567E+01 | 1 | 3.28567E+01 | 3.920E+00 | | 0.23 |
| $t_A$ | 1 | 3.49750E+02 | 1 | 3.49750E+02 | 4.172E+01 | ** | 3.21 |
| | 2 | 2.32421E+00 | 0 | 2.32421E+00 | 0.000E+00 | | 0.00 |
| $t_B$ | 1 | 1.51583E+02 | 1 | 1.51583E+02 | 1.808E+01 | ** | 1.35 |
| | 2 | 3.28609E+00 | 0 | 3.28609E+00 | 0.000E+00 | | 0.00 |
| $t_C$ | 1 | 2.30287E+03 | 1 | 2.30287E+03 | 2.747E+02 | ** | 21.60 |
| | 2 | 9.40078E+01 | 1 | 9.40078E+01 | 1.121E+01 | ** | 0.81 |
| $t_D$ | 1 | 8.13900E+02 | 1 | 8.13900E+02 | 9.709E+01 | ** | 7.58 |
| | 2 | 1.45538E+01 | 0 | 1.45538E+01 | 0.000E+00 | | 0.00 |
| $t_C*t_D$ | | 4.05736E+01 | 0 | 1.01434E+01 | 0.000E+00 | | 0.00 |
| Error | 1 | 1.17359E+02 | 14 | 8.38279E+00 | | | 2.10 |
| Total | | 1.06229E+04 | 26 | | | | 100.00 |

$$\partial d_{max} / \partial t_A = -22.04 \quad (mm/mm)$$
$$\partial d_{max} / \partial t_B = -14.51 \quad (mm/mm)$$
$$\partial d_{max} / \partial t_C = -56.56 + 197.92(t_C - 1.4) \quad (mm/mm)$$
$$\partial d_{max} / \partial t_D = -33.62 \quad (mm/mm)$$
$$\partial d_{max} / \partial \sigma_A = -0.1314 + 0.00342(\sigma_A - 245) \quad (mm/MPa)$$
$$\partial d_{max} / \partial \sigma_B = -0.03218 \quad (mm/MPa)$$
$$\partial d_{max} / \partial \sigma_C = -0.3144 + 0.004148(\sigma_C - 245) \quad (mm/MPa)$$
$$\partial d_{max} / \partial \sigma_D = -0.1830 \quad (mm/MPa)$$

## 3.2 Evaluation of Dispersion

The evaluation of the dispersion in the maximum displacement was evaluated based on the assumption that the design factors would vary without any correlation to each other. To calculate the standard deviations ($\sigma_f$) of respective design factors, the coefficient of variation was specified to be 3% for the sheet thickness and 5% for yield stress. The equation (20) is an estimation expression for the variance $Var[d_{max}]$ of the maximum displacement which is calculated by using the above values and sensitivities. The square root of the equation (20) is an estimation expression for the standard deviation $\sigma_{dmax}$ of the maximum displacement.

$$\begin{aligned} Var[d_{max}] = {} & (-0.6612 t_A)^2 + (-0.4353 t_B)^2 \\ & + [\{-1.697 + 5.937(t_C - 1.4)\} t_C]^2 + (-1.009 t_D)^2 \\ & + [\{-0.00657 + 0.0001715(\sigma_A - 245\} \sigma_A]^2 \\ & + (-0.001609 \sigma_B)^2 \\ & + [\{-0.01572 + 0.0002074(\sigma_C - 245)\} \sigma_C]^2 \\ & + (-0.009147 \sigma_D)^2 \end{aligned} \quad (20)$$

Any designer can proceed his work exactly and efficiently with consideration of the dispersion by making use of these results .

## 3.3 Optimum Calculation

To minimize the material cost of the frame, optimum calculation was made by using SQP method. The estimation expression of the maximum displacement and that of dispersion were used as a behavior inequality constraint function. As the value for constraint conditions, an assumed value was set to prevent the personal injury by deformation (maximum displacement). In this calculation, all the design factors were assumed to be continuous variables.

