

Structural optimization: where we've been and where we're going

G. N. Vanderplaats

*Vanderplaats Research & Development, Inc., 1767 S. 8th Street,
Suite 200, Colorado Springs, CO 80906 USA*

EMail: vanderplaats@vma.com

Abstract

Structural optimization techniques are discussed, from the initial concept to modern approximation techniques. The present state of the art will be reviewed and future needs are addressed to indicate some of the challenges that lie ahead. It is concluded that the challenge now is to assimilate this technology into the practicing design environment.

1 Where We've Been

Any review of the state of the art in structural optimization is necessarily subjective. One person's view is offered here. Reference 1 contains a more extensive technical discussion of the present state of the art, along with numerous references. The goal is to understand where we've been, where we are, and where we are going.

1.1 Early Works

Structural optimization has been a topic of interest for over 100 years, beginning with the early works of Maxwell in 1869 and Michell in 1904. In the 1940s and early 1950s, considerable analytical work was done on component optimization as represented by such works as Shanley's Weight-Strength Analysis of Aircraft Structures.

Development of linear programming techniques by Dantzig, together with the advent of the digital computer, led to the application of mathematical programming techniques to the plastic design of beam and frame structures as described by Heyman in 1951.

1.2 Integration of Finite Element Analysis and Numerical Optimization

Schmit² in 1960 was the first to offer a comprehensive statement of the use of mathematical programming techniques to solve the nonlinear, inequality constrained problem of designing elastic structures under a multiplicity of loading conditions.

Numerical optimization solves the general problem: Find the set of design variables, $\mathbf{X}$, that will³;

$$\text{Minimize } F(\mathbf{X}) \quad (1)$$

Subject to:

$$g_j(\mathbf{X}) \leq 0 \quad j=1,m \quad (2)$$

$$X_i^L \leq X_i \leq X_i^U \quad i=1,n \quad (3)$$

The function, $F(\mathbf{X})$, is referred to as the objective or merit function and is dependent on the values of the design variables, $\mathbf{X}$, which themselves include member dimensions or shape variables of a structure as examples. The limits on the design variables, given in Equation 3, are referred to as side constraints and are used simply to limit the region of search for the optimum.

The $g_j(\mathbf{X})$ are referred to as constraints, and they provide bounds on various response quantities. The most common constraint is the limits imposed on stresses at various points within the structure.

Schmit recognized that this form of defining the design problem was precisely what was needed to find the minimum weight of structures, where finite element analysis is used to calculate the needed responses.

The finite element method solves for nodal displacements in the structure using the familiar relationship;

$$[\mathbf{K}]\mathbf{u} = \mathbf{P} \quad (4)$$

Once the displacements are calculated, the stresses in the members are recovered from those. Schmit solved the minimum weight problem for the now classical 3-bar truss and showed that the optimum design was not fully stressed.

This pioneering work led to a great deal of research in the early 1960s and beyond, and it was soon recognized that gradient based optimization methods were most efficient for solution of the optimization task.

1.3 Sensitivity Analysis

Gradients of many responses require the gradient of the displacements with respect to the design variables. Differentiating Equation 4 we get;

$$\frac{\partial \mathbf{u}}{\partial X_i} = [\mathbf{K}]^{-1} \left\{ \frac{\partial \mathbf{P}}{\partial X_i} - \frac{\partial \mathbf{K}}{\partial X_i} \mathbf{u} \right\} \quad (5)$$

Because we have already decomposed $[\mathbf{K}]$, the gradient of loads is zero (for static loads) and the gradient of the stiffness matrix with respect to the design variables is easily calculated, the needed information is readily available.

Using the standard finite element method for analysis, there are two key approaches to gradient computations, the direct method given by Equation 5 and the adjoint method. If a large number of gradients are needed, the direct method will be most efficient, whereas if the number of gradients needed is small, the adjoint method is most efficient. In practice, this decision should be made by the finite element code at the time of the computation in order to maintain maximum efficiency.

