An integrated model for crankshaft optimal design

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Abstract

A computational capability is developed for the optimal design of a crankshaft to satisfy deflection, fatigue, vibration, and balancing requirements using 3-D modelling and finite element analysis. The model shape is optimised based on the results of the mathematical programming used to find the minimum weight of the crankshaft. The optimal results are compared for geometric programming and the GRG program. The finite element analysis predicts deflection, principal stresses, shear stresses and fatigue life. All of these values are below the critical value.

Keywords: optimal design, crankshaft, geometric programming, generalised reduced gradient, solid modelling, finite element method, sum of squares of error.

1 Introduction

Today’s automotive industries are faced with a number of issues, which require them to be responsive in order to be competitive. To be competitive, one has to produce components with low cost and high quality. The advent of high performance computers, CAD tools and Optimisation techniques has helped realize the demand of global market. With the help of Optimisation techniques and numerical methods, one can design a component, create a solid model using CAD tools, simulate the operating conditions and find out if the component meets the expectations and feasibility before starting the actual production, thereby saving time and resources. The general considerations [1] in designing a crankshaft are; type of loads and stresses caused by it, selection of material, motion of parts or kinematics of the crankshaft, form and size of parts, convenient and economical features like minimization of wear, and use of standard parts. Failure of the Crankshaft will result in the failure of the engine. A
A typical crankshaft is shown in Fig.2. There are two main crankshaft types: 1) Crankshafts with center crank, 2) Crankshafts with side or overhung crank. Side or overhung crankshafts are particularly suited for transmitting huge torque and with engines having only one cylinder, whereas crankshafts with center crank can be used to transmit high powers in engines with more than one cylinder. The other advantage of the side crank is that it requires only two bearings thereby reducing the chances of misalignment, which is the major cause in most crankshaft failures. For the sake of simplicity and also for safety, the shaft is considered supported at the centers of the bearings, and all the forces and reactions are assumed to be acting at these points. Qualitatively speaking, the loads on a crankshaft result in stress due to bending, torsion and shear throughout its entire length. The complex geometry involved would make accurate stress computations impossible even if the loads were accurately known among the factors that determine the stress on crankshaft are speed and acceleration of the piston. It is therefore proper to examine the various aspects of the relationships between crankshaft rpm, piston speed and acceleration, connecting rod length and crankshaft radius [1]. Crankshafts are made from steel forging, or are cast in steel, when high specific output is desired, forged steel is the preferred material. Grey iron is used only for small, low-cost engines. Nodular cast iron is used to manufacture wide range of automotive components because it is strong, ductile, tough and extremely shock resistant.

Figure 1: Center crank crankshaft.

2 Optimal design technique

In optimising the weight of the crankshaft, geometric Programming & generalized reduced gradient techniques are used and results compared. In order to obtain a more explicit method of design, the overall objective must be more clearly defined and all undesirable effects must be avoided. Good problem formulation is the key to the success of an optimisation study.

2.1 Geometric programming

An important approach to deal with constrained geometric programs with high degree of difficulty is to use Kochenberger’s principle, which suggests addition of slack variables to all inequality constraints, and executing this equivalence through a penalty function methodology. Kochenberger’s principle is explained below.
Consider a posynomial geometric program of the following form:

\[
\begin{align*}
\text{minimize } & \quad y_0(x) = \sum_{t=1}^{T_0} c_{0t} \prod_{n=1}^{N} x_n^{a_{0tn}} \\
\text{subjected to } & \quad y_m(x) = \sum_{t=1}^{T_m} \prod_{n=1}^{N} x_n^{a_{mnt}} \leq 1, \quad m = 1,2,\ldots,M \\
\end{align*}
\]

