Aeroelastic analysis and sensitivity of the fluter speed of long span suspension bridges with distributed computing

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Abstract

One of the main difficulties in the design of long span suspension bridges is the stability under wind loads and examples of failures due to flutter have been experienced in the past. Current methods of aeroelastic studies, which rely not only on experimental data but also on computer analysis, open the door for adding sensitivity analysis techniques with the objective of helping engineers in the design of such challenging constructions. In this paper, a methodology aimed at that objective is implemented in a distributed computing environment and an example, corresponding to the Great Belt suspension bridge with a span of 1624 m, is included showing the capabilities of the approach developed.

1 Introduction

Suspension bridges have been always a challenge for civil engineering designers. When the length of span of such structures increases the main difficulties for the design are not those coming from service loads as vehicles or trains, but seismic or wind actions. In the case of wind loads it is very well known the disastrous effect produced by a windstorm in the Tacoma Narrows Bridge that collapsed in 1940.

Nevertheless the advances in the knowledge of how wind pressures interact with cable supported bridges have led to the possibility of building longer and longer suspension bridges. Today the world record corresponds to the bridge over the Akashi strait in Japan with a span length of 1991 m and a bridge over the Messina strait in Italy with 3300 m of span length is currently under project.

In both cases wind behaviour of the bridge is one of the main concern for the final design and this circumstance will be identical in every long span bridge to

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be projected in the future. Therefore efforts devoted to identify properly wind structure interaction in such structures are very necessary.

Figure 1: Collapse of the Tacoma Narrows Bridge.

Figure 2: Akashi strait bridge.

Figure 3: Proposed Messina strait bridge.

2 Hybrid methods of aeroelastic studies

In the past the mainly, and almost unique, methodology to find out the behaviour of suspension bridges under wind flow was to test a reduced model of the whole
bridge in boundary layer wind tunnels, which were large and expensive facilities where by properly modelling both structure and wind flow, the safety of the model against instabilities as flutter could be investigated.

![Figure 4: Wind tunnel testing of Great Belt Bridge.](image)

The increasing length of bridges under consideration nowadays and in a near future and the convenience of maintaining an adequate scale of the model, that should not exceed 1/200, leads to the necessity of create larger wind tunnels to an extent that could become prohibitive in budget terms.

Because of that and alternative based on a different approach has been implemented during the last decade, which does not require expensive testing facilities.

The method develops in two phases, the first one, which is experimental, is carried out in small wind tunnels, where reduced model of a segment of the deck is tested under wind flow. The aim of this step is to identify the relationship between wind forces on the deck and displacement and velocities of it, according with the Scalan [1] formulation.

![Figure 5: Experimental step of hybrid method.](image)

The relationship of wind forces on the deck $L$: lift, $D$: drag and $M$: moment, and deck displacements $v$, $w$, $\phi_x$ and velocities $\dot{v}$, $\dot{w}$, $\dot{\phi}_x$ is
As indicated, the objective of the experiment is to find out the values of coefficients $A_i, H_i, P_i \ (i = 1, \ldots, 6)$.

Expression (1) can be presented in matrix notation as

$$P = K_a u + C_a \dot{u} \quad (2)$$

Second step in this approach is to produce a finite element model of the bridge and carry out a dynamic analysis using the wind loads defined in expression (2). The dynamic equilibrium can be written as

$$M \ddot{u} + C \dot{u} + K u = K_a u + C_a \dot{u} \quad (3)$$

by using modal descomposition equation (3) is finally transformed into an eigenvalue problem.

$$(A - \mu I) w_\mu = 0 \quad (4)$$

Equation (4) provides a set of pairs of conjugate complex eigenvalues $\mu_i \ (i = 1, n)$, being $n$ the number of modes included in the analysis.

$$\mu_i = \alpha_i + i \beta_i , \quad \overline{\mu_i} = \alpha_i - i \beta_i \quad (5)$$
Real part $\alpha_j$ is related to the modal damping. Therefore positive damping values are required for the structure to be stable under wind flow. If $\alpha_j$ becomes zero for a given mode $\alpha_j$ under a specific wind speed, then flutter instability can arise and such speed $U_j$ is the maximum speed the bridge can withstand.

Therefore obtaining flutter speed $U_j$ is made by solving the eigenvalue problem indicated in (4) according with the flowchart of figure 7 and then, proceed iteratively increasing values of wind speed $U$ until obtaining zero value for a given real part $\alpha_j$. Such procedure is shown in figure 8.

Figure 7: Flow diagram of eigenvalue problem.

