The optimal shape of curvilinear fibers in FRC

P. Procházka¹ & N. Starikov²
¹Czech Association of Concrete Engineers, Prague, Czech Republic
²CTU Prague, Civil Engineering, Prague, Czech Republic

Abstract

In the problem of cracking of composite structures of several sorts, the pullout problem has frequently been solved. In some previous papers by the authors, curvilinear fibers reinforcing concrete mainly during its curing process have been studied. It was shown that the fibers of Dramix type proved to have a higher local bearing capacity than the straight fibers. The problem was solved in 3D on a unit cell assuming a periodicity of fiber placing. The linear behavior of concrete was taken into account and the pullout test was simulated by the finite element method. In this paper the optimal slope of fibers is sought. Using similar computations as those which were carried out in the above-mentioned papers, the optimum conditions have been added. They obey the typical Mohr-Coulomb law along the fiber-concrete interfacial zones and the compressive strength in the concrete cannot be exceeded at any point. The study is carried out on different unit cells, i.e. different positioning of fibers is considered. A different fiber ratio is also considered to show the mechanical behavior and to limit analysis of the composite aggregate.

1 Introduction

In many branches of structural engineering fiber reinforced concrete has been used with different types of fibers. Because this study was financially supported from a grant project, which is concentrated on geomechanical structures, principally on reinforcement of tunnel linings and foundations, the steel fibers are considered in this paper. They play very important role particularly during curing process of concrete, as they suppress local cracking and warping in the composite structure and avoid a possibility of corrosion of reinforcing steel rods. Very important fact follows from plenty of experimental studies: since the steel fibers are more stiff then concrete itself, pullout problem has to be solved when
requiring knowledge about the bearing capacity of the structure. In this study 3D finite elements are used to solution of the problem on unit cells together with substructuring of the domain describing the unit cell. The unit impulses are applied to nodes of the 20-degree of freedom elements to create influence matrices. This approach is effective from the point of view of iterative method, which has to be applied to the solution of contact problem. Because the optimization of slope of Dramix type fibers is required discrete problems are successively solved for different unit cells. Four typical unit cells are studied and discrete fiber slopes are solved, namely admissible pullout force is determined from the set 0, 10, 20, 30, 40, 45, 50, 60, 70, 80, and 90 degrees of angle between the axial coordinate and the slope. Interpolation is available, as the results provided from these computations admit approximately continuous curve describing the influence of the slope.

2 Pullout problem of steel FRC beam

The pullout problem has frequently been solved in a problem of cracking of composite structures of several sorts. Previously, several numerical studies were carried out using the FEM, [1], and the BEM, [2], and the results were compared with experimental results from available literature with a good agreement. In the sections from the available literature the topics have mainly been focused on experiments and the theory has based on different hypotheses, i.e., the results from the theoretical considerations were very approximate. A remarkable feature in [1] and [2] of the mathematical solution occurred: a cracking was initiated not only at the face of the fiber-matrix system, but also inside the trial body. As the previous problems were concentrated only on straight fibers and the nature of the material used was based on classical composites with epoxy matrices and high bearing capacity and durability, a natural question arises: what happens when the matrix is created from concrete and the aggregate behave realistic, i.e., the shape of the fibers is no more straight and the concrete matrix is taken into account. The answer to this question is the objective of this section. For more details see [3].

In this study of a lag model of the system concrete matrix - steel fibers of a special shape the geometry of the problem is considered as three-dimensional and finite isoparametric elements with 20 degrees of freedom are used together with Uzawa's algorithm, [4], for solving a contact problem fiber - matrix.

The debonding or “slipping” (jumps of responding points of the matrix and fibers boundaries in the tangential direction to the interface) is caused due to Mohr-Coulomb law. Since no fiber gripping force from matrix is expected, the general Mohr-Coulomb friction law is restricted to the exclusion of shear stresses exceeding the shear strength in the tangential direction and the tension exceeding the tension strength is also excluded along the fiber - matrix interface. In our study the grip in normal direction to the fibers is neglected.