The approach is to construct the following augmented program by adding a slack variable to each constraint, a weighted reciprocal of each slack to the objective function, and an additional weighting constant \( \delta \) to each constraint. The reformulated mathematical model is

\[
\begin{align*}
\text{minimize } & \quad y_0(x) = \sum_{t=1}^{T_0} c_{0t} \prod_{n=1}^{N} x_n^{a_{0tn}} + \sum_{m=1}^{M} Bx_{k+N}^{-1} \\
\text{subject to } & \quad y_m(x) = \delta \left[ \sum_{t=1}^{T_m} \prod_{n=1}^{N} x_n^{a_{mnt}} \leq 1 \right] + Bx_{m+N} = 1, \quad m = 1,\ldots,M, \quad B \geq 0, \quad 0 < \delta \leq 1
\end{align*}
\]

The dual of the original problem is given by

\[
d(w) = \prod_{m=0}^{M} \prod_{t=1}^{T_m} \left[ \frac{c_{mt}w_{m0}}{w_{mt}} \right]^{w_{mt}}
\]

where

\[
w_{m0} = \sum_{t=1}^{T_m} w_{mt}, \quad m = 1,2,\ldots,M \\
w_{00} = 1, \quad \sum_{t=0}^{T_0} w_{0t} = 1, \quad \sum_{m=0}^{M} \sum_{t=1}^{T_m} a_{mnt}w_{mt} = 0, \quad \sum_{m=0}^{M} w_{mt} \geq 0
\]

To form the dual, it is necessary to add additional variables to the dual program. The advantage of using this formulation is that if an optimum solution exists, the constraints will all be tight at the optimum. We found that acceptable experimental results could be obtained through the solution of the above formulation with \( B=0.001 \) and \( \delta=1 \).

### 2.2 Generalized reduced gradient method (GRG)

Generalized reduced gradient method [7] is used to solve non-linear problems. The principle of GRG is based on *Implicit Variable Elimination*, according to which, a problem involving non-linear equality constraints can be solved explicitly and used to eliminate variables. This methodology not only helps in reduction in the number of variables, but also reduces the number of constraints.

A constraint \( h_1(x) = 0 \) can be solved to yield, \( x_k = \phi(x,x,x,\ldots,x_{k-1},\ldots,x_N) \). The GRG technique is well known and this algorithm is not presented to save space.
3 Mathematical model for optimum design of crankshaft

The model for minimization of the weight of the crankshaft is developed as

\[
\text{Minimize } \left[ 5574.d_s^2 + 5574.d_p^2 + 374.88.d_s.t + 374.88.d_p.t + 6688.d_p^2.t + 5574.d_w^2.t - 3834.d_p^2.t \right]
\]

subjected to constraints:

\[
0.833.L_s + 0.833.L_p + 1.667.t \leq 1
\]

\[
\begin{bmatrix} 0.014.L_s^3.d_s - 4 + 0.014.L_p^3.d_p - 4 + 0.014.d_s^3 - 4 + 0.042.L_s^2.L_p.d_s - 4 + 0.042.d_p^2.t.d_s - 4 + 0.042.L_p^2.t.d_s - 4 + 0.042.L_p^2.t.d_s - 4 + 0.084.L_s.L_p.t.d_s - 4 \\ 0.042.L_s^2.L_p.d_s - 4 + 0.042.L_s^2.d_s - 4 + 0.042.L_p^2.t.d_s - 4 + 0.042.L_p^2.t.d_s - 4 + 0.084.L_s.L_p.t.d_s - 4 \\ 6.379.10^{-6}.d_p^{-6}.(L_s + L_p + 2.t)^2 + 2.455.10^{-3}.d_p^{-6} \leq 1 \\ 1.842.10^{-8}(L_s + L_p + 2.t)^2 + 6.632.10^{-11}.d_p^{-6} \geq 1 \\ 2.893.10^{-7}(L_s + L_p + 2.t)^2 + 1.042.10^{-9}.d_p^{-6} \leq 1 \\ 1.783.10^{6}.d_s^{-4} \geq 1 \\ 0.067.d_s.d_w^{-2}.t^{-1} + 0.067.d_p.d_w^{-2}.t^{-1} + 1.2.d_p^{-2}.d_w^{-2}.t - 0.688.d_p^{-2}.d_w^{-2}.t^{-1} = 1
\end{bmatrix}
\]

The degree of difficulty (DOD) in solving the above model is given by the equation,

\[
DOD = T - (N + 1)
\]

where \( T \) is the number of terms in the mathematical model and \( N \) is the number of variables. For the above model, the degree of difficulty is 74.

