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Transfer matrix-analog beam method to calculate natural frequencies of elastic composite beams

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Abstract

The principal aim of this paper is to present a numerical model for predicting the natural frequencies of elastic composite beams, like bridges, with different intermediate conditions. The prediction of natural frequencies is carried out using the Transfer Matrix- Analog Beam Method [TMABM]. The effects of intermediate conditions such as elastic supports and hinge have been included in the presented model. Elastic composite beams are composed of an upper slab and a lower beam connected at the interface by shear transmitting studs. Parametric studies on the effects of various geometric and material parameters on the fundamental natural frequencies of elastic composite beam are conducted. The obtained results show that these parameters must be taken into account when analyzing the free and forced vibrations or designing these types of composite beams. The developed model can also be applied to calculate the natural frequencies of multi span composite bridges with various intermediate conditions.

1 Introduction

The determination of free vibration frequencies of elastically supported composite beams with point masses fixed to them rigidly or elastically features prominently in the analysis of many contemporary engineering structures. Several authors have investigated the free vibration characteristics of composite beams [1-3] but only a few have taken into account the effects of shear deformation. Glabisz [4] used a coherent algorithm for the exact determination of deformation. Glabisz [4] used a coherent algorithm for the exact determination of the free vibration frequency of beam systems. His proposed algorithm is based on exact solution for segments distinguished in beams, which when boundary conditions are taken into account allow one to construct digitally a map of the variation in the frequency of the beams' vibration. Chung et all [5] presented an exact solution method for the natural vibrations and a Rayleigh-Ritz method with polynomials having the property of Timoshenko beam-column functions as trial functions for vibrations and dynamic sensitivity. Kukla[6] used the Green function method to proposed exact solution of the problem for 10 combinations of classical boundary conditions. Lee [7] used the energy method (Rayleigh-Ritz method) to develop a method to calculate the natural frequencies of thin orthotropic composite shells.

The basic idea of the analog beam method is to replace the real beam with an analog beam where all the shear deformation is concentrated in a thin horizontal layer. When the correct stiffness is assigned to this layer it is possible to get the real beam, and its analog, to behave the same way in the overall sense [8].

Ellakany, Elawadly, and Elhamaky [9] presented the Transfer Matrix- Analog Beam Method [TMABM] to determine the natural frequencies of simply elastic composite beam. Using TMABM they obtained the exact field matrix (flexibility matrix) for the elastic composite beam element with uniformly distributed mass. In extended work Ellakany, Elawadly and A. Ellakany [10] presented a mathematical model for predicting the natural frequencies of multi span elastic composite beams with intermediate rigid support in addition to the effect of shear restraint at the end supports.

The objective of this paper is to present a numerical model for predicting the natural frequencies of composite beams with intermediate hinge and elastic supports. Natural frequency prediction is carried out using TMABM. The developed models presented in [9], [10] and in this paper can be applied to calculate the natural frequencies of most types of bridges, which are mainly, consist of steel girders and concrete slab.

2 Mathematical model

The model used in this analysis is a composite steel-concrete beams which are composed of a concrete slab and a steel beam, connected at their interface by a shear transmitting device such as studs, as shown in Figure.1.

The purpose of the shear studs is to transmit the horizontal shear force between the slab and the beam. To illustrate this model, let us consider a beam of length (ℓ), with the following properties that are constant over the length: cross-sectional area (A), second moment of area (I), and mass per unit length (μ). The slope (dw/dx) of the centerline of the beam is affected by both the bending moment and the shear force. The action of the bending moment rotates the face of the cross-section through an angle (ψ), and from there the shearing action turns the center line to adopt the slope (dw/dx), the angle of the face of the beam remaining unchanged (Figure 2).

