Structural shape optimization of a body of revolution

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Abstract

In this paper, a solution procedure is presented for the structural shape optimization of a body of revolution which uses the iterative method to define the moving boundary. The structural shape optimization of the cross shaft of an universal joint is used to show its effectiveness.

1 Introduction

The structural shape optimization has been extensively discussed[1]-[3]. Among these literatures various reanalysis methods were used. These methods had directly influence on the efficiency of the shape optimization.

The technique of reanalysis of the entire structure is conceptually simple and can be quite efficient in a relatively small problem. But for a large structure, it may adversely affect the efficiency of computation.

The technique of substructuring is an another method of reanalysis, which is more efficient than reanalysis of the entire structure. But the disadvantage of this technique is that the computer program is more complicated due to the increasing file handling and data transfers.

In this paper, a solution procedure is presented for the structural shape optimization of a body of revolution, which combines the iterative method and reanalysis method for the structural analysis.

The body of revolution subjected to optimization consists of two connected parts, say parts A
and B. Part A has a well-defined exterior boundary and part B has a varying exterior boundary which is to be optimized to achieve a goal of engineering design. The boundary between these two parts is moving due to the interaction of these two parts under load. In this procedure, the shape optimization is carried out in the part which has varying exterior boundary, namely part B. Then a reanalysis of the entire structure, parts A and B, is performed to determine the current moving boundary. Thereafter this newly defined moving boundary is used as the boundary of part B and part B is again optimized. This iterative process is carried out a few times until the moving boundary is no longer moving. Then the solution obtained is the optimum shape of the body.

The method is especially suitable for the optimization of stress concentration factor. For this kind of optimization the shape of the body is locally adjusted and therefore the solution can converge easily.

2 Application to a Mechanical Element

Universal joints are important parts in machinery. They are most commonly used in automobiles and rolling mills. The cross shaft is the main element of an universal joint. This key mechanical element has been designed by using the traditional strength formula of mechanics of material. Failure often happens at the chamfer of the cross shaft due to the stress concentration and fatigue in the service life. For the purpose of reducing stress concentration, the aforementioned solution procedure is applied on the design of the cross shaft which is a body of revolution as show in figure 1. Due to the symmetry of the structure and loads, the top half of a quarter of the cross shaft may be used as the calculation model as shown figure 2, which has parts A and B, and a moving boundary $L_4$ between them.

2.1 Selection of Element

In order to accommodate large stress gradients and to model geometric curve boundary, the isoparametric hexahedral element of twenty nodes is chosen. The faces of the element are quadratic surfaces and the edges of the faces are quadratic curves. The accuracy of this finite element approximation is very high at Gauss points.

The goal of optimization in this problem is to
minimize the stress concentration factor for the stresses at each and every nodal point of the body. Thus the stresses at Gauss points are extrapolated to nodes of each element. Since a node may be shared by several elements and the extrapolated stresses at the node may be different from the surrounding elements, the stress at this node is set equal to the average of the extrapolated stresses.

In subsequent cycles of computation, similar mesh subdivisions are automatically generated and the finite element analysis and optimization procedure are jointly performed.

2.2 Formulation of the Optimization Problem

2.2.1 Objective Function

According to the known failure mode of the universal joint, the stress concentration factor is chosen as the objective function. Thus mathematically it can be expressed as

\[
Z = \text{MIN} \left[ \text{MAX}\left(\frac{\sigma_p}{\sigma_n}\right) \right] \quad R \in \Omega
\]

where: \( p \) is an arbitrary node point within the subdomain \( R \). \( \sigma_n \) is the average stress. \( \Omega \) is the total domain of the design envelope. \( R \) is a subdomain of the design envelope. \( \sigma_p \) is the design stress at point \( p \).

2.2.2 Design Variables

As shown in figure 2, \( L_1 \) and \( L_3 \) are treated as the frozen boundaries. \( L_2 \) is treated as the design boundary. \( L_4 \) is the moving boundary which is determined iteratively. Thus design variables are the \( y \) coordinates of the design boundary.

2.2.3 Constraints

Since a stress concentration problem is dealing with a phenomenon involving small changes in a localized region of the body. The behavior properties of the body in a global sense will not greatly affected. The only constraints are lower and upper bounds of the variables. Thus, the problem may be referred as a constrained optimization with constraints of the
linear type.

\[ g_m \leq 0 \quad m=1, \ldots, M \]

\( g_m \) are linear functions of variables and \( M \) is the total number of side constraints.

### 2.2.4 Method of Optimization

For the problem under discussion, the constraints are linear and the objective function is highly nonlinear. The internal penalty function method is used for the structural shape optimization. The internal penalty function transforms a constrained optimization problem into an unconstrained one and takes the following equation form which was given by Fiacco and McCormick[4]

\[
\Phi(x, r) = F - r \sum_{j=1}^{m} \frac{1}{g_j} + \frac{1}{r} \sum_{k=1}^{t} L_j^2
\]

where \( \phi(x, r) \) is the internal penalty function, \( F \) is the objective function, \( g_j \) are inequality constraints, \( L_j \) are the equality constraints and \( r \) represents a parameter with a decreasing sequence of values. Powell’s method[5] is used for unconstrained optimization, and an initial feasible point is found by random method. The example results show that for highly nonlinear objective function (such as the stress) with linear constraints, internal penalty function method is extremely effective and it’s easy to converge to the optimum solution. However for a nonlinear or linear objective function (such as the structure weight) with highly nonlinear constraints (such as the stress constraints), the internal penalty function method has difficulty to converge.

### 3 Numerical Example

To show the application of the aforementioned process, a typical universal joint is selected as an example to demonstrate the effectiveness of the program.

First, assuming the boundary \( L_4 \) is fixed, the design boundary \( L_2 \) of part B is obtained by shape optimization procedure. Then by analyzing the struc-
ture as a whole, the displacement of the moving boundary \( L^4 \) is determined. Thereafter this newly acquired boundary \( L^4 \) is used as the boundary of part B, and the design boundary \( L^2 \) is again obtained by optimization procedure. This iterative process is repeated until the moving boundary is stop to move. Then the optimum solution is reached. Figure 3 shows the successive optimization cycles. Curve 1 represents initial curve. Curve 2 represents first iterative optimum solution. Curve 3 represents the second iterative optimum solution. The value of initial objective function (\( \sigma_x \)) is 7715 kg/cm\(^2\). The value of second iterative optimization objective function (\( \sigma_x \)) is 6694 kg/cm\(^2\). the objective function decreases about 10 %.

4 Conclusion

The shape optimization problem of a body of revolution has been solved by using iterative method. This method is especially effective for the shape optimization problems which modifies the local boundary. Internal penalty method is proved to be a good method for the optimization problem in which objective function is highly nonlinear and constraints are linear.

Reference

Figure 3