Analysis and design of cable net structures through optimization techniques
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ABSTRACT

This paper shows that numerical optimization methods provide a comprehensive and rigorous basis not only for design but also for a variety of very practical analysis problems associated with grossly non-linear cable structures. The static analysis is achieved through unconstrained optimization of the total potential energy stored in the structure. The pretension design is set in a multicriteria optimization context. A minimax solution is found by means of an entropy-based optimization algorithm. Illustrative examples are solved.

INTRODUCTION

The behaviour of cable net structures is characterised by a combination of geometrical and physical non-linearities. Geometric non-linearities arise from the fact that such structures equilibrate applied loading by large changes of shape but small strains. Physical non-linearities are caused by the fact that cables are able to carry only tension forces and become slack under compressive loads. Further non-linearity may arise from possible yielding of the tension cables.

The analysis of these non-linear structures involves two iterative calculations: one to determine the zero configuration, which results from the application of pretensioning to an untensioned net, and one to determine the final configuration, resulting from the application of service loading upon the prestressed structure. The design of cable net structures consists of finding the best distribution of prestress to achieve satisfactory performance in terms of cable stress and joint displacement levels, and is even more complex than the analysis problem. The overall theme of this paper is to show that computer-based numerical optimization methods provide a comprehensive and rigorous basis for practical analysis and design problems associated with grossly nonlinear cable net structures.

The static analysis of cable net structures is achieved through direct unconstrained optimization of the total potential energy stored in the
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structure. This approach to analysing cable net structures has been described by Buchholdt [1] and later developed by Sufian [2] and Sufian and Templeman [3] to include pretensioning analysis through specification of cable element shortnesses representing externally applied pretensioning forces. The zero configuration is found by introducing desired pretension forces via end cable elements in specified locations. These end elements are assumed to be slightly shorter than geometrically necessary by specified shortnesses over which they are stretched into position during the process of pretensioning. This analysis approach is described in detail in Refs. [2,3] and is used in the present work. A brief outline of this method is given in the next Section of this paper.

Nodal displacements can usually be reduced by increasing the levels of pretensioning forces. However, this requires the use of larger diameter cables, more robust clamps and anchors and much stiffer supporting structures, hence, a more expensive structure. For this reason, pretensioning is one of the most important features to be accounted for in designing cable nets and is an ideal candidate for optimization.

As a design goal, it is desirable to find as low a level of prestressing as possible which satisfies the performance requirements for cable net structures. This paper develops a numerical method which seeks the minimum level and optimum distribution of prestress whilst satisfying prescribed limits upon cable stresses and nodal deflections under any given load condition which the structure encounters during its service life. This is achieved by posing an optimization problem in terms of shortness variables for which values are sought. design is posed in a vector (multicriteria) optimization format in which normalised constraint goals, governing the target behavioural restrictions, are assumed as objective functions to be minimized. The formulated minimax design problem is conveniently converted into a convex scalar optimization function by means of an entropy-based technique [4,5] which can be solved using any unconstrained optimization algorithm.

STATIC ANALYSIS

The analysis approach is to consider the determination of equilibrium as a minimization process of the total potential energy. The total potential energy, \( \pi \), in any loaded structure is the summation of the strain energy, \( S \), stored in the structure and the potential energy, \( U \), of the external loads and may be expressed as

\[
\pi = S + U
\]  

(1)

The condition for equilibrium can be mathematically expressed as

\[
\partial \pi = 0
\]  

(2)

Cable nets subjected to pretensioning

The zero configuration geometry and pretensioning forces can be calculated using the minimum energy method under the following conditions

1. The unstressed configuration geometry is known and is pretensioned by stretching any of the end cable elements.
2. These end cable elements are assumed to be initially shorter than is geometrically necessary to stretch the cables on to the supports.

Consider a shortened member, $k$, connected to adjoining inner members, $i$, at node $j$ in space as shown in Figure 1. The end member, $k$, is of initial length $L_{nk}$ which is shorter by an unknown amount $\delta L_{nk}$ than the physical distance between joints $j$ and $m$. Applying some pretension force $P_k$ to member $k$ stretches this member on to support $m$ so that the cable system pretension forces are then $T_r$ in members $i$ and $T_{ok}$ in member $k$. The free joints $n$ and $j$ will displace from their initial coordinates $(x_r, y_r, z_r)$ and $(x_{nj}, y_{nj}, z_{nj})$ to a new equilibrium position $(x_{on}, y_{on}, z_{on})$ and $(x_{oj}, y_{oj}, z_{oj})$ respectively.