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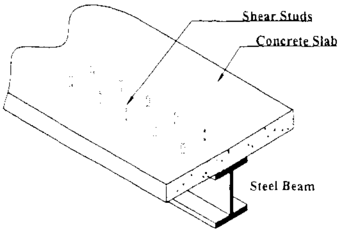

Figure 1: Composite beam cross-section

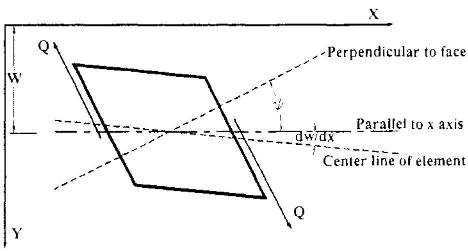

Figure 2: Displacements of the composite beam

2.1 Field transfer matrix

The total bending moment (*M*) can be decomposed into two components [9]

$$M = M_t + M_c \tag{1}$$

Where M_t is the bending moment in the beam from what can be called its "truss action", while M_c represents the combined bending moment from the individual beam action of the sub-beams acting independently.

$$M_t = (EI)_t \frac{d\psi}{dx} \quad , \quad \text{and} \quad M_c = -(EI)_c \frac{d^2 w}{dx^2} \tag{2}$$

Where: $(EI)_t$ and $(EI)_c$ are the bending stiffness for the truss component and for the beam component. The horizontal shear force in the shear layer, q per unit length, which acts at the interface between the sub-beams, can be expressed as

$$q = kh(\psi + \frac{dw}{dx}) \tag{3}$$

Where: h is the distance between the centroids of the sub-beams, and k is the shear stiffness of the shear layer.
In a similar way, the total shear force Q can also be thought of as

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$$Q = kh^2(\psi + \frac{dw}{dx}) - (EI)_c \frac{d^3w}{dx^3} \tag{4}$$

Substitution in eqn (1) yield

$$M = \frac{(EI)_t}{kh^2}\frac{dQ}{dx} + \frac{(EI)_t(EI)_c}{kh^2}\frac{d^4w}{dx^4} - EI\frac{d^2w}{dx^2} \tag{5}$$

Where: $EI = (EI)_t + (EI)_c$

If a sinusoidal variation of w with circular frequency ω, is assumed, then

$$w(x,t) = W(x)\sin[\omega t] \tag{6}$$

Taking the second derivative of eqn (5) with respect to x and combined it with eqns (6) and applying the equilibrium considerations gives the governing equation as

$$\frac{d^6W}{dx^6} - \frac{kh^2EI}{(EI)_c(EI)_t}\frac{d^4W}{dx^4} - \frac{\omega^2\mu}{(EI)_c}\frac{d^2W}{dx^2} + \frac{\omega^2\mu kh^2}{(EI)_c(EI)_t}W = 0 \tag{7}$$

Its solution is of the form

$$w = \overline{C}e^{\lambda x/\ell}\text{, where } \overline{C} \text{ is constant.} \tag{8}$$

Substitution from eqn (8) in eqn (7) leads to the characteristic equation in λ:

$$\lambda^6 - C_1\lambda^4 - C_2\lambda^2 + C_3 = 0 \tag{9}$$

Where $C_1 = \frac{kh^2EI\ell^2}{(EI)_c(EI)_t}$, $C_2 = \frac{\omega^2\mu\ell^4}{(EI)_c}$, $C_3 = \frac{\omega^2\mu kh^2\ell^6}{(EI)_c(EI)_t}$

The roots of eqn (9) are $\pm\lambda_1$, $\pm\lambda_2$, and $\pm\lambda_3$ [9]

Concerning the roots of the characteristic equation, the deflection W can be expressed as:

$$\begin{aligned} W = &-\alpha_1A_1\sin\left(\lambda_1\frac{x}{\ell}\right) + \alpha_1A_2\cos\left(\lambda_1\frac{x}{\ell}\right) + \alpha_2A_3\sinh\left(\lambda_2\frac{x}{\ell}\right) \\ &+ \alpha_2A_4\cosh\left(\lambda_2\frac{x}{\ell}\right) + \alpha_3A_5\sinh\left(\lambda_3\frac{x}{\ell}\right) + \alpha_3A_6\cosh\left(\lambda_3\frac{x}{\ell}\right) \end{aligned} \tag{10}$$

