The MINLP optimisation of composite I-beams

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Abstract

The paper presents the optimization of composite I-beams, designed to be connected together from structural steel I sections and reinforced concrete slabs. The optimization of composite beams has been performed by the Mixed-Integer Non-linear Programming (MINLP) approach. The MINLP performs a discrete optimization of different materials and standard dimensions, while continuous parameters are simultaneously calculated inside the continuous space. As the discrete/continuous optimization problem of the composite beams is non-convex and highly non-linear, the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm has been used for the optimization. The accompanied Linked Two-Phased MINLP Strategy accelerated the convergence of the mentioned algorithm.

1 Introduction

The paper presents the optimization of composite I-beams, designed to be connected together from welded structural steel I sections and reinforced concrete slabs. The optimization of composite beams has been performed by the Mixed-Integer Non-linear Programming (MINLP) approach. The MINLP performs a discrete optimization of different materials and standard dimensions,
while continuous parameters are simultaneously calculated inside the continuous space.

The MINLP approach handles with continuous and discrete variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (dimensions, stresses, strains, weights, costs, etc.), discrete variables are used to express discrete alternatives of different materials and standard dimensions. Since continuous and discrete optimizations are carried out simultaneously, the MINLP approach also finds optimal continuous parameters, materials and standard dimensions simultaneously.

The MINLP discrete/continuous optimization problems are in most cases comprehensive, non-convex and highly non-linear. The MINLP optimization approach is proposed to be performed through three steps. The first one includes the generation of a mechanical superstructure of different material and standard dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The Modified Outer-Approximation/Equality-Relaxation algorithm\textsuperscript{1,2} and the Linked Two-Phased MINLP strategy\textsuperscript{3} (LTP) have been proposed to solve this discrete/continuous class of the optimization. The MINLP optimization starts with the discrete material optimization, while standard dimensions are temporary relaxed into continuous parameters. Material and continuous parameter optimization is combinatorially easier to solve and accumulates good global linear approximation of the superstructure model representation. When the optimal material is found, standard dimensions are re-established and the simultaneous material and standard dimension optimization is then continued based on the obtained linear global approximation until the optimal solution is found.

2 Composite I-beams

Simply supported composite I-beams have been designed according to ENV 1993 (Eurocode 3)\textsuperscript{4} and ENV 1994 (Eurocode 4)\textsuperscript{5}. Beams have been checked for the conditions at the ultimate limit and the serviceability limit states. The class of steel cross-sections considered was the Class 1. Beams are subjected to the permanent load (self-weight) and to the uniformly distributed variable load. When the ultimate limit state was considered (plastic analysis), the effects of the bending moment, shear force, shear buckling, the interaction between bending and shear, shear lag effect and shear connections were taken into account. In the case of the serviceability limit state we calculated deflections by using elastic analysis. Creep and shrinkage of the concrete were also taken into account. Many different structural steel grades, reinforcing steel grades and concrete strength classes have been considered for the design.

The self-manufacturing labour and material costs of composite I-beam have been accounted for in the economical type of the objective function. Thus, the
task of the optimisation is to find the optimal concrete and steel materials as well as the optimal geometry and standard dimensions with respect to the minimum self-manufacturing costs, subjected to the given design, material, stress, deflection and stability constraints.

![Figure 1: Vertical cross-section of composite I-beam](image)

The MINLP optimization approach requires the generation of an MINLP mechanical superstructure composed of various material and standard dimension design alternatives that are all candidates for a feasible and optimal solution. The main goal is to find within the given composite superstructure a feasible structure that is optimal with respect to material, standard dimension and all defined continuous parameters (weight, cost, etc.).

### 3 The MINLP model formulation

It is assumed that a general non-linear and non-convex discrete/continuous optimization problem can be formulated as an MINLP problem in the form:

\[
\begin{align*}
\min \quad & z = c^T y + f(x) \\
\text{s.t.} \quad & h(x) = 0 \\
& g(x) \leq 0 \\
& By + Cx \leq b \\
& x \in X = \{x \in \mathbb{R}^n : x^{\text{LO}} \leq x \leq x^{\text{UP}}\} \\
& y \in Y = \{0, 1\}^m
\end{align*}
\]

(MINLP)
where \( \mathbf{x} \) is a vector of continuous variables specified in the compact set \( X \) and \( \mathbf{y} \) is a vector of discrete, mostly binary 0-1 variables. Functions \( f(\mathbf{x}) \), \( h(\mathbf{x}) \), and \( g(\mathbf{x}) \) are non-linear functions involved in the objective function \( z \), equality and inequality constraints, respectively. Finally, \( \mathbf{B} \mathbf{y} + \mathbf{C} \mathbf{x} \leq \mathbf{b} \) represents a subset of mixed linear equality/inequality constraints.