2.1 Formulation of the contact problem

Consider a problem of two elastic bodies. The first body (fiber) occupies in undeformed state the domain $\Omega'$ with the boundary $\Gamma'$ (rectangular
parallelepiped) and the second body (concrete matrix) occupies the domain with the boundary $\Gamma''$ (curvilinear parallelepiped).

There is no external load considered, but the pullout force at the face of the rectangular parallelepiped. Generally, it is assumed that the tension exceeding the tensile strength in the normal direction to the fiber-matrix interface $\Gamma$ is not admitted, so that both bodies may disconnect (they mutually debond) in certain region of the common boundary $\Gamma'$, which is a part of $\Gamma'$ and $\Gamma''$. In order to hold the mathematical purity let us consider tensile strength at the contact to be zero. This fact may be overcome by simple introducing the tension strength but theoretical (not practical) difficulties may occur. The Mohr-Coulomb contact conditions are taken into account. More complex study concerning the contact conditions was carried out in [1]. A reader can find there that four zones can be distinguished on the straight rod to be pulled out of a surrounding medium. Along the first zone, additional debond can occur. The second zone is a firm bond, while in the third zone a slipping in the tangential direction is possible. The zone belonging to the face (the fourth zone) is a typical opening crack. Similar situation is observed in curvilinear reinforcement. Any of the four zones can disappear but the third, which is the zone in the undeformed state.

The load is considered in the following way. The pullout traction $p$ is applied at the face of the fiber in the axial direction, prescribed by a constant value. There is symmetry about the vertical plane containing the horizontal axis $x$ regarding both geometry and loading, so that only one half of the unit cell may be solved.

Displacements are described by the vector function $\mathbf{u} = \{u,v,w\}$ of the variable $\mathbf{x} = \{x,y,z\}$. Denote $\Omega \equiv \Omega' \cup \Omega''$. The restriction of any function to $\Omega'$ or $\Omega''$ is denoted, respectively, by one prime or by two primes, e.g., $u \cap \Omega' = u'$ and $u \cap \Omega'' = u''$.

On the boundary displacements and tractions are prescribed in such a manner that a periodicity is assumed at all sides of a "unit cell", The loading is given in both domains due to the pull-out traction $P$.

Denote the set of admissible displacements $\mathbf{u}$ on $\Omega$ satisfying the essential boundary conditions.

Consider Hooke’s law in the form:

$$
\varepsilon_{ij}(\mathbf{u}') = L_{ijkl}' \sigma_{kl}' \quad \varepsilon_{ij}(\mathbf{u}^\ast) = L_{ijkl}'' \sigma_{kl}''
$$

(1)

where $L'$ and $L''$ are the material stiffness matrices of the fiber and the concrete, respectively.

When assuming "small deformation" theory, it may be satisfactory to formulate the essential boundary condition on the contact boundary as follows (Signorini's conditions):

$$
[u]_n = u_n^* - u_n^0 < 0 \quad \text{a.e. on } \Gamma
$$

(2)

Suppose that we disconnect both bodies under consideration, but keep the stress and deformation state in them "frozen". Then the vector of contact
tractions $\mathbf{p} = \{p_x, p_y, p_z\}$ must be introduced and their equilibrium (action and reaction law) says that:

$$\mathbf{p} = \mathbf{p}^* + \mathbf{p}^\prime = 0$$  \hspace{1cm} (3)$$

Let us write the total energy $J$ of both bodies assuming them separately:

$$J(\mathbf{u}, \mathbf{p}) = \Pi(\mathbf{u}) - J(\mathbf{u}, \mathbf{p}),$$  \hspace{1cm} (4)$$

where

$$\Pi(\mathbf{u}) = \frac{1}{2} a(\mathbf{u}, \mathbf{u}) - \int_{\Gamma_0} \mathbf{p}^\prime \cdot \mathbf{u}^\prime \, d\Gamma_0 - P\varphi \quad I(\mathbf{p}, \mathbf{u}) = \int_{\Gamma} (p_n[u]_n + p_t[u]_t) \, d\Gamma$$