**Total potential energy function**

The total potential energy function, $\pi$, is given by

$$\pi = \frac{1}{2} \sum_{i=1}^{I} (K_i e_{oi}^2) + \frac{1}{2} \sum_{k=1}^{ne} (K_k e_{ok}^2) - \sum_{k=1}^{ne} (P_k \delta L_{rk})$$

where $I = (nm - ne)$, $nm$ is the total number of cable elements in the net structure and $ne$ is the total number of end cable elements stretched, $K(=EA/L)$ is the axial rigidity, $e_i$ and $e_k$ are the elongations of cable elements $i$ and $k$ and can be expressed in terms of the nodal coordinates thus

$$e_{oi} = L_{oi} - L_{ri}$$

$$e_{ok} = L_{ok} - L_{rk}$$
where $L_{ri}$ and $L_{oi}$ are untensioned and tensioned lengths of member $i$ respectively and are given by

$$L_{ri} = \left\{ (x_{rn} - x_{rj})^2 + (y_{rn} - y_{rj})^2 + (z_{rn} - z_{rj})^2 \right\}^{1/2}$$

(6)

$$L_{oi} = \left\{ (x_{on} - x_{oj})^2 + (y_{on} - y_{oj})^2 + (z_{on} - z_{oj})^2 \right\}^{1/2}$$

(7)

Similarly

$$L_{rk} = \left\{ (x_{m} - x_{rj})^2 + (y_{m} - y_{rj})^2 + (z_{m} - z_{rj})^2 \right\}^{1/2} - \delta L_{rk}$$

(8)

$$L_{ok} = \left\{ (x_{m} - x_{oj})^2 + (y_{m} - y_{oj})^2 + (z_{m} - z_{oj})^2 \right\}^{1/2}$$

(9)

where $(x_m, y_m, z_m)$ are the coordinates of the fixed joint $m$. Equation (3) is a function of two sets of unknown variables: the first set contains the three free joint coordinates $(x_o, y_o, z_o)$ and the second set contains the end member shortnesses $\delta L_{rk}$. The derivatives of $\pi$ with respect to these two sets of unknowns with respect to the unknowns are given [2,3] by

$$\frac{\partial \pi}{\partial \delta L_{rk}} = K_k e_{ok} - P_k$$

(10)

$$\frac{\partial \pi}{\partial x_{oj}} = \sum_{i=1}^{nj} K_{e_{oi}} \frac{(x_{oj} - x_{on})_i}{L_{oi}}$$

(11)

with similar derivatives of $\pi$ with respect to $y_{oj}$ and $z_{oj}$. $nj$ is the number of cable elements meeting at joint $j$ and $x_{on}, y_{on}$ and $z_{on}$ are the coordinates of the joint connecting the remote end of cable element $i$. If an end member, $k$, meets joint $j$ then $x_{on}, y_{on}, z_{on}$ in Equations (11) become $x_m, y_m, z_m$ respectively.

At the stationary point of $\pi$, setting Equations (10) and (11) to zero gives the equilibrium condition of the pretensioned cable net structure. If $P_k$ are specified, Equations (10) and (11) are solved to find the required initial shortnesses $\delta L_{rk}$ and the free joint coordinates $(x_o, y_o, z_o)$. However, in practice it is more practical for the initial shortnesses to be specified than the pretension forces $P_k$. Therefore, if $\delta L_{rk}$ are specified, Equations (10) and (11) can be solved to give values for coordinates $(x_o, y_o, z_o)$ and the pretension forces $P_k$ necessary to satisfy equilibrium.

In this work Equation (3) was minimized directly to determine equilibrium, rather than using Equations (10) and (11) which are non-linear and hard to solve. Examining Equation (3) further, it can be seen that if the initial shortnesses $\delta L_{rk}$ of end members are specified the first two terms become functions of the unknown coordinates only and the third term is effectively constant. Consequently the minimization process of Equation (3) can be considerably simplified by minimizing

$$\pi = \frac{1}{2} \sum_{i=1}^{l} (K_{e_{oi}}^2) + \frac{1}{2} \sum_{k=1}^{ne} (K_{k e_{ok}}^2)$$

(12)

over the free joint coordinates only.
Response to static loading

The applied load on a cable net structure is assumed to act at the nodes only. Once the zero configuration and the pretension forces in the cables have been found, the final configuration and member forces can be worked out in the same way as for prestressing, using the zero configuration as datum.