Eqn (10) can be now used to obtained Q, W', ψ, M_t and M_c, which are represented in the following matrix form:

$$
\begin{bmatrix} W \\ W' \\ \psi \\ M_t \\ M_c \\ Q \end{bmatrix} =
\begin{bmatrix}
-\alpha_1 \sin(\frac{\lambda_1 x}{\ell}) & \alpha_1 \cos(\frac{\lambda_1 x}{\ell}) & \alpha_2 \sinh(\frac{\lambda_2 x}{\ell}) & \alpha_2 \cosh(\frac{\lambda_2 x}{\ell}) & \alpha_3 \sinh(\frac{\lambda_3 x}{\ell}) & \alpha_3 \cosh(\frac{\lambda_3 x}{\ell}) \\
-\frac{\lambda_1 \alpha_1}{\ell} \cos(\frac{\lambda_1 x}{\ell}) & -\frac{\lambda_1 \alpha_1}{\ell} \sin(\frac{\lambda_1 x}{\ell}) & \frac{\lambda_2 \alpha_2}{\ell} \cosh(\frac{\lambda_2 x}{\ell}) & \frac{\lambda_2 \alpha_2}{\ell} \sinh(\frac{\lambda_2 x}{\ell}) & \frac{\lambda_3 \alpha_3}{\ell} \cosh(\frac{\lambda_3 x}{\ell}) & \frac{\lambda_3 \alpha_3}{\ell} \sinh(\frac{\lambda_3 x}{\ell}) \\
\beta_1 \cos(\frac{\lambda_1 x}{\ell}) & \beta_1 \sin(\frac{\lambda_1 x}{\ell}) & \beta_2 \cosh(\frac{\lambda_2 x}{\ell}) & \beta_2 \sinh(\frac{\lambda_2 x}{\ell}) & \beta_3 \cosh(\frac{\lambda_3 x}{\ell}) & \beta_3 \sinh(\frac{\lambda_3 x}{\ell}) \\
-\gamma_1 \sin(\frac{\lambda_1 x}{\ell}) & \gamma_1 \cos(\frac{\lambda_1 x}{\ell}) & \gamma_2 \sinh(\frac{\lambda_2 x}{\ell}) & \gamma_2 \cosh(\frac{\lambda_2 x}{\ell}) & \gamma_3 \sinh(\frac{\lambda_3 x}{\ell}) & \gamma_3 \cosh(\frac{\lambda_3 x}{\ell}) \\
\delta_1 \sin(\frac{\lambda_1 x}{\ell}) & -\delta_1 \cos(\frac{\lambda_1 x}{\ell}) & \delta_2 \sinh(\frac{\lambda_2 x}{\ell}) & \delta_2 \cosh(\frac{\lambda_2 x}{\ell}) & \delta_3 \sinh(\frac{\lambda_3 x}{\ell}) & \delta_3 \cosh(\frac{\lambda_3 x}{\ell}) \\
\cos(\frac{\lambda_1 x}{\ell}) & \sin(\frac{\lambda_1 x}{\ell}) & \cosh(\frac{\lambda_2 x}{\ell}) & \sinh(\frac{\lambda_2 x}{\ell}) & \cosh(\frac{\lambda_3 x}{\ell}) & \sinh(\frac{\lambda_3 x}{\ell})
\end{bmatrix}
\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}
$$