$$a(\mathbf{u}, \mathbf{u}) = \int_{\Omega} \left( \sigma^\prime \cdot \varepsilon^\prime + \sigma^\prime \cdot \varepsilon^\prime \right) \, d\Omega$$  \hspace{1cm} (5)$$

where $\sigma$ and $\varepsilon$ are the stresses and the strains in both bodies, $\Gamma_0$ is the outer boundary of the unit cell, $p_n$ is the normal traction, $[u]_n$ is the jump in displacements, $p_t$ is the traction in tangential direction and $[u]_t$ is the jump of displacements in tangential direction of the interface.

The Mohr-Coulomb contact conditions read as:

$$[u]_n < 0, \quad p_n < 0, \quad [u]_n p_n = 0, \quad |p_t| < k(p_n)c, \quad \text{if} \quad |p_t| > k(p_n)c \quad \text{then} \quad |p_t| = c$$  \hspace{1cm} (6)$$

and $k$ is Heaviside's function, $c$ is the cohesion (shear strength):

Formulation of the contact problem of pull out can be declared as: Find minimum $\mathbf{u}$ and maximum $\mathbf{p}$ of $J$ obeying (3) and (6). Such a posted formulation leads to the well-known Uzawa's algorithm; see [4], for example. In this way all examples were solved. Although the Coulomb influence is absent in the law (6), a slip part can occur, Heaviside's function is applied in the formulas (6) because of assuring a natural requirement: once the tensile strength is attained, no shear forces occur either at the trial point.

### 3 Optimization problem

Natural question occurs when studying the problem of behavior of curvilinear fibers in concrete matrix. The slope of the fibers in the curvilinear part can change and the response of this change will obviously have an impact to the bearing capacity of the composite aggregate. If we leave the vertical distance between horizontal parts of the fibers constant, see experimental part of this paper, the slope of the middle part can change and the design parameter of the
optimization is the angle of the slope. From experimental tests it has been proved that the extraction of the fibers out of the concrete are caused by penetrating of the fibers into the concrete due to attaining some admissible stress (compressive stress) or slipping of the fibers along the interfacial zones with the concrete. The optimization criteria follow this observation of experimental results. The first criterion will require that the admissible force cannot exceed the criterion of extraction of the fibers due to violation of Mohr-Coulomb hypothesis. This case prevails particularly in the case of straight or almost straight fibers. The second criterion states that near the fibers no stress attains the compressive strength.

Also from studies carried out in [3] it immediately follows that this can occur most probably at point 10 in pictures presented in [3], i.e. at the lower part of the fibers right in front of the slope, see also Fig. 1. This point is decisive for assessment whether the compressive stress is attained or not. For curvilinear fibers the optimization problem turns to limit analysis.

4 Examples

Four examples were tested to show the behavior of different shapes of fibers, which are pulled out of the concrete. In each example the stainless wire fibers are considered, for which $E' = 179$ GPa, $v' = 0.3$, $E'' = 41$ GPa, $v'' = 0.16$. The shear bond strength $c = 4.35$ MPa. The results from the examples are displayed in compact form and described as Experiment-1 to Experiment-4. First, the vertical cut crossing the fibers in the ‘unit cell’ illustrates the geometry of the aggregate in Fig. 1. The vertical size is always identical with the horizontal depth. From the geometry one can calculate the fiber volume fraction. The fiber volume fractions are selected in the range from 0.8 to 1.5 percent.

The vertical cut illustrated of fibers is depicted in Fig. 1.

In Fig. 2 vertical cut of unit cells together with finite element meshing shows the fibers with slope of $45^0$. For fibers, which hold the distance in vertical direction of horizontal parts, the slope changes from $0^0$ to $90^0$.