The expression for the total potential energy function is given by

\[ \pi = \sum_{i}^{n}(s_{o} + T_{o}e + \frac{1}{2}ke^{2}) - \sum_{j}^{n}(F_{xj}\delta_{xj} + F_{yj}\delta_{yj} + F_{zj}\delta_{zj}) \]  

(13)

where \( e \) is the elongation of a cable element due to static service loads, \( T_{o} \) is the initial pretension forces in the cable element, \( s_{o} \) is the strain energy in the cable element due to pretension force and \((F_{xj}, F_{yj}, F_{zj})\) and \((\delta_{xj}, \delta_{yj}, \delta_{zj})\) denote the components of the applied load at joint \( j \) and the displacements of joint \( j \) respectively. \( n_{f} \) is the total number of loaded joints.

The elongations and nodal deflections can be expressed in terms of the variable joints coordinates thus for element \( i \)

\[ e_{i} = L_{fi} - L_{oi} \]

(14)

where \( L_{oi} \) and \( L_{fi} \) are the prestressed and loaded lengths which can be related to the global coordinates as

\[ L_{oi} = \left\{(x_{on} - x_{oj})^{2} + (y_{on} - y_{oj})^{2} + (z_{on} - z_{oj})^{2}\right\}^{1/2} \]

(15)

\[ L_{fi} = \left\{(x_{fn} - x_{fg})^{2} + (y_{fn} - y_{fg})^{2} + (z_{fn} - z_{fg})^{2}\right\}^{1/2} \]

(16)

Finally, the nodal deflections can also be expressed in terms of the global coordinates in the following way

\[ \delta_{xj} = x_{fg} - x_{oj}; \ \delta_{yj} = y_{fg} - y_{oj}; \ \delta_{zj} = z_{fg} - z_{oj} \]

(17)

Hence Equation (13) is expressed in terms of the known coordinates of the zero configuration and the unknown joint coordinate variables over which the equation is minimized for equilibrium to find the final configuration.

Derivatives of the total energy function with respect to the variable coordinates \( x_{f}, y_{f} \) and \( z_{f} \) at joint \( j \) can be expressed as:

\[ \frac{\partial \pi}{\partial x_{fj}} = \sum_{i=1}^{nj}(T_{o} + Ke)_{i} \frac{(x_{fg} - x_{fn})_{i}}{L_{fi}} - F_{xj} \]

(18)

with similar derivatives of \( \pi \) with respect to \( y_{fj} \) and \( z_{fj} \). where \( x_{m}, y_{m} \) and \( z_{m} \) are the coordinates of the joint connecting the remote end of cable element \( i \).
Minimization of the total potential energy

The minimization of the total energy functions is treated as an unconstrained optimization problem and is expressed in the following form

\[
\min_{X} \pi(X)
\]  

(19)

where \( \pi(X) \) is the total energy function expressed in terms of, \( X \), the joints coordinate variable vector. Checks on the cable strains during the optimization process are incorporated and the strain energy terms corresponding to slack cable elements are removed when necessary.

Problem (19) is a straightforward unconstrained non-linear optimization problem for which a variety of solution methods are available. The one used in the present work was the conjugate gradient method [6].

PRETENSION DESIGN

As was noted earlier, the magnitude of the nodal displacements may be reduced by increasing the levels of prestressing forces in the cable net. As a design goal, it is desirable to find as low a level of prestressing as possible which satisfies the performance requirements for a cable net structure.

One important performance requirement for a cable net is that all of the cable elements remain stressed under the various loading cases. The prestress should be neither too large nor too small but sufficient to maintain a tensile force under all conditions. The other important requirement for a pretensioned cable net is that no excessive deflection of the joints should occur under any of the loading cases that the structure may encounter during its service life.

Minimax problem formulation

Assume that an initially untensioned (reference) configuration of a cable net structure is given and the cross sectional areas of the cable elements are known. Pretensioning is carried out by specifying shortnesses in any of the end cable elements as described earlier. A combination of shortness magnitudes and shortened cable elements positions which give the lowest possible prestressing forces is desirable. If the total number of end cable elements chosen to be stretched is denoted by \( ne \) and the shortnesses in these elements denoted by \( \delta L_i \) then, \( \delta L \) is a vector of end cable element shortnesses, \( \delta L_i, i = 1, \ldots, ne \). These are design variables for the optimization.

