Weight minimization of dynamically loaded 3-D foundation on layered medium

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Abstract

Minimum-weight design of a machine foundation on an inhomogeneous soil is considered. The soil model under the base of block corresponds to a layer with linearly varying properties overlying a uniform half-space. Furthermore, the block may be surrounded by a backfill. The optimal designs of dynamically loaded rectangular machine foundation, including coupled sliding-rocking vibrations, are found for a number of realistic inhomogeneous supporting media. The results obtained in the case of the layered soil are compared with those for a uniform half-space. It illustrates the problem of adequate modelling of the nature of the soil profile providing an insight into the action of the soil-foundation-machine system from the point of view of performance and safety.

1 Introduction

Dynamic response of foundations has attracted the attention of engineers since the classical work of Lamb\textsuperscript{1} published in 1904. In the last decade, the considerable progress has been made in several areas of analysis and design of machine foundations.\textsuperscript{2,3,4}

The design involves determination of the vibration characteristics of the machine-foundation-soil system based on machine, block and soil data. Because of very small dynamic displacements allowable in design, a linear elastodynamics analysis of foundation response is generally carried out. The dynamic soil properties are a crucial input parameter governing the predicted behaviour of the system. Significant advancements in the area of soil dynamics made recently afford possibilities for determination of soil parameters in a realistic manner after careful evaluation of the field or laboratory test data.\textsuperscript{5,6}
Dynamic properties of soil medium are usually described in terms of complex-valued and frequency dependent impedance functions of a homogeneous half-space. However, the homogeneous elastic half-space of soil underestimates the real vibration amplitudes, particularly for the vertical and horizontal modes of vibration. It is a crucial problem from the point of view of performance and safety of the machine-foundation structural system. In real soil the confining pressure increase with depth due to the overburden even for deep structurally homogeneous deposits of uniform sand and clay. Then, the shear wave velocity increase with depth depending upon the type of soils.

In general, the engineering decision-making process is a trial-and-error procedure that requires a systematic using of principle of soil engineering, soil dynamics and theory of vibration. A safe and economical foundation block can be found by computer-aided engineering decisions. The problems of optimum design of a vertically vibrating machine foundation on an inhomogeneous soil have been studied by the authors where the soil model corresponding to a layer with constant elastic properties overlying elastic half-space was considered.

In this paper, the problem is studied for a more general model of soil corresponding to a layer with linearly varying shear wave velocity overlying a uniform half-space and for a more complex dynamic load (coupled horizontal and rocking vibrations). The results in each case are compared with those obtained for uniform supporting medium. It provides insight into the action of the machine-block-soil system concerning the safety and durability.

2 Analysis model

A dynamic system consists of a machine on a rigid rectangular foundation perfectly bonded to a layered inelastic half-space. The block may be partially or totally embedded in the ground, Figure 1. Dynamic soil-foundation interaction is performed by a substructure method, in which the system is divided into two substructures. The governing equations are developed separately and then are combined due to conditions of equilibrium and compatibility at the foundation-soil interface. One of the key elements in the formulation of the linear soil-structure interaction problem is the determination of the impedance functions representing the dynamic force-displacement relationships for a rigid massless foundation bonded to a layered half-space. To find the functions, a mixed boundary-value problem in elastodynamics must be solved, in which displacements are prescribed on the contact area between the foundation and the soil, and tractions vanish at the free surface of the medium. In this paper the mixed boundary-value problem has been reduced to the numerical solution of Fredholm integral equations of the first kind with the Green’s functions for a layered medium as kernels. Backfill surrounding the foundation is modelled as an independent layer overlying the layered medium below the base of block. Finally, the horizontal, coupling, rocking, vertical and
torsional impedance functions of a supporting layered medium are expressed in general as $K(i\omega) = K_1(\omega) + iK_2(\omega) = k(\omega) + i\omega c(\omega)$ in which $k(\omega) = K_1(\omega)$ is the true stiffness and $c(\omega) = K_2(\omega)/\omega$ is the coefficient of damping; $\omega$ is a circular frequency and $i^2 = -1$. The equivalent stiffness and damping coefficients depend on the frequency and they are also affected by soil material damping.