or

$$\boldsymbol{Z}(x) = \boldsymbol{B}(x)\ \boldsymbol{a} \tag{11}$$

The expressions of α_i, β_i, γ_i, and δ_i (where $i = 1,2,3$) are presented in ref. [9]. At the left edge of beam element $(x = 0)$ we have $\boldsymbol{Z}\ (x) = \boldsymbol{Z}_{i-1}$, and at the right edge of beam element $(x = \ell)$, we have $\boldsymbol{Z}\ (x) = \boldsymbol{Z}_i$. Using these conditions to eliminate $\boldsymbol{a}$, hence the transfer matrix of element i relating between the state vectors $\boldsymbol{Z}_i$ and $\boldsymbol{Z}_{i-1}$ is

$$\boldsymbol{Z}_i = \boldsymbol{FM}_i\, \boldsymbol{Z}_{i-1} \tag{12}$$

Where: $\boldsymbol{FM}_i = \boldsymbol{B}(\ell)\ \boldsymbol{B}^{-1}(0)$ and $\boldsymbol{FM}_i$ is called the field transfer matrix for an element i of elastic composite beam. For more details regarding the above method, see reference [9].

2.2 Transfer matrix scheme

The actual beam is divided into N elements and $N+1$ stations (Nodes), the matrix relating the right state vector to the left state vector of an element (i) is $\boldsymbol{FM}_i$, while the matrix relating the right state vector to the left state vector of a station (i) is known as point matrix $\boldsymbol{PM}_i$. Both the field and the point matrices are calculated for each element and station. The point matrix is a (6×6) identity matrix except for the case of intermediate elastic support, which will be seen later in this paper. The relation between the state vector $\boldsymbol{Z}_{S_N}$ at support S_N and the state vector $\boldsymbol{Z}_{S_o}$ at S_o, using transfer matrix method is:

$$\boldsymbol{Z}_{S_N} = \boldsymbol{T}\ \boldsymbol{Z}_{S_o} \tag{13}$$

where: $\boldsymbol{T} = \boldsymbol{FM}_{N\text{-}1}\ \left(\prod_{i=N-1}^{1} \boldsymbol{PM}_i\ \ \boldsymbol{FM}_i \right)$, which is called over-all transfer matrix.

Expanding eqn (13) gives six equations, by applying the boundary conditions to these equations the frequency determinant can be easily obtained [11].

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2.3 Elastic composite beams with intermediate hinge

Consider the beam model shown in Figure 3, which has rigid support S_o at the left end while fixed at the other end. In this case, there is an internal discontinuity due to the change of the slope ϕ at the hinge H.

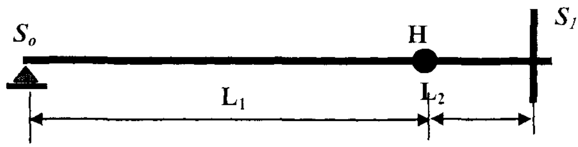

Figure 3: Case of elastic beam with intermediate hinge

Corresponding to this unknown discontinuity, the additional condition (M = 0.0) has to be imposed at the hinge H. Using the transfer matrix scheme, eqn (13), the relationship relating $\boldsymbol{Z}_H^L$ to $\boldsymbol{Z}_{S_o}$ can be written as:

$$\begin{bmatrix} W \\ W' \\ \psi \\ M_c \\ M_t \\ Q \end{bmatrix}_H^L = \begin{bmatrix} T_{12} & T_{13} & T_{16} \\ T_{22} & T_{23} & T_{26} \\ T_{32} & T_{33} & T_{36} \\ T_{42} & T_{43} & T_{46} \\ T_{52} & T_{53} & T_{56} \\ T_{62} & T_{63} & T_{66} \end{bmatrix} \begin{bmatrix} W_o' \\ \psi_o \\ Q_o \end{bmatrix} \tag{14}$$