One set of goals arises from the requirement that under any dead or static service loading, the displacement of the joints should all be as small as possible. A question then arises as to which configuration the displacements should be measured relative to: the initial reference configuration or the zero configuration. In this work it was decided that a maximum desirable value of the displacements \( \delta_{max} \) of any joint under applied, dead or static service loads from the zero configuration should be imposed. If \( \delta_{xj}, \delta_{yj} \) and \( \delta_{zj} \) are the displacement components of joint \( j \) from the zero configuration due to applied load, then the displacement goals are
The transverse displacements in the direction of the applied loads are large in magnitude compared to those in the longitudinal directions, and therefore are more critical. It is, therefore, only necessary to constrain the displacements in the direction in which the loads are applied. If a sign convention is chosen such that the y axis represents the transverse direction in which the load is applied, then out of constraints (20) only displacement goals (21) for joint \( j \) need be considered.

\[
\delta_{yj} \leq \delta_{\text{max}}
\]

(21)

It is only necessary to limit the displacements of those joints which are prone to undergo large deflections.

Imposing a lower limit on the total forces in a cable ensures that cable elements do not become slack. If \( T_{\text{min}} \) is the value of a minimum desirable tensile force in any cable element, a minimum tension force goal can be written in the form

\[
T_{fi} \geq T_{\text{min}}
\]

(22)

where \( T_f \) is the final force in a particular cable element \( i \). Similarly, the maximum force in any cable element may be constrained not to exceed a desirable maximum value \( T_{\text{max}} \) by means of the goal

\[
T_{fi} \leq T_{\text{max}}
\]

(23)

The optimization method used in this work requires that all these goals should be cast in a normalized form. The goals (21), (22) and (23) may be written in the normalized form as follows:

\[
G_j(\delta L) = \frac{\delta_{yj}}{\delta_{\text{max}}} - 1 \leq 0 \quad j = 1, \ldots, J
\]

(24)

\[
G_k(\delta L) = \frac{T_{\text{min}}}{T_{fi}} - 1 \leq 0 \quad k = J + 1, \ldots, J + nm + 1
\]

(25)

\[
G_i(\delta L) = \frac{T_{fi}}{T_{\text{max}}} - 1 \leq 0 \quad i = J + nm + 2, \ldots, J + 2nm
\]

(26)

where \( J \) is the total number of joint displacement restrictions and \( nm \) is the number of cable elements.

It is desirable to change the values of the variable vector \( \delta L \) in such a way that the values of all the elements of the goals are made as small as possible. This can be achieved by minimizing the maximum of the goal element values over variables \( \delta L \) thus casting the problem in a minimax format which may be defined as

\[
\text{Minimize } \max_{\delta L} \ G_i(\delta L)
\]

(27)

in which \( G_i, i = 1, \ldots, J + 2nm \), are the goal functions (24) to (26).
Minimax optimization

Problem (27) is discontinuous and non-differentiable which makes its numerical solution by direct means difficult. In this work the solution to the minimax problem (27) is found indirectly by the unconstrained minimization of a scalar function which is continuous, and thus considerably easier to solve. The following theorem [4,5] can be proved.

Theorem

The vector \( \mathbf{x} \) which solves the vector minimax problem

\[
\text{Minimize} \quad \text{Maximum} \quad \langle \mathbf{G}_i(x) \rangle \quad \text{for} \quad i = 1, \ldots, I
\]

where \( \mathbf{G} \) is a vector of dimensionless goal functions, is generated by solving the scalar optimization problem

\[
\text{Minimize} \quad \left( \frac{1}{p} \right) \ln \left[ \sum_{i=1}^{I} \exp[\mathbf{pG}_i(x)] \right]
\]

with positive values of the parameter \( p \) increasing towards infinity.

The minimax optimization problem (27) for the cable net structure with goals defined by Equations (24) to (26) was solved by the scalar minimization problem:

\[
\text{Minimize} \quad F_p = \left( \frac{1}{p} \right) \ln \left[ \sum_{j=1}^{M} \exp[\mathbf{pG}_j(\Delta L)] \right]
\]

over an increasing positive sequence of \( p \), where \( M = J + 2nm \).

Scalar optimization

Problem (30) may, in theory, be solved by a variety of conventional scalar optimization methods. Derivatives of \( F_p \) with respect to all variables \( \Delta L \) may be formed but contain the first derivatives of the goal functions \( \mathbf{G}_j(\Delta L) \). Some of these goals are implicit functions of \( \Delta L \) and their derivatives are consequently very hard to obtain by algebraic means. A possible alternative to analytical derivatives is the use of numerical derivatives. However, in this cable net problem this would require \( ne + 1 \) complete analyses of the cable net under all loading cases in order to calculate numerical first derivatives for use in a first order solution method for problem (30). This would be enormously time consuming.