3 Statement of the problem

The problem of optimum design of vibrating 3-D machine foundation coupled to layered half-space can be stated as in Box 1. The above optimization problem is a standard nonlinear programming problem.\(^\text{17}\) To solve this problem an iterative application of a sequential linear programming (SLP) has been applied.\(^\text{18}\) Linear approximation of nonlinear functions is accomplished by replacing the nonlinear functions of the problem with their first-order Taylor series.\(^\text{19}\) Then this linearized problem is solved with using Simplex algorithm.\(^\text{18,20}\) The solution to the optimization problem needs sensitivities of the objective function $W$ and behaviour constraints (see Box 1.) with respect to the design variable $D_j$. The finite difference method (FDM) is adopted to obtain the gradients of the objective function $W$ and behaviour constraints. In order to control a stability and convergence of the algorithm a set of move limits is added to the constraints of the SLP problem. These move limits are specified as: $\Delta D_{\min} \leq \Delta D \leq \Delta D_{\max}$, where: $\Delta D_{\min}$ and $\Delta D_{\max}$ are the lower and upper limits on design changes respectively.
Box 1. Problem statement

Find the vector of design variables $\mathbf{D}$ such that

$$W(\mathbf{D}) - \text{weight (mass) of the concrete block} \rightarrow \min$$

subject to behaviour constraints:

- on total vertical vibration amplitude $A_v^{\text{max}} \leq A_v^{\text{feas}}$
- on total horizontal vibration amplitude $A_h^{\text{max}} \leq A_h^{\text{feas}}$
- on amplitude of dynamic mean normal pressure $\sigma \leq \sigma^{\text{feas}}$

and side constraints $\mathbf{D}_l \leq \mathbf{D} \leq \mathbf{D}_u$,

where:

$\mathbf{D}$ - are dimensions of the foundation base
$\mathbf{D}_l$ and $\mathbf{D}_u$ - are the lower and upper limiting values of a vector $\mathbf{D}$ of design variables, and

symbols with index feas mean allowable values of behaviour constraints.

4 Numerical examples

To illustrate the effect of soil inhomogeneity, a rigid rectangular machine foundation on layered inelastic soil is taken into consideration, Figure 1. The foundation is excited by a single cylinder reciprocating engine when counterweights are not installed (see Box 2).

The soil under the base of block consists of a layer of constant thickness $H_L$ bonded to an underlying uniform half-space. The inelastic layer is characterized by a shear modulus linearly varying with depth $G_1(z)$, density $\rho_1$, Poisson's ratio $\nu_1$, and hysteretic damping constants $\zeta_{1s}$ and $\zeta_{1p}$ for distortional and dilatational waves, respectively.

The underlying homogeneous inelastic half-space is characterized by the constant shear modulus $G_2$, density $\rho_2$, Poisson's ratio $\nu_2$, and hysteretic damping constants $\zeta_{2s}$ and $\zeta_{2p}$ for distortional and dilatational waves, respectively. The backfill is characterized by its constant shear modulus $G_b$, density $\rho_b$, Poisson's ratio $\nu_b$, and hysteretic damping constants $\zeta_{bs}$, $\zeta_{bp}$ for distortional and dilatational waves, respectively.

The design data parameters and limiting values of constraints are summarized in Box 2. The calculations were performed for two values of embedment $Z_E$, nine values of layer thickness $H_L$, one value of shear modulus $G_1(0)$ at the top of the layer and nine values of contrast ratio $CR = \sqrt{G_1(H_L)/G_1(0)}$, where $G_1(H_L)$ is the shear modulus at the bottom of the layer.
Increase of the layer thickness and contrast ratio changes the stiffness and damping coefficients of the inhomogeneous soil medium compared with the uniform one. Furthermore, the backfill surrounding the foundation alters its dynamic response by increasing the stiffness and damping coefficients of the subsoil. The results of the optimization process are summarized in Table 1 and Table 2 where the case CR = 1 corresponds to a uniform supporting medium characterized by the layer properties. The effect of the subsoil inhomogeneity and depth of embedment on the objective function is shown in Figure 2. To optimal rectangular shape of block corresponds the appropriate stiffness and damping coefficients of the supporting medium. They are shown in Figure 3, Figure 4 and Figure 5 (reference point: the centroid of the foundation basemat-soil interface).