Noting that the first, the fourth, and the fifth columns of the over all transfer matrix are adopted because ($M_t = M_c = W = 0.0$) at the support S_o.
Setting the boundary condition $M = 0.0$ ($M_t = -M_c$) at hinge H, we can obtain W_o' as a function of ψ_o and Q_o. The deflection, moments and shear force at hinge H are continuous, while the slopes have a change with an angle φ. Applying the boundary conditions at hinge H we can eliminate the initial unknown W_o' and introduce the new unknown φ. The state vector $\boldsymbol{Z}_H^R$ can be then expressed in the term of ψ_o, Q_o and φ as

$$\begin{bmatrix} W \\ W' \\ \psi \\ M_t \\ M_c \\ Q \end{bmatrix}_H^R = \begin{bmatrix} F_{11} & F_{21} & 0 \\ F_{21} & F_{22} & 1 \\ F_{31} & F_{32} & -1 \\ F_{41} & F_{42} & 0 \\ F_{51} & F_{52} & 0 \\ F_{61} & F_{62} & 0 \end{bmatrix} \begin{bmatrix} \psi_o \\ Q_o \\ \varphi \end{bmatrix} \tag{15}$$

Where: $F_{ij} = T_{ij+1} - T_{i1}\dfrac{(T_{42} + T_{52})}{(T_{41} + T_{51})}$

Using the transfer matrix scheme the state vector at the right support S_1 can be found as:

$$\begin{bmatrix} W \\ W' \\ \psi \\ M_c \\ M_t \\ Q \end{bmatrix}_{S_1} = \begin{bmatrix} c'_{11} & c'_{12} & c'_{13} \\ c'_{21} & c'_{22} & c'_{23} \\ c'_{31} & c'_{32} & c'_{33} \\ c'_{41} & c'_{42} & c'_{43} \\ c'_{51} & c'_{52} & c'_{53} \\ c'_{61} & c'_{62} & c'_{63} \end{bmatrix} \begin{bmatrix} \psi_o \\ Q_o \\ \varphi \end{bmatrix} \tag{16}$$

By applying the boundary conditions at S_1 the frequency determinate can be easily obtained and the values of the natural frequencies can be calculated.
Another alternative to set $M = 0.0$ is to consider that $M_t = M_c = 0.0$ at hinge H. In this case the two unknowns W'_o and ψ_o at support S_o can be obtained separately and eliminated from the matrix equation. However, two additional unknowns at hinge H will be introduced. These two unknowns correspond to the change of slope of beam bending and the change of slope of the face rotation. Following the same previous procedure, the state vector at the right side of hinge H can be written as

$$\begin{bmatrix} W \\ W' \\ \psi \\ M_t \\ M_c \\ Q \end{bmatrix}_{H}^{R} = \begin{bmatrix} G_{11} & 0 & 0 \\ G_{21} & 0 & 1 \\ G_{31} & 0 & 1 \\ G_{41} & 0 & 0 \\ G_{51} & 0 & 0 \\ G_{61} & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_o \\ \alpha_1 \\ \alpha_2 \end{bmatrix} \tag{17}$$

Where: $G_{i1} = T_{i1} \times UU + T_{i2} \times U + T_{i3}$, $UU = \dfrac{T_{42} \times U + T_{43}}{T_{41}}$ and $U = \dfrac{T_{43} \times T_{51} - T_{41} \times T_{53}}{T_{41} \times T_{52} - T_{42} \times T_{51}}$

2.4 Elastic composite beams with intermediate elastic support

In the actual system of beam structures like bridges, the intermediate support can be classified as rigid or elastic support according to its vertical displacement. The support, which has a high length with small cross-section area and small modulus of elasticity or which has any relatively small vertical displacement, is considered as elastic support. The elastic support can be modeled as spring with constant stiffness $k_{es} = E_{es} A_{es} / \ell_{es}$, Figure 4. From the free body diagram of elastic support, which is modeled as spring, the relations of the state vector elements at both sides of elastic support will be as:

$W^R_{es} = W^L_{es}$, $W'^R_{es} = W'^L_{es}$, $\psi^R_{es} = \psi^L_{es}$, $M^R_{tes} = M^L_{tes}$, $M^R_{ces} = M^L_{tes}$ and $Q^R_{es} = Q^L_{es} + k_{es} W$.