The solution to problem (30) may be achieved by other methods which do not require derivative values and thus the complication of their evaluation can be avoided. The choice of unconstrained optimization methods that can be used to solve problem (30) is, therefore, restricted to those in which derivative evaluations are not required. Optimization methods which do not require derivative values are less efficient than those which require derivatives. However, the increased efficiency which could be achieved by first order optimization methods is outweighed by the complication involved in evaluating the first derivatives. In the
present work, therefore, a zeroth order optimization method, namely the nonlinear Simplex method [7], was used to solve problem (30).

Optimization Design algorithm
The main steps in the algorithm used for the optimization of cable net structures can be summarised as follows

- Step 1: Input data: reference configuration, material properties, all loading cases and an initial set of shortnesses as the starting point
- Step 2: Choose a value of \( p \) for solving problem (28)
- Step 3: Analyse the net structure for pretensioning plus each applied loading system using the minimum energy method
- Step 4: Find the numerical values of all goal functions for use in problem (28)
- Step 5: Solve problem (28) by any zeroth optimization method (such as the simplex method). This gives new estimates for all end cable element shortnesses.
- Step 6: If iteration have converged stop iteration and go to the next step otherwise return to step 3
- Step 7: Retain the new design and use it as a new starting point and increase the value of \( p \) then repeat the optimization by returning to Step 3. If the difference between the starting point and the new design is small stop the algorithm and use the new design as the optimum.

The minimax optimization algorithm requires a sequence of positive values of \( p \) increasing towards infinity. In the present work a value in the range \( 10^4 \) to \( 10^6 \) was used for the solution of problem (28). Further algorithmic details and computational aspects can be found in Ref.[2].

DESIGN EXAMPLES
The above optimization procedure has been used to find the optimal shortness set and the corresponding pretension state for two cable net structures. The first net model examined is simple and has been solved in order to illustrate the proposed design algorithm and to validate and confirm the accuracy of the results. The method is then used to find the optimum distribution of prestressing forces in a more realistic cable net structure.

Example 1: Cable structure
A simple five member cable structure is shown in Figure 2. Before performing the design calculation it is appropriate to examine the behaviour of the loaded cable structure under different prestressing states. The structure is to carry a point load of \( 60 \text{kN} \) in the vertical direction at the free joint, 2, as shown in Figure 2. If the prestressing procedure is restricted to the stretching of cable elements 4 and 5 only by equal
shortness then, it would be desirable to know the final force distribution and nodal deflection under varying shortness magnitudes.

To this end the loaded net was analysed for increasing shortness values specified in elements 4 and 5. The resulting nodal deflection and force distribution are depicted graphically by Figure 3a, curve 1, and Figure 3b respectively. Curve 1, in Figure 3a reveals that no significant reduction in the joint displacement can be achieved for shortnesses greater than 0.004m. The minimum displacement possible is about 0.0021m. Examination of the final force distribution results depicted by Figure 3b, reveals that for shortness values less than 0.004m cable elements 1 and 2 become slack and for values greater than 0.004m element 3 remains slack in the final configuration. This means that no shortness solution exists which satisfies the non-slackening condition. For this criterion to be satisfied other members need to be stretched together with members 4 and 5. The effect of stretching member 3 along with members 4 and 5 has been examined by re-analysing the cable structure subjected to varying shortness magnitudes specified in elements 4 and 5 combined with shortnesses of $\delta L_3 = 0.004m$ and 0.008m at element 3. Displacement results are shown in Figure 3a, curves 2 and 3, and the final force distribution in the members are shown in Figure 3c and 3d.

If limits upon the joint displacement and maximum and minimum member forces are imposed such that $\delta_{j_{\text{max}}} \leq 0.00155m$, $T_{r_{\text{max}}} \leq 25kN$ and $T_{r_{\text{max}}} \geq 10kN$ then, a close examination of Figure 3 reveals that an optimum shortness solution which simultaneously satisfies all of these goals does not exist. However, the best possible solution whereby the degree to which these goals are violated is a minimum may be predicted by the use of Figure 3. The optimum solution lies in the region marked A in Figure 3c, in which $\delta L_3$, $\delta L_4$, and $\delta L_4$ have values of approximately 0.004m.
Figure 3. Force and displacement variation with changes in shortness magnitudes
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For the above simple example with only two design variables, it has been possible to predict the optimum solution with reasonable accuracy. This provides a means of checking the design result obtained using the present optimization method.