Minimize

\[
y_p(x) = 5574.x_s^2 + 5574.x_p^2 + 374.88.x_s.x_t + 374.88.x_p.x_t + 6688.x_s^2.x_t + 5574.x_s^2.x_t - 3834.x_p^2.x_t
\]

subject to

\[
0.833.x_2 + 0.833.x_4 + 1.667.x_5 \leq 1
\]

\[
\begin{bmatrix} 0.014.x_2^3.x_1^{-4} + 0.014.x_2^3.x_1^{-4} + 0.014.x_3^3.x_1^{-4} + 0.042.x_2^2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} \\
0.042.x_2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} + 0.042.x_2.x_s.x_1^{-4} + 0.084.x_2.x_s.x_1^{-4} \\
6.379.10^{-6}.x_3^{-6}.(x_2 + x_4 + 2.x_8)^2 + 2.455.10^{-3}.x_3^{-6} \geq 1 \\ 1.842.10^{-8}.(x_2 + x_4 + 2.x_5)^2 + 6.632.10^{-11}.x_3^{-6} \geq 1 \\ 2.893.10^{-7}.(x_2 + x_4 + 2.x_5)^2 + 1.042.10^{-9}.x_3^{-6} \leq 1 \end{bmatrix}
\]
Using the principle enunciated above, the solution to the above program is given by

\[ x_1^* = 67.7; \quad x_2^* = 33.33; \quad x_3^* = 54.6; \quad x_4^* = 26.4; \quad x_5^* = 21.3; \quad x_6^* = 98.7 \]

Table 1 shows dimensions obtained from GP and GRG techniques.

<table>
<thead>
<tr>
<th>Dimension (mm)</th>
<th>Original Design</th>
<th>Design using GP</th>
<th>Design using GRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal Diameter</td>
<td>69.901</td>
<td>67.7</td>
<td>67.635</td>
</tr>
<tr>
<td>Pin Diameter</td>
<td>60.9</td>
<td>54.6</td>
<td>54.4</td>
</tr>
<tr>
<td>Journal Length</td>
<td>31.45</td>
<td>33.33</td>
<td>33.15</td>
</tr>
<tr>
<td>Pin Length</td>
<td>30.45</td>
<td>30.07</td>
<td>30.6</td>
</tr>
<tr>
<td>Cheek Thickness</td>
<td>26.213</td>
<td>19.2</td>
<td>19.125</td>
</tr>
<tr>
<td>Dia of Counter weight</td>
<td>107</td>
<td>98.7</td>
<td>98.592</td>
</tr>
</tbody>
</table>

The sum of squares of errors (SSE) was calculated for both 157.499 and 171.209 for GP and GRG respectively. From SSE point of view, Results of GP are better compared to the results obtained from GRG. However, the dimensions obtained from both the techniques will be input into the constraint equations to see which results satisfy the constraints better. Table 2 gives the comparison of the various constraints.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Desired Value</th>
<th>Value using GP</th>
<th>Value using GRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Spacing (m)</td>
<td>0.102</td>
<td>0.102</td>
<td>0.102</td>
</tr>
<tr>
<td>Deflection (µm)</td>
<td>&lt;5</td>
<td>0.6157</td>
<td>0.613</td>
</tr>
<tr>
<td>Vibration (rad/sec)</td>
<td>&gt;10677</td>
<td>101800</td>
<td>104719</td>
</tr>
<tr>
<td>Fatigue Safety Factor</td>
<td>&gt;2</td>
<td>2.004</td>
<td>2</td>
</tr>
<tr>
<td>Balancing Error (%)</td>
<td>0</td>
<td>0.1</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

From table 2 the following observation is made:
Engine Spacing constraint is satisfied and are active in both cases
Deflection constraint is satisfied but inactive in both cases
Vibration constraint is satisfied but inactive in both cases
Fatigue Safety constraint is satisfied but inactive in both cases
Balancing constraint is not satisfied in both cases. However, the balancing error is least in the case of GRG.
4 Forging considerations for crankshaft

A list of parameters and the additional tolerances for forging are described below. In designing the crankshaft, following parameters should be taken into consideration [16] Parting line ii) Draft iii) Ribs, bosses, webs and recesses IV) Machining allowance.