1.3 The Period of Triumph and Tragedy

The decade of the 60s saw extensive research in structural optimization, but virtually no practical applications. Indeed, by the end of the 60s it was apparent that design problems were limited to perhaps 10 variables and even these tasks often required over 100 finite element analyses. Given the slowness of computers (by today's standards), together with the continually increasing size of the finite element models, it was clear that this technology was reaching a dead end. This observation was dramatically offered by Galletly, Berke and Gibson⁴ when they called the 1960s the "Period of Triumph and Tragedy" for structural optimization. Furthermore, discretized optimality criteria methods presented by Venkayya, based on earlier analytical work by Prager and co-workers, offered an efficient method for solution of problems with large numbers of design variables. Optimality criterion methods were shown to be efficient for large problems, but were limited in the number of constraints that could be simultaneously considered.

1.4 Approximation Concepts

The use of mathematical programming for structural optimization breathed new life in 1974 when Schmit and his co-workers published the concept of approximation techniques for structural synthesis⁵.

The basic concept can be understood by considering a simple rod in tension and approximate the stress in the element as a linear function of the "reciprocal" of the original design variable, 'A.' This is a much higher quality approximation than we would have by simply linearizing with respect to 'A'.

Stress $\sigma = \frac{F}{A}$. In general, this is nonlinear and implicit in A.

Let $X = 1/A$. Now

$$\sigma = FX \quad \text{More Linear} \quad (6)$$

$$\sigma = \sigma^0 + \nabla \sigma \delta X \quad \text{Linear, Explicit} \quad (7)$$

The idea is that we create an approximation to the key response based on physics of the problem at hand. We then use this approximation during the optimization phase, instead of calling the finite element analysis whenever we need to calculate the stress. Now, the objective function, being A^*L , becomes L/X , which is clearly nonlinear. However, the objective function is easily calculated, along with its sensitivities.

In 1977, Fleury and Sander recognized that there is a direct relationship between optimality criteria methods and mathematical programming, noting that optimality criteria can be viewed as mathematical programming in dual space. This further established the validity of approximation concepts as a tool for large problems.

Using approximations, it is not necessary to approximate all response quantities considered in the optimization process. We only need to approximate those constraints that are critical or near critical for this step in the optimization process. This is referred to as “constraint screening” or ‘constraint deletion.’ For example, we begin by deleting from present consideration all constraints that are more negative (satisfied) than -0.3. Next, we consider regions in the structure. If we have a very fine finite element model, many elements in a small region of the structure will have nearly the same stress. It is not necessary to retain all of these stresses in our approximation because the most critical of them will be representative of the responses in that region. Therefore, we retain only a subset of these constraints.

The result of these simple concepts is that we could now optimize structures using, typically, as few as ten detailed finite element analyses, even for large numbers of design variables. Indeed, for statically determinate structures, this approximation for stress (or displacement) is precise, so only one detailed finite element analysis is needed to reach the optimum. The optimizer may still require a large number of function evaluations, but these are explicit and very cheap to evaluate.

The original approximation methods are quite good for problems where the element stiffness matrix is a product of the original design variable (‘A’ for rods, ‘t’ for membranes) and a geometric matrix. However, this approximation is not as good for cases such as beam elements or for shape optimization.

The difficulties in applying approximation techniques to general responses was somewhat overcome with the use of “conservative approximations.” This concept was first proposed by Starnes and Haftka and later refined by Fleury and Braibant. The basic concept is that we wish to create a conservative approximation to the response in question, so when we analyze the proposed design, the constraints are less likely to be violated.

2 Where We Are Today

References 6 - 8 offer a general review of the state of the art in 1980. From these reviews, it is clear that the state of the art was reasonably well developed at that time, but very few real applications could be found. It is also clear that these authors were optimistic about the future, but that nearly twenty years later, their expectations have not been met.

During the 1980's, and continuing today, second generation approximation techniques began to evolve. These include the use of intermediate variables, force approximations for stress constraints, and Rayleigh quotient approximations for frequency constraints, as examples. These methods dramatically improve the quality of the approximations, but are more difficult to incorporate into existing analysis codes.