Design calculations have been carried out for the above example. A starting point $\delta L = 0.001$ was used. An initial value of $p = 10$ was subsequently increased until no further improvement to the solution was obtained. The solutions obtained for the different $p$ values are given in Table 1. It can be seen from Table 1 that the solution is fairly insensitive to changes in the $p$ value. It is noted that the function $F_p$ monotonically decreases as $p$ increases. Significant improvement to the solution ceases at $p = 10^4$ and the optimum design is in agreement with that predicted from the above analysis.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\delta L_{14}$</th>
<th>$\delta L_{13}$</th>
<th>$T_{f_{\text{max}}}(kN)$</th>
<th>$T_{f_{\text{min}}}(kN)$</th>
<th>$s_{p_{\text{max}}}(kN)$</th>
<th>$G_{i_{\text{max}}}$</th>
<th>$F_p$</th>
</tr>
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<tr>
<td>$10^1$</td>
<td>4.0078</td>
<td>4.1594</td>
<td>9.910</td>
<td>28.149</td>
<td>0.15356</td>
<td>0.123024</td>
<td>0.255929</td>
</tr>
<tr>
<td>$10^2$</td>
<td>3.8491</td>
<td>4.0503</td>
<td>9.212</td>
<td>27.418</td>
<td>0.15356</td>
<td>0.097291</td>
<td>0.109929</td>
</tr>
<tr>
<td>$10^3$</td>
<td>3.8365</td>
<td>4.0366</td>
<td>9.140</td>
<td>27.380</td>
<td>0.15356</td>
<td>0.095194</td>
<td>0.096482</td>
</tr>
<tr>
<td>$10^4$</td>
<td>3.8268</td>
<td>4.0379</td>
<td>9.126</td>
<td>27.310</td>
<td>0.15356</td>
<td>0.096250</td>
<td>0.096332</td>
</tr>
<tr>
<td>$10^5$</td>
<td>3.8262</td>
<td>4.0377</td>
<td>9.122</td>
<td>27.298</td>
<td>0.15356</td>
<td>0.096270</td>
<td>0.096283</td>
</tr>
<tr>
<td>$10^6$</td>
<td>3.8262</td>
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<td>0.096271</td>
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<tr>
<td>$10^7$</td>
<td>3.8262</td>
<td>4.0377</td>
<td>9.122</td>
<td>27.298</td>
<td>0.15356</td>
<td>0.096270</td>
<td>0.096270</td>
</tr>
</tbody>
</table>

Table 1. Effect of $p$ on the solution for example 1

A solution which satisfies all the target goals does not exist for this particular example. A design which satisfies any two limit conditions greatly violates the third. The optimum solution given in Table 1 is the best compromise in which all the critical goals are exceeded by similar proportions.

Example 2: Hyperbolic paraboloid

The hyperbolic paraboloid cable net, shown in Figure 4 and used by Krishna [8] for analysis, has been used here as a design example for which the optimum distribution of prestressing forces was determined. The same model has been previously used as a design example by Cinquini and Contro [9] and Simoes and Templeman [10]. However, the extent to which comparison can be made is limited due to differences in the specification of the design problems by the respective authors. The extensional stiffnesses (EA) of the sagging and hogging cables are 293.6 MN and 197.5 MN respectively. Three loading cases were designed for and are detailed in Refs. [2,8,9,10]. Load case I is a vertical loading of 100kN at every joint in the net. Load cases II and III are non-symmetric but turn out to be non-critical.

Symmetry of the cable layout implies that this net can have eight different end cable element shortnesses $\delta L_i (i = 1, ..., 8)$ as design variables as shown in Figure 5a. Imposing a symmetrical shortness distribution in
Figure 4. Hyperbolic paraboloid
this manner has the advantage of a reduced number of design variables and consequently less computational effort. However, for problems in which geometrical symmetry of the structure is not accompanied by symmetrical layout of applied loading, imposing symmetrical shortness distributions may not result in an optimum design. Since, for the present problem, load cases II and III are not symmetric, the most logical number of design variables to have is eleven, as shown in Figure 5b. Design calculations have been carried out using both eight and eleven shortness variables and the results from both calculations were found to be similar. This is due to Load case I being the dominant load in the sense that it produced all the active goals. Results of calculations with eight variables only have been quoted for the purpose of the following discussion.

To appraise the practical benefit arising from the proposed method, it is appropriate to evaluate the final optimized design with respect to an initial design. To this end an initial design in which a uniform pre-stress distribution of about 800 kN in all the outer cable elements, as given in Krishna [8], is considered. The maximum and minimum cable element forces and maximum nodal deflection resulting from this design are tabulated in Table 2, column 1. The values in bracket are the equivalent end cable prestress forces corresponding to the given shortnesses in cm.