![Forging Terminology Diagram]

Figure 2: Forging terminology as applied to crankshaft.

The general consideration for any forging is that the ratio of the rib height (H) to the rib thickness (T) should not be greater than 6. The rib thickness should be less than or equal to the web thickness in order to avoid the process defects.

*Parting line:* Parting line is preferred to lie in one plane perpendicular to the axis of the die motion. The angle of the surface at the parting line from the primary parting plane should not exceed 75°.

*DRAFT:* It is the necessary taper on the side of the forging to allow removal from the dies. Minimum draft angle for high tolerance forgings can be 0 deg. ± 0.5 deg. Standard draft angles should be at least 1 deg. ± 0.5 deg.

*Rib, Bosses, Webs, and Recesses:* It is easier to manage metal flow when the ribs and bosses are not too high and narrow, and the web is relatively thick and uniform.

*Machining Allowance:* Extra metal should be provided to at the critical machined surfaces away from the grain flow pattern that occurs in the flash region near the parting line. Generally, the machine allowance should be 0.06 inch for each machined surface.

*Radii and Fillet:* Larger radii and fillets are desired at corners. Radii should blend tangent with draft surfaces. Forging radii are designed with die design and requirements in consideration. Tolerances for minimum radii are given in table 3.

*Wall:* Wall thickness of 0.20 over a span of 4.5 inches is considered minimum.

*Surface Quality:* Typical surface roughness of forged components is specified at 64 RMS or rougher.
On application of the above considerations to the dimensional values obtained, new values are obtained. These values are given in table 4.

### Table 3: Tolerances for fillet radii.

<table>
<thead>
<tr>
<th>Rib or Wall Height</th>
<th>Minimum Fillet Radii</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 1.5 inch</td>
<td>0.13 inch</td>
</tr>
<tr>
<td>1.50 – 3.00 inch</td>
<td>0.19 inch</td>
</tr>
<tr>
<td>≥ 3.0 inch</td>
<td>0.25 inch</td>
</tr>
</tbody>
</table>

### Table 4: Dimensions with forging considerations.

<table>
<thead>
<tr>
<th>Dimension (mm)</th>
<th>Dimensions After Optimal Design.</th>
<th>Dimensions with Forging Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal</td>
<td>67.635</td>
<td>67.955</td>
</tr>
<tr>
<td>Pin</td>
<td>54.4</td>
<td>54.72</td>
</tr>
<tr>
<td>Total Length</td>
<td>647.1</td>
<td>651.09</td>
</tr>
</tbody>
</table>

## 5 Solid modelling of crankshaft

The 3-D modelling [17] of crankshaft has been presented in detail for completeness. The solid model of the crank-throw based on the optimal dimensions shown above in table 4 is created. Solid model is created using I-DEAS CAD software. We have used “variational geometry” approach for solid modelling. This approach starts with sketching sections to define the shape of the part and modify the dimensions later. The master model is starting is starting point and a shared information source containing the geometric definition of the part in the concurrent design. The solid model is used for many down stream applications such as mass property calculation, kinematic analysis, stress analysis & dynamic analysis. The solid model is ready and can be used to analyse the crankshaft using the simulation task with SDRC I-DEAS. The Crank-Throw is copied, rotated and joined to the original throw to form a 6-throw Crankshaft with firing order 1-6,2-5,3-4 separated at 120 degrees.