Figure 8: Flow diagram of flutter speed.

3 Sensitivity analysis of flutter speed

Given the significance of flutter speed in the design of long span bridges it seems very important to develop methodologies helping to improve the capacity of the designer in the decision process. Astonishingly no formal procedure of sensitivity analysis optimization method has been used in the past in the design.
of world record suspension or cable-stayed bridges. Thus a research project aimed to introduce such methodologies in the design of cable-supported bridges that will be built in the future has been carried out by the authors.

The first step was to work out the formulation of sensitivity analysis of flutter speed with respect to any design variable. Remembering that at flutter speed \( \alpha_j \) vanishes equation (3) can be written as

\[
(A - i\beta_j I)w_\mu = 0
\]  

(6)

The parameter \( \beta_j \) is a function of \( U_f \) expressed by

\[
\beta_j = \frac{K_f U_f}{\beta}
\]

(7)

substituting in (6)

\[
\left( A - i\frac{K_f U_f}{\beta} I \right)w_\mu = 0
\]

(8)

Derivation of (8) with regards to any design variable \( x \) was carried out in reference [3] and after a quite lengthy procedure finally the sensitivity of \( U_f \) with regards to a design variable \( x \) turns out

\[
\frac{dU_f}{dx} = \frac{-\text{Im}(g_K h_{4x})}{\text{Im}(g_K g_U)}
\]

(9)

being

\[
g_K = h_{4K} + \frac{iU_f}{B} v_\mu^t I w_\mu \quad g_U = h_{4U} + \frac{iK_f}{B} v_\mu^t I w_\mu \quad h_{4x} = v_\mu^t \frac{\partial A}{\partial x} w_\mu
\]

(10)

where \( v_\mu \) and \( w_\mu \) are eigenvectors accomplishing the following equations

\[
v_\mu (A - i\beta_j I) = 0 \quad \text{or} \quad (A - i\beta_j I)w_\mu = 0
\]

(11)

The set of design values used in the analysis must be adapted to the bridge engineers’ requirements. Two different conceptions can be selected. One is

![Bridge deck representation](image)

Figure 9: Bridge deck representation.
composed by the set of inertia moduli and area of the deck and the other by a group of equivalent thicknesses of the deck plates providing identical mechanical parameters, in other words, set of design variables could be written as

\[ X = [I_x, I_y, I_z, A] \quad \text{or} \quad X = [e_1, e_2, e_3] \quad (12) \]

4 Distributed computing.

The computer phases of the aeroelastic studies and sensitivity analysis described in the previous paragraphs require, in summary, the following evaluations.

a) Computer evaluation of natural eigenresponses.

b) Computer evaluation of aeroelastic eigenresponses.

c) Computer evaluation of sensitivity analysis.

While the calculation of natural frequencies of a structural model only can be made one by one, the evaluation of b) and c) responses can be carried out in parallel.

Therefore there is a big chance to take advantage of capabilities of distributed computing. This pioneer approach has been implemented by the authors using a cluster of personal computers with a front end CPU that manages the task load amongst each individual computer. Figure 10 shows the scheme of computing facilities.

![Distributed computing environment](image)

Figure 10: Distributed computing environment.

5 Application example.

The described methodology has been applied to an important bridge, the Great Belt suspension bridge, that with a central span of 1610 m is the world second longest span length. Figure 11 shows a view of the bridge and the finite element model used for the analysis.

In Table 1 the geometrical and mechanical parameters of the bridge are presented.
Up to eighteen modes were included in the aeroelastic analysis and the natural frequencies for such modes appear in Table 2 along with the type of displacement associated.

The flutter speed obtained in the aeroelastic analysis was $U_f = 62.02 \text{ m/seg}$, and the sensitivity analysis with regards to the inertia moduli appear in Table 3, that includes numerical results of the sensitivities considering the set of 18 modes and those values obtained considering only two modes in the analysis, namely the first vertical and torsional modes. Such comparison has been made because more conventional flutter analysis only considered these two natural
modes. The results of Table 3 clearly state that there are great differences between the values of the sensitivities when two and 18 modes are included in the analysis. Obviously as the addition of more modes makes more precise the model it can be concluded that for long span bridges, as the example posed, the designer cannot rely on aeroelastic analysis based in a short number of modes.

Table 2: Natural frequencies.