It would be desirable to improve this design in order to reduce the maximum final forces in the cable elements and the maximum nodal displacement, but ensuring at the same time that the forces do not fall below a specified bound as a safety measure against slackening. In an attempt to achieve this, three design problems, 1, 2, 3, corresponding to different combination of permissible force and displacement target limits were examined.
Problem 1
In this problem, the extreme force and displacement values, $T_{f_{\text{max}}} = 1740kN$, $T_{f_{\text{min}}} = 318kN$ and $\delta_{r_{\text{max}}} = 0.85m$, resulting from the non-linear analysis of the net structure when subjected to uniform prestressing forces of $800kN$, were used as target constraints. Bounds on the vertical displacements of joints 7, 12, 13, 14, and 19 were considered to control the global deformability of the structure. The shortness vector ($\delta L_i$) corresponding to the uniform prestressing force of $800kN$ was considered to be the most appropriate starting point for the design iteration.

The final design shown in Table 2, column 2, was obtained after two iterations of the design problem, with $p = 10^3$. Further iterations with updated starting points produced no further significant improvements. It can be seen that the optimized shortnesses give a solution in which the final maximum force and nodal displacement are smaller and the minimum force is larger than those of the uniform prestress design. Changes in these values from the specified targets are all in the same proportion, as indicated by the calculated goal values shown in brackets.

<table>
<thead>
<tr>
<th>Uniform prest.</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start point</td>
<td>$\delta L_i$</td>
<td>$\delta L_i$</td>
<td>$\delta L_i$</td>
</tr>
<tr>
<td>$\delta L_1(cm)$</td>
<td>3.70 (800)</td>
<td>3.916 (842)</td>
<td>3.480 (748)</td>
</tr>
<tr>
<td>$\delta L_2$</td>
<td>7.45 (800)</td>
<td>6.469 (709)</td>
<td>0.663 (723)</td>
</tr>
<tr>
<td>$\delta L_3$</td>
<td>11.14 (800)</td>
<td>10.870 (769)</td>
<td>10.010 (751)</td>
</tr>
<tr>
<td>$\delta L_4$</td>
<td>14.60 (800)</td>
<td>12.60 (829)</td>
<td>13.760 (789)</td>
</tr>
<tr>
<td>$\delta L_5$</td>
<td>2.40 (800)</td>
<td>0.376 (434)</td>
<td>2.770 (896)</td>
</tr>
<tr>
<td>$\delta L_6$</td>
<td>4.93 (800)</td>
<td>4.044 (705)</td>
<td>6.070 (924)</td>
</tr>
<tr>
<td>$\delta L_7$</td>
<td>7.43 (800)</td>
<td>7.510 (810)</td>
<td>7.440 (769)</td>
</tr>
<tr>
<td>$\delta L_8$</td>
<td>9.76 (800)</td>
<td>9.329 (769)</td>
<td>8.640 (701)</td>
</tr>
<tr>
<td>$T_{f_{\text{max}}}(kN)$</td>
<td>1740</td>
<td>1705 (-0.019)</td>
<td>1665 (0.11)</td>
</tr>
<tr>
<td>$T_{f_{\text{min}}}(kN)$</td>
<td>318</td>
<td>324 (-0.0185)</td>
<td>305 (0.10)</td>
</tr>
<tr>
<td>$\delta_{y_{\text{max}}}(kN)$</td>
<td>0.85</td>
<td>0.834 (-0.0191)</td>
<td>0.88 (0.10)</td>
</tr>
</tbody>
</table>

$F_p$ | 0.015869 | 0.02735

Table 2. Hyperbolic net design results

Problem 2
For this problem, target values for the maximum and minimum forces were 1500$kN$ and 335$kN$ respectively, and for the maximum nodal displacement was 0.8$m$. These targets are more stringent than those before, i.e. cable elements may not be as highly or lightly loaded as in problem 1 and the joints may not displace to the same degree.
Table 2, column 3, gives the final design results obtained after two iterations of the design problem initiated from starting point $\delta L_i$ (see Table 2, column 1) again with $p = 10^5$. It can be seen from these results that the target bounds have not been achieved and have all been proportionately violated as indicated by the goal values. However, the overall objective, of reducing the maximum force and displacement and increasing the minimum force, with respect to the uniform design, has been partly met by the reduced final maximum force. However, this is accompanied by an increase in nodal displacement and a reduction in minimum cable force.

It is apparent from the above results that excessive reduction in the cable forces lead to larger deflections. Therefore, to achieve a reduced degree of displacement, it may be necessary to impose a more liberal maximum force constraint. To this end the following problem was solved.