## 6 Finite element analysis of crankshaft

The purpose of performing finite element analysis is to check for possible failure modes in the crankshaft. Finite element method [18] is a numerical method that creates elements by subdividing the domain of the solid structure. These elements are assembled at nodes. Within the domain of each element we assume a simple general solution to the governing equations. Application of the FE method results in a finite set of algebraic equations for unknowns.
The Strain displacement equations are given below the stress-strain relationship for an isotropic homogeneous material are given below

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{x,y} \\
\tau_{x,z} \\
\tau_{y,z}
\end{bmatrix} \begin{bmatrix}
1-\nu & \nu & 0 & 0 & 0 \\
\nu & 1-\nu & \nu & 0 & 0 \\
\nu & \nu & 1-\nu & 0 & 0 \\
0 & 0 & 0 & 1-2\nu & 0 \\
0 & 0 & 0 & 0 & 1-2\nu \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{x,y} \\
\gamma_{x,z} \\
\gamma_{y,z}
\end{bmatrix}
\]

In the above equations, \(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{x,y}, \gamma_{x,z}, \gamma_{y,z}, \nu, w, \sigma_x, \sigma_z, \tau_{x,y}, \tau_{x,z}, \) and \(\tau_{y,z}\) have usual connotation of design engineering.

The geometry is divided into 271363 tetrahedral elements and total 581505 nodes are created using solid meshing process. In this case, the end faces of the crank-throw are set to fixed conditions in all the axes, except in z-axis, where it is free to rotate. Initially a static force of 23000 N is applied on the face of the pin. It takes 32.6 minutes to solve 101823 solutions. Once the solution to the static stress is found, we can switch the task from Model solution to Durability & Dynamics.

7 Result

We have selected 1045/225 Nodular Cast Iron. Table 5 shows the comparison of dimensions prior to and after Optimal Design.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-Optimal Design</th>
<th>Optimal Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal Diameter (mm)</td>
<td>69.901</td>
<td>67.7</td>
</tr>
<tr>
<td>Journal Length (mm)</td>
<td>31.45</td>
<td>33.33</td>
</tr>
<tr>
<td>Pin Diameter (mm)</td>
<td>60.9</td>
<td>54.6</td>
</tr>
<tr>
<td>Pin Length (mm)</td>
<td>30.45</td>
<td>26.4</td>
</tr>
<tr>
<td>Web Thickness (mm)</td>
<td>26.213</td>
<td>21.3</td>
</tr>
<tr>
<td>Counterweight Diameter (mm)</td>
<td>107</td>
<td>98.7</td>
</tr>
<tr>
<td>Weight of Each Throw (kg)</td>
<td>5.981</td>
<td>4.991</td>
</tr>
</tbody>
</table>

A weight reduction of about 16.5% is achieved from the Optimal Design. Due to the dynamic force acting on the instant center of the crankshaft a deflection of
0-6 \( \mu \)m, principal stress distribution of -5.19E06 to 1.69E07, Von Mises stress distribution of 1.15E03 to 4.01E07, Shear stress distribution of 6.02E02 to 2.31E07 and Fatigue life between 10E30 to 10E36 cycles. Were estimated. All the stresses lie well below the yield stress of 4.74E08. The desired deflection was of the order of 1 \( \mu \)m. The maximum deflection obtained by FEA is 6\( \mu \)m. The life cycle of 10E36 corresponds to Infinite life.

8 Conclusion

The paper presents the theoretical investigation in optimal design of crankshaft. The crankshaft is one of the most important components of automotive system and its design and manufacture is very challenging. In particular, we conclude the following:
1. A basic mathematical model for optimal design of crankshaft was developed along with several real world constraints.
2. Two optimisation techniques were compared. The Crank throw would weigh 3.605 Kg based on the results of the GP technique and 3.4239 based on the results of the GRG technique as compared to the original weight of 4.723 Kg.
3. All constraints were tested for both methods and GRG technique gave better results.
4. All the optimal design parameters were modified based on real world forging considerations.

All the stresses were well below the safety limit and the life of the crankshaft under the dynamic operating conditions was estimated to be infinite.

References