(1) Design factors: Thickness and yield stress of all members (8 factors)
(2) Objective function: Cost of materials (function of thickness and yield stress)

$$\text{Cost} = 0.012331\left\{\left(25.4t_A - t_A^2\right)\left(\frac{\sigma_A}{4.9} + 70\right) + \left(25.4t_B - t_B^2\right)\left(\frac{\sigma_B}{4.9} + 70\right) + \left(25.4t_C - t_C^2\right)\left(\frac{\sigma_C}{4.9} + 70\right) + 0.109272t_D\left(\frac{\sigma_D}{4.9} + 70\right)\right\}(yen) \quad (21)$$

(3) Constraints:
- Max. displacement $d_{max}$ [estimation expression (19)]+
  Standard deviation $\sigma_{dmax}$ [square root of expression (20)] $\leqq$ 50 *mm*
- Thickness (1.2 *mm* $\leqq t \leqq$ 1.6 *mm*)
- Yield stress (196 *MPa* $\leqq \sigma_y \leqq$ 294 *MPa*)

(4) Results of Optimization
- Objective value: 153.8 *yen*
- Max. displacement $d_{max}$ + Standard deviation $\sigma_{dmax}$
  = 44.7 *mm* + 5.3 *mm* = 50 *mm*
- Variable: $t_A$ = 1.2 *mm* $t_B$ = 1.2 *mm* $t_C$ = 1.29 *mm* $t_D$ = 1.6 *mm*
  $\sigma_A$ = 239 *MPa* $\sigma_B$ = 196 *MPa* $\sigma_C$ = 283 *MPa* $\sigma_D$ = 294 *MPa*
- Convergence frequency: 14 *times*

Because of its simple mathematical expression, this optimum calculation was able to carried out immediately at once on a personal computer.

### 3.4 Evaluation of Structural Reliability

Evaluation of structural reliability was evaluated by using the second moment method. Evaluation was carried out for combined variables as calculated by the optimum computation. The performance function (22) was set by using the estimation expression (19) of maximum displacement and by assuming that this displacement loses its function at 50 *mm* as the limit value.

$$Z = 50\,mm - \text{max. displacement } d_{max} \text{ (estimation expression (19))} \quad (22)$$

#### 3.4.1 Evaluation by FOSM Method

The reliability index $\beta$ by FOSM method becomes $\beta = 1$ by mean of the expectation $E[Z]$ = 5.3 *mm*, the standard deviation $\sigma_Z$ = 5.3 *mm* in the equation (22), and the equation (12).

The upper bound of probability of failure under this condition is $P_f < 0.5$ from the equation (18).

#### 3.4.2 Evaluation by AFOSM Method

The reliability index $\beta^*$ by AFOSM method was calculated by using the SQP method into the equation (23). $Y$ in this equation is a variable normalized by the equation (14). $Z(Y)$ is the equation (22) as transformed into the function of $Y$.

$$\text{Objective function: } \sqrt{Y_{t_A}^{\ 2} + Y_{t_B}^{\ 2} + Y_{t_C}^{\ 2} + Y_{t_D}^{\ 2} + Y_{\sigma_A}^{\ 2} + Y_{\sigma_B}^{\ 2} + Y_{\sigma_C}^{\ 2} + Y_{\sigma_D}^{\ 2}} \rightarrow \text{Minimum value} = \beta^*$$

$$\text{Equality constraint: } \quad Z(Y) = 0 \tag{23}$$

Reliability index (AFOSM method) $\beta^* = 0.972$

# 4 Conclusion

As a conclusion the authors describe the characteristics of the statistical optimization method:

(1) Applicable to nonlinear problems

(2) The statistical optimization method is an efficient method that can be applied with less labor of structural analysis.

(3) This method is very practical because it is possible to make use of existing programs for structural analysis and optimization.

(4) This method serves as a tool to get quantitative information useful for design.

(5) Integrated design analysis and evaluation system

(6) Since the optimum calculation by this method uses the estimation expressions equivalent to the regression expressions, the optimal solutions is not strict but approximate one, but the calculation can be carried out within very short computing time.

(7) The accuracy of estimation expression depends on how to set the number and range of levels of design factors. Therefore, to set the problem a designer is required to have some knowledge of the phenomena.

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