Table 1. Optimal design variables $D_1 \times D_2$ [m×m] for surface foundation

<table>
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<tr>
<th>CR</th>
<th>HL=2 [m]</th>
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<th>6 [m]</th>
<th>8 [m]</th>
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Table 2. Optimal design variables $D_1 \times D_2$ [m×m] for embedded foundation (embedment ZE=1.0)

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Box 2. Input data and limiting values of constraints

(1) **machine data:**

mass of machine = 1500.0 kg,
mass moment of inertia of the machine about an axis passing through its centre of mass = 2500 kgm$^2$,
total rotating mass $M_A = 7.25$ kg,
total mass due to piston and crank rod $M_B = 13.15$ kg,
crank length $r = 0.15$ m,
length of connecting rod $l = 0.45$ m,
operating speed = 600 rpm ($\omega = 20\pi$ rad/s),
unbalanced forces (vertical and horizontal):

\[ F_v = M_A \omega^2 \sin \omega t, \]
\[ F_h = (M_A + M_B) \omega^2 \cos \omega t + M_B (r^2 \omega^2/l) \cos 2\omega t. \]

(2) **block data:**
density of concrete block = 2400.0 kg/m$^3$,
height $H_B = 1.5$ m,
thickness of base slab $H_P = 0.5$ m,
constant dimensions of top part of block: $Y_D = 1.5$ m, $X_D = 1.5$ m,

(3) **backfill data:**

thickness of backfill layer $Z_E = 0.0$ and 1.0 m,
dynamic shear modulus $G_b = 30000.0$ kN/m$^2$,
density $\rho_b = 1350.0$ kg/m$^3$,
Poisson’s ratio $\nu_1 = 0.25$,
hysteretic damping constants $\zeta_{bs} = \zeta_{bp} = 0.05$.

(4) **soil below the base of block:**

layer

thickness $H_L = 2, 3, 4, 5, 6, 7, 8, 9$ and 10 m,
shear modulus $G_1(0) = 60000.0$ kN/m$^2$, $G_1(H_L) = CR^2 \times G_1(0)$

\[ CR = 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0 \]
Poison’s ratio $\nu_1 = 0.33$,
density $\rho_1 = 1800.0$ kg/m$^3$,
hysteretic damping constants $\zeta_{1s} = \zeta_{1p} = 0.05$.

underlying half-space

shear modulus $G_2 = 1.13 G_1(H_L)$
Poison’s ratio $\nu_2 = \nu_1$,
density $\rho_2 = 1.13 \rho_1$,
hysteretic damping constants $\zeta_{2s} = \zeta_{2p} = 0.03$.

(5) **limiting values of constraints:**
total vertical displacement amplitude limit: $A_{v,\text{feas}} = 6.0 \times 10^{-6}$ m,
total horizontal displacement amplitude limit: $A_{h,\text{feas}} = 9.0 \times 10^{-6}$ m,
stresses in the soil limit $\sigma_{\text{feas}} = 150.0$ kN/m$^2$,
size limits: $1.5$ m $\leq D_1 \leq 3.5$ m, $1.5$ m $\leq D_2 \leq 3.5$ m.
a) b)

Figure 2: Objective function $W$ [kg] versus $HL = 2.0 \pm 10.0$ [m] and $CR = 1.0 \pm 3.0$: a) - surface foundation, b) - embedded foundation

1.25\cdot 10^9
1.05\cdot 10^9
8.5\cdot 10^8
6.5\cdot 10^8
4.5\cdot 10^8
2.5\cdot 10^8

1.25\cdot 10^9
1.05\cdot 10^9
8.5\cdot 10^8
6.5\cdot 10^8
4.5\cdot 10^8
2.5\cdot 10^8

a) b)

Figure 3: a) Vertical stiffness [N/m] and b) vertical damping [Ns/m] coefficients of layered medium versus $HL = 2.0 \pm 10.0$ [m] and $CR = 1.0 \pm 3.0$ for frequency of primary motion component (surface foundation - optimal design)
Figure 4: a) Rocking stiffness [Nm/rad] and b) rocking damping [Nms/rad] coefficients of layered medium versus HL = 2.0\pm10.0 [m] and CR = 1.0\pm3.0 for frequency of primary motion component (surface foundation - optimal design)

Figure 5: a) Horizontal stiffness [N/m] and b) horizontal damping [Ns/m] coefficients of layered medium versus HL = 2.0\pm10.0 [m] and CR = 1.0\pm3.0 for frequency of primary motion component (surface foundation - optimal design)
5 Conclusions

Soil layering considerably affects the optimal design of the surface rectangular foundation subjected to unbalanced forces due to a single-cylinder reciprocating machine generating vertical and coupled sliding-rocking vibrations.

This effect in the case of embedded foundation is of minor importance. However, as more surface area of the vibrating embedded block is in contact with the soil, more energy is transmitted into the surrounding soil. The energy transmitted in this manner may have adverse effects on adjoining structures.

Characteristics of the machine-foundation-soil system to be adequately reflected in the design procedure of machine foundations refer to the shape of the block-soil interface, backfilling, frequencies of excitation and local soil conditions.

References