2.1 Second Generation Approximations

Second generation approximation techniques make the intermediate variables more complicated functions of the design variables, and also use intermediate responses, rather than the original responses during gradient computations (see reference 1 for a numerous references. This is best understood by considering stress constraints for beam elements. Consider a simple rectangular beam element of width B and height, H . These are the physical design variables that the engineer wishes to determine. Assume we have a simple stress constraint calculated at the outer surface. Then;

$$\sigma = \frac{Mc}{I} \pm \frac{P}{A} \quad (8)$$

where $c=H/2$, $I=BH^3/12$ and $A=BH$ are simple, but nonlinear, functions of B and H . M is the bending moment and P is the axial force. A traditional linearization would be to create a Taylor series approximation to stress. However, it is clear that stress is highly nonlinear in the design variables, B and H , and so very small move limits would be necessary during the solution of the approximate problem.

Now consider how we might better approximate the stress. First, we treat A and I as intermediate variables. Next, we calculate the gradients of M and P (intermediate responses) with respect to A and I and create a Taylor series expansion.

When the optimizer requires the value of stress, we first calculate A and I explicitly as functions of B and H . Then, we calculate the member end forces, M and P using the approximation. Finally, we recover the stress in the usual fashion.

With the use of such intermediate variables and responses, we achieve two important goals. First, we allow the engineer to treat the physical dimensions, B and H , as design variables. Second, we retain a great deal of the nonlinearity of

the original problem explicitly. This allows us to make very large changes in the design variables during a given design cycle.

In a similar way, other responses can be approximated using our insight into the mathematical nature of the particular response. For example, eigenvalues may be approximated by what is called the Rayleigh quotient approximation as proposed by Canfield;

$$\lambda_k = \frac{U}{T} = \frac{\Phi_k^T K \Phi_k}{\Phi_k^T M \Phi_k} \quad (9)$$

We now create an approximation to the numerator, U , and denominator, T , independently and use the result to estimate λ_k . Note that we can use intermediate variables, such as section properties to create these approximations, just as for stress in the previous example.

2.2 Basis Vectors

The basic concept here is that we may provide several candidate designs and then find the best linear combination of these designs to achieve the overall design objective⁹. In the case of shape optimization, such basis vectors can be used to control internal nodes to retain a reasonable mesh in the finite element model, and thus reduce the need for remeshing the analysis model during the optimization process.

For example, let vectors $\mathbf{Y}^i$ define coordinates that can be changed, where X_i are the design variables. Then the resulting shape is

$$\mathbf{Y} = X_1 \mathbf{Y}^1 + X_2 \mathbf{Y}^2 + \dots + X_N \mathbf{Y}^N \quad (10)$$

A simple example of this approach for shape optimization is the optimum shape of a hole in a plate. If we treat only the positions around the hole as design variables, we are limited to very small changes in the design before it is necessary to remesh the analysis model. However, by including the positions of the interior nodes in the basis vector, we can change the external shape much more before remeshing is needed.

Creation of basis vectors for shape optimization and the issues of automatic mesh generation and refinement are key issues in developing reliable general purpose shape optimization software.

2.3 The Overall Process

Using approximation concepts, the basic program structure is shown in Figure 1 and described as follows;

1. Analyze the initial proposed design as a full finite element analysis.



2. Evaluate all constraint functions and rank them according to criticality. Retain only the critical and potentially critical constraints for further consideration during this design cycle.
3. Call the sensitivity analysis to calculate gradients of the retained set of constraints. These may be calculated as gradients of intermediate responses in terms of intermediate variables.
4. Using these gradients, construct approximations and create an optimization problem to be solved by a general purpose optimization code, and solve it. Here, the approximation could be linear, or may be modified in various ways. During this approximate optimization, move limits are imposed on the design variables to insure the reliability of the approximation.
5. Update the analysis data and call the analysis program again to evaluate the quality of the proposed design, If the solution has converged to an acceptable optimum, terminate. Otherwise repeat from step 2.

From these five steps, it is clear that the analysis and optimization tasks are quite closely coupled. This provides the greatest possible efficiency, but at the expense of major development costs. The overall process consists of an outer loop and an inner loop.

The outer loop consists of analysis, constraint deletion, gradient calculations, and creation of the approximate problem. The inner loop consists of actually solving the approximate optimization problem. Typically about ten design cycles are required, while perhaps 20 or more iterations are required to solve each approximate problem. Thus, the key is to create approximations of very high quality to reduce the number of design cycles (full finite element analyses) and for the approximate functions to be rapidly evaluated to reduce the cost of optimization in the inner loop.