<table>
<thead>
<tr>
<th>Mode Nº</th>
<th>Frequency (Hz)</th>
<th>Period (sec)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.052</td>
<td>19.231</td>
<td>Lateral (symm.)</td>
</tr>
<tr>
<td>2</td>
<td>0.096</td>
<td>10.417</td>
<td>Vertical (symm.)</td>
</tr>
<tr>
<td>3</td>
<td>0.110</td>
<td>9.091</td>
<td>Vertical (anti symm.)</td>
</tr>
<tr>
<td>4</td>
<td>0.124</td>
<td>8.065</td>
<td>Lateral (anti symm.)</td>
</tr>
<tr>
<td>5</td>
<td>0.129</td>
<td>7.752</td>
<td>Vertical (symm.)</td>
</tr>
<tr>
<td>6</td>
<td>0.176</td>
<td>5.682</td>
<td>Vertical (anti symm.)</td>
</tr>
<tr>
<td>10</td>
<td>0.197</td>
<td>5.076</td>
<td>Vertical (anti symm.)</td>
</tr>
<tr>
<td>12</td>
<td>0.217</td>
<td>4.608</td>
<td>Vertical (symm.)</td>
</tr>
<tr>
<td>13</td>
<td>0.220</td>
<td>4.545</td>
<td>Lateral (symm.)</td>
</tr>
<tr>
<td>18</td>
<td>0.248</td>
<td>4.032</td>
<td>Vertical (symm.)</td>
</tr>
<tr>
<td>19</td>
<td>0.278</td>
<td>3.597</td>
<td>Vertical (anti symm.)</td>
</tr>
<tr>
<td>21</td>
<td>0.289</td>
<td>3.460</td>
<td>Vertical (anti symm.)</td>
</tr>
<tr>
<td>22</td>
<td>0.300</td>
<td>3.333</td>
<td>Torsional (symm.)</td>
</tr>
<tr>
<td>27</td>
<td>0.337</td>
<td>2.967</td>
<td>Vertical (symm.)</td>
</tr>
<tr>
<td>28</td>
<td>0.351</td>
<td>2.849</td>
<td>Lateral-Torsional (anti symm.)</td>
</tr>
<tr>
<td>29</td>
<td>0.387</td>
<td>2.584</td>
<td>Lateral (symm.)</td>
</tr>
<tr>
<td>30</td>
<td>0.392</td>
<td>2.551</td>
<td>Torsional (anti symm.)</td>
</tr>
<tr>
<td>37</td>
<td>0.447</td>
<td>2.237</td>
<td>Torsional (symm.)</td>
</tr>
</tbody>
</table>

Table 3: Sensitivities of $U_f$ with regards to inertia moduli.

<table>
<thead>
<tr>
<th>Number of modes</th>
<th>$dU_f / I_y$</th>
<th>$dU_f / I_z$</th>
<th>$dU_f / I_x$</th>
<th>$dU_f / dA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.2496</td>
<td>-0.0817</td>
<td>2.20</td>
<td>7.159</td>
</tr>
<tr>
<td>18</td>
<td>-0.2418</td>
<td>-0.1847</td>
<td>2.415</td>
<td>23.296</td>
</tr>
</tbody>
</table>

An important issue arose by the results of the sensitivities is that the sign of the sensitivity of $U_f$ with regards to $I_y$, that represent the stiffness with respect to vertical bending, is negative and that means that to increase flutter speed would require to diminish $I_y$. But such class of change weaken the deck bridge against gravitatory loads and therefore optimization of suspension bridges including static and aeroelastic constraints becomes a multiobjective
optimization problem with conflicting objective functions that is a work in progress in this research group.

Table 4 presents the sensibilities of flutter speed with respect to the equivalent plate thicknesses defined for the bridge deck. Again, the numerical results for the aeroelastic analysis considering only two modes and a larger number up to eighteen are very different and the last ones must be considered as correct.

<table>
<thead>
<tr>
<th>Number of modes</th>
<th>$dU_f / d_e$</th>
<th>$dU_f / d_{e2}$</th>
<th>$dU_f / d_{e3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>81,52</td>
<td>331,06</td>
<td>2624,59</td>
</tr>
<tr>
<td>18</td>
<td>333,79</td>
<td>609,91</td>
<td>2887,09</td>
</tr>
</tbody>
</table>

6 Conclusions

The following conclusions can be extracted from this research:

— Long span suspension bridges require the use of the most advanced design methodologies due to their social, economical and structural significance.

— Sensitivity analysis is a powerful technique to help bridge designer in the definition of this class of complicated structures.

— The aeroelastic studies and sensitivity analysis are very demanding computer time methods that can be benefited by the use of distributed computing.

— Design optimization of cable-supported bridges under static and aeroelastic constraints is a multiobjective optimization problem that deserves to be carefully studied.

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References