**Problem 3**

This problem had the least stringent maximum force limit among the problems studied, $T_{\text{max}}$ was 2000$k N$. $T_{\text{min}}$ and $\delta_{\text{max}}$ were as for problem 2. In addition to solving this problem using the usual starting point $\delta L_i$, the problem was also solved using a second initial point $\delta L_i$. The shortness vector values of $\delta L_i$ are shown in Table 3. Both calculation results are tabulated in Table 2 for comparison purposes. As anticipated, two different design solutions resulted corresponding to the two different starting sets of shortnesses, as can be seen from Table 2. This reinforces the subjective conclusion that the final design is dependent upon the position of the initial trial point.

<table>
<thead>
<tr>
<th>Starting point</th>
<th>$\delta L_1$</th>
<th>$\delta L_2$</th>
<th>$\delta L_3$</th>
<th>$\delta L_4$</th>
<th>$\delta L_5$</th>
<th>$\delta L_6$</th>
<th>$\delta L_7$</th>
<th>$\delta L_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta L_i$</td>
<td>3.70</td>
<td>7.45</td>
<td>11.14</td>
<td>14.60</td>
<td>2.40</td>
<td>4.93</td>
<td>7.43</td>
<td>9.76</td>
</tr>
<tr>
<td>$\delta L_i$</td>
<td>1.00</td>
<td>5.00</td>
<td>10.00</td>
<td>15.60</td>
<td>5.00</td>
<td>3.00</td>
<td>7.00</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Table 3. Starting shortnesses for hyperbolic net optimization

In the context of the original aim of attempting to improve the results with respect to the uniform prestress design, it can be seen that this objective has been met in reducing the displacement and increasing the minimum force. This, however, results in an increased maximum cable force. Viewing the results in the context of achieving the targets set for this particular problem, it is clearly seen that both solutions are within the specified bounds.

There are several points of interest to be noted from these solutions. $\delta L_i$ gives a better solution than $\delta L_i$ as indicated by the lower $F_{\rho}$ value. Moreover, a relatively lower degree of prestress in the design with $\delta L_i$ achieves a smaller nodal displacement. This is contrary to what might intuitively be expected to occur and gives some insight into the complex and highly non-linear nature of cable net structures. The last point worth
noting is that the extreme force and displacement values fall within the set bounds to disproportionate extents, the maximum force and joint displacement constraints being the active goals, as indicated by the calculated goal values in Table 2. The solutions given in Table 2 for problems 1, 2, and 3 all correspond to different target values. They can all be interpreted, therefore, as compromise (or Pareto optimal) solutions of the design problem.

DISCUSSION AND CONCLUSION

The design examples studied in this paper and others presented in Ref.[2] give rise to several discussion points about the present design method and the behaviour of cable net structures. It has been shown that the optimization of cable net structures is a particularly difficult problem. A completely automatic optimum design program is currently not an achievable goal. The use of optimization in a user-interactive mode has been shown to result in significant prestress reductions.

The non-linear analysis of pretensioned cable net structures involves considerable computation because the shape-finding problem must be done iteratively. At least two shape-finding problems must be solved in each analysis: first, to find the shape of the zero configuration under pretensioning forces only; second, to find the shape for the final configuration for each applied load case. Because the non-linear analysis can only be done numerically, it is not possible to get closed form algebraic expressions for any of the functions needed in the optimization model. Consequently the optimization must be based upon models which use numerical function values and do not employ first derivative values.

The minimax formulation adopted in this work was found to be satisfactory in that it allowed the simultaneous optimization and control of the different engineering goals. The entropy-based approach to solving the minimax optimization formulations proved to be very successful in transforming the problem to a scalar optimization problem. The solution of the scalar problem was found to be insensitive to changes in values of the parameter $p$. A sufficiently high $p$ value of about $10^6$ was quite adequate to ensure a smooth convergence to an optimum solution.

The examples solved in this paper provided considerable insight into the behaviour of prestressed cable net structures. Joint displacements and individual cable element forces were sometimes very sensitive to comparatively small changes in the initial pretensioning forces. It was difficult to predict which cable element might become slack or which joint would have the largest displacement. Very different pretensioning operations produce very similar prestress force distribution. This is disturbing from a design point of view. It suggests that many local optima may exist in the design optimization problem. A good design solution depends on a good choice of an initial starting point. This presents a major disadvantage in that time must be devoted to choosing the initial shortness set. However, viewed in the context of an overall strategy which aims at taking an initial design for a cable net structure and making successive improvements to that design until the rate of improvement becomes...
too small to warrant further computational effort, the present algorithm was very successful.

REFERENCES