Reference 1 offers several examples to demonstrate the present state of the art. Thanedar and Chirehdast¹⁰ offer recent examples of both sizing and shape optimization in the automotive industry applied to realistic design tasks.

3 Where Are We Going

In order to make structural optimization a widespread reality, we must consider not just technical issues. Indeed it is this author's experience that the key is "politics." Specifically, people become comfortable doing what they did last year and the year before. They (and more importantly, their management) must now be convinced that new technology can help their corporation and improve the "bottom line." The evidence that structural optimization can do these things is compelling. Software costs are not relevant. Software is very cheap. The issue is getting it used.



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3.1 Education

A key problem that this author has lamented for over 25 years is that most universities do not teach the fundamentals of optimization, much less structural optimization. Thus, few graduates are equipped to use it in their everyday work.

A second problem in gaining more widespread use of optimization is that (related to the education problem) most applications are in the research or advanced design departments of corporations. They would serve their respective corporations well by identifying company specific needs, aggressively working with software vendors to provide the best possible technology, transferring this to the engineering design departments.

3.2 Enhanced Software

Numerous commercial structural optimization capabilities are now available, using various levels of sophistication. The providers can be expected to continue to develop and enhance these programs.

Of paramount importance to more widespread use of structural optimization are ease of use issues. CAE and related graphical interface developers have been slow to add the needed design model creation capabilities to their products. As customer pressure increases, we can expect that the graphical interface developers will take optimization more seriously and provide the needed tools. This will, in turn, dramatically increase the use of optimization over a broad range of industries.

3.3 Very Large Scale Problems

Using approximation techniques, the actual optimization process typically accounts for only a few percent of the overall computational effort. However, as structural optimization becomes more widely used, the number of design variables will continue to increase. Most modern optimization algorithms solve a sub-problem for finding the search direction. If there are large numbers of variables and active constraints, this sub-problem can become quite time consuming and memory intensive. Thus there is a clear need to develop algorithms that will solve such problems efficiently and reliably. The goal should be to create optimization algorithms that can handle tens of thousands of variables with an equal number of active constraints.

3.4 Topology Optimization

Topology optimization has received increasing recent interest. Here, the goal is to begin with a block of material and use topology optimization to determine the basic configuration. On a theoretical level, it is clear that the "optimum topology" will be a truss. However, most structures of interest here are shell or solid structures. Therefore, upon finding a basic configuration with topology

optimization, the model must be refined to create a realistic structure. Clearly, considerable research remains to be done on issues of finding a basic topology, automatically convert the result to a realistic finite element model, and refine it with available sizing and shape optimization software.

3.5 Computer Architecture

Developments in computer architecture offer opportunities for both enhanced efficiency of analysis and new optimization algorithms. For example, massively parallel processors make it possible to create element stiffness and mass matrices very quickly, as well as offering opportunities for iterative analysis schemes. Distributed processors make it possible to gain near theoretical improvements in sensitivity analysis while making use of processors that may otherwise be idle. While continual changes in computer architecture offer opportunities for enhanced computational efficiency, single processor systems are still the most popular systems and will remain so for some time. Thus, there is limited motivation for software vendors to capitalize on these newer systems. This provides a time for proper research to develop a whole new class of analysis and optimization techniques.

3.6 Optimization Using Nonlinear Analysis

Finally, the technology already available can provide important insights into expanding optimization into other areas. For structural design, optimization based on material and/or geometric nonlinear analysis is a key topic. However, this technology should not be limited to structures and the current emphasis on multidisciplinary analysis and optimization must continue to seek ways to efficiently combine many disciplines.

4 Summary

Structural optimization technology is now reasonably mature and these methods have been added to most commercial finite element codes. Various methods are employed, ranging from simple coupling between the analysis and optimization to quite sophisticated use of approximation techniques.

Despite the fact that modern optimization methods are nearly as old as the finite element method, the acceptance of optimization as a commonly used design tool lags far behind the use of finite element methods for analysis.

The key issue now is to make this an everyday design tool. To achieve this, universities must begin to teach it as a design method, software vendors (particularly graphical interface providers) must dramatically improve ease of use, and corporations must invest in the needed training to make optimization a standard practice. The combination of these things will combine to dramatically reduce product development time, while at the same time improving quality.



It is time to move aggressively to get structural optimization out of the research department and into the design department.

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