This general MINLP model formulation has been adapted for the optimization of composite beams. In the context of structural optimization, continuous variables \( \mathbf{x} \) define structural parameters (dimensions, strains, stresses, costs, weight...) and binary variables \( \mathbf{y} \) represent the choice of concrete/steel materials and standard dimensions. The economical objective function \( z \) involves fixed costs charges in the term \( e \mathbf{y} \) for manufacturing, while the dimension dependant costs are included in the function \( f(\mathbf{x}) \). Non-linear equality and inequality constraints \( h(\mathbf{x}) = 0 \), \( g(\mathbf{x}) \leq 0 \) and the bounds of the continuous variables represent the rigorous system of the design, loading, stress, deflection, stability, etc. constraints known from the structural analysis. Logical constraints that must be fulfilled for discrete decisions and structure configurations, which are selected from within the superstructure, are given by \( \mathbf{B} \mathbf{y} + \mathbf{C} \mathbf{x} \leq \mathbf{b} \). These constraints describe relations between binary variables and define materials and standard dimensions.

4 The OA/ER algorithm and the LTP strategy

For the solution of non-linear and non-convex problems we used the Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm\(^1\), in which many modifications like deactivation of linearizations, decomposition and deactivation of the objective function linearization, use of the penalty function, use of the upper bound on the objective function to be minimized as well as a global convexity test and a validation of the outer approximations have been applied for the master problem.

The OA/ER algorithm consists of solving an alternative sequence of Non-linear Programming optimisation subproblems (NLP) and Mixed-Integer Linear Programming master problems (MILP). The former corresponds to the optimization of parameters for a mechanical structure with fixed materials and standard dimensions and yields an upper bound to the objective to be minimized. The latter involves a global approximation to the superstructure of alternatives in which new concrete/steel materials and standard dimensions are identified so that its lower bound does not exceed the current best upper bound. The search is terminated when the predicted lower bound exceeds the upper bound.

The optimal solution of complex non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to be obtained. Therefore, the Linked Two-Phased (LTP) strategy\(^3\) has been used to accelerate the convergence of the OA/ER algorithm. The optimization is
performed sequentially in two different phases. The MINLP optimization starts with the discrete material optimization, while standard dimensions are temporary relaxed into continuous parameters. Material and continuous parameter optimization is combinatorically easier to solve and accumulates good global linear approximation of the superstructure model representation. When the optimal material is found, standard dimensions are re-established and the simultaneous material and standard dimension optimization is then continued based on the obtained linear global approximation until the optimal solution is found.

5 The example

Here, presented is an example of the optimization of a composite I-beam, subjected to the self-weight and to the uniformly distributed imposed load of 2.5 kN/m². The beam is simply supported on a span of 25 m. The optimisation has been performed by the MINLP optimization approach. The composite superstructure has been generated in which all possible structures are embedded by different material and standard dimension variation. The superstructure comprises 6 different concrete strengths (C25, C30, C35, C40, C45, C50), 3 different structural steel grades (Fe 360, Fe 430, Fe 510), 48 various standard reinforcing steel sections as well as 9 different standard thicknesses of sheet-iron plates (from 8 mm to 40 mm) for webs and flanges separately. The material and labour costs for composite beams considered are shown in Table 1.

Table 1: Material and labour costs

| Material costs for concrete C 25/30 | 85.00 EUR/m³ |
| Material costs for reinforcing steel S 400 | 0.70 EUR/kg |
| Material costs for structural steel Fe 360 | 0.338 EUR/kg |
| Material costs for structural steel Fe 430 | 0.364 EUR/kg |
| Material costs for structural steel Fe 510 | 0.390 EUR/kg |
| Sheet-iron cutting costs | 2.00 EUR/m² |
| Welding costs | 3.00 EUR/m² |
| Anti-corrosion resistant painting | 7.50 EUR/m² |
| Panelling costs | 12.00 EUR/m² |

The MINLP optimization model of the composite beams has been developed. As an interface for mathematical modelling and data inputs/outputs GAMS⁶ (General Algebraic Modelling System), a high level language, has been used. The self manufacturing costs of material, sheet-iron cutting, welding, anti-corrosion resistant painting and panelling have been accounted for in the
economical type of the objective function, subjected to the given design, material, stress, deflection and stability constraints.

![Figure 2: The optimal cross-section of the composite I-beam](image)

As the model is non-convex and highly non-linear, the Modified OA/ER algorithm accompanied with the LTP strategy has been used for the optimization. Structural synthesis of the gate was carried out by our MINLP computer package TOP7 (Topology Optimization Program). MINOS has been used to solve the NLP subproblems and OSL to solve the MILP master problems.

The optimal result of 43.67 EUR/m² was hereby obtained in the 3rd MINLP iteration, see Figure 2. It should be noted that without the LTP strategy no feasible solution was obtained. Beside the optimal self-manufacturing costs, the optimal concrete strength C25/30, steel grade Fe 430, intermediate distance between I sections, depth of the slab and optimal standard thicknesses of webs and flanges have been obtained.

### 6 Conclusions

The paper presents the optimization of composite I-beams, designed to be connected together from structural steel I sections and reinforced concrete slabs. The optimization of composite beams has been performed by the Mixed-Integer Non-linear Programming (MINLP) approach. The modified OA/ER algorithm together with the LTP MINLP strategy has been used.
Beside the optimal self-manufacturing costs of composite beams (material, welding, sheet-iron cutting, anti-corrosion resistant painting and panelling), the optimal concrete strength and steel grade, the optimal continuous dimensions including the intermediate distances between I sections and the depth of the slab as well as standard thicknesses of webs and flanges have been obtained.

References