Since the optimization problem strongly depends on the meshing, some types are shown in Fig. 3 in the neighborhood of fibers. The meshing for successively 10, 20, 30 and 90 degrees is illustrated in Fig. 3. Although the mesh changes sometimes irregularly, the results seem to be relatively reliable and reasonable. The optimization procedure leads to the admissible forces $P$ acting at the face of the sample, the distribution of which is seen in Figs. 4 to 7. In Fig. 4 results from Experiment 1 are depicted, in Fig. 5 the admissible forces from Experiment 2 are
described, in Fig. 6 from Experiment 3 and in Fig. 7 forces from Experiment 4 are described, all in dependence of slope angles, which are considered from 0 to 90 degrees.

Figure 2: Four examples of fiber position in unit cells with FEM meshing.

Figure 3: Details of meshing in neighborhood of fibers.
Experiment 1 considers the volume fraction ratio about 0.6 percent and larger distances between fibers in vertical direction. Long debond zone, but relatively short sliding zones are recorded in this example. Due to the bending of the fiber the axial contact forces are lower in the upper part and vice versa for lower part. This observation is valid for all experiments. Sometimes an additional debond is seen at the points in from of the slope for slopes starting at 40 degrees. Sudden drop of transversal tractions indicates occurrence of a crack.
In Experiment 2 we consider relatively higher volume fraction ratio, about 1.5. On the other hand, dense distribution of fibers in vertical direction is introduced. The unit cell is the smallest from all four cases. Basically, higher stresses are observed at the decisive point. The debonding region is relatively short.

Experiment 3 has extremely low volume fraction ratio, namely 0.5, and very long distances between fibers in vertical direction. It leads to decreasing of stresses concentrating at the point of observation. The admissible force appears to be much higher then in the previous case.

Experiment 4 has similar geometry as the first one, and the behavior of the admissible force is also similar. In the latter two cases it is necessary to count with long debonding region in the front part of the fibers.

Although the admissible forces are different in different cases, the angles of optimal slopes are similar and their values range from 55 to 60 degrees. This remarkable observation leads to conclusion that 45 degrees, on which the computations in [3] were carried out and which possesses the Dramix type fibers should be in optimal case improved.

It was shown that in the case of high volume fiber ratio the admissible pullout force is basically lower then in the case of course distribution of fibers. On the other hand, it is necessary to note that the pullout force in this case bears distributed loading, i.e., the tensile force being applied to a cross section of the beam is distributed in such a way that smaller part of the overall loading is applied to this force.

5 Conclusions

While in the most of papers dealing with pullout problems the fibers are straight, in this paper a more advantageous shape of fibers is taken into account. The cohesion of fibers and the concrete matrix is considered and the optimal pullout force is studied. Large test examples were treated and several interfacial conditions were applied. From these examples four experiments are selected to show, first, the material behavior of the interface in the case of the shape envisaged, and second, a possibility of the application of the numerical method introduced at the beginning of this section. In comparison with the papers [1] and [2] and the papers cited in [1] and [2] it uniquely follows that for the cement-based matrix with steel fibers the shape plays very important role for the interfacial mechanical behavior and, consequently, for the bearing capacity of the aggregate.

The optimal slope of Dramix type fibers was the principal goal of this paper. It has been shown that the shape of the fibers has very important role in the bearing capacity of the fiber-concrete composite. From typical examples of unit cells numerical studies showed important impacts on the behavior of the aggregate. First, the slope ranging from 55 to 60 degrees seems to be optimal under condition that has been taken into consideration, i.e., the pullout force damages the composite either because of failure of contact conditions or due to attaining the admissible stress (compressive strength). Second, the maximum
value of admissible force is the higher the course the density of distribution of fibers is. But it is worth noting that the force on the overall cross section is approximately regularly distributed on fibers.

More accurate analysis is under preparation: the mechanical behavior of fibers is considered as linear elastic, but the material of concrete obeys von Mises – Huber – Hencky elastic-plastic criterion. This will desire more sophisticated treatment. It will start with generalized Transformation field analysis, which involves an influence of eigenparameters, see, e.g., [5], [6].

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References