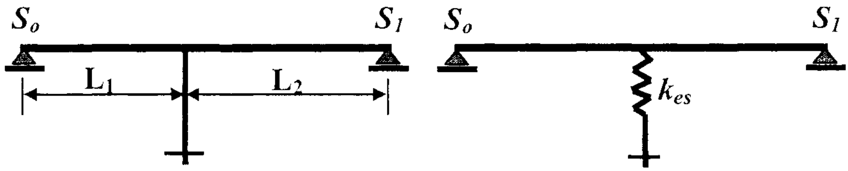

Figure 4: Elastic composite beam with intermediate elastic support

From the previous relations, the point matrix $\boldsymbol{PM_{es}}$ relating the state vectors at both sides of the elastic support can be written as

$$PM_{es} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ k_{es} & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

3 Numerical results

The failure or the maximum response of any dynamical beam systems, like bridges, depend on the natural frequencies as compared with the frequencies of the applied load which may be due to earthquake, sudden or moving loads. Among the different parameters on which the natural frequencies strongly depend are the shear stiffness k and the ratio η ($\eta = EI_t/EI_c$). Hence, the effects of k and η on the natural frequencies of elastic composite beams are studied in the following two cases.

In the first case a beam with an intermediate hinge is considered, Figure 3. The data used in the analysis are as follows: $L_1 = 8$ ms, $L_2 = 2$ ms, $EI=10^7$ N.m^2 and μ =2000 kg/m. The support conditions are rigid at the left side and completely fixed at the other one. While, in the second case a beam with intermediate elastic support is analyzed, Figure 4, the following data are considered: $L_1 = L_2 = 5$ ms, $EI=10^7$ N.m^2 and μ=2000 kg/m . The stiffness of the intermediate elastic support, $K_{es,}$ takes the values of 10^3 ,10^5 and 10^7 N/m. The values of k range from 0.1 to 10^7 N/m^2, while the values of η are 1.0, 1.6 and 2.0. The natural frequencies for each case at the scanned values of k and η are presented in Tables 1-3.

It can be directly concluded from the tables that the higher the values of k and η, the higher the value of natural frequency. This means that these values must be taken into account when calculating the natural frequencies of this type of beam. It is noticeable that when M_t is assumed to equal $-M_c$ at hinge H, the second and the third natural frequencies start at higher values for low values of shear stiffness k. While for the situation at which both M_c and M_t are zeros, the natural

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Table 1 Natural frequencies of beam with intermediate hinge(M_c=M_t=0.0)

k (N/m²)	η=1.0			η=1.6			η=2.0		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.1	7.25	16.92	35.12	7.413	18.78	38.63	7.495	19.58	40.21
10^2	7.27	17.35	35.48	7.466	19.20	39.36	7.512	20.06	40.98
10^4	7.31	19.08	36.10	8.085	21.02	40.22	8.421	22.60	41.98
10^5	7,37	21.72	38.20	8.150	24.36	42.83	8.475	25.43	44.97
10^6	7.61	22.62	40.14	8.403	25.12	44.53	8.770	26.15	46.35
10^7	8.17	21.91	41.31	8.933	23.17	45.44	9.474	23.85	42.52

Table 2 Natural frequencies of beam with intermediate hinge (M_c = -M_t)

k (N/m²)	η=1.0			η=1.6			η=2.0		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.1	7.166	25.32	53.62	7.740	27.89	57.92	8.270	29.79	61.22
10^2	7.166	25.32	53.65	7.74	27.89	57.95	8.275	29.82	61.00
10^4	7.410	26.90	56.90	8.21	30.26	63.72	8.600	32.30	66.61
10^5	7.456	21.70	36.40	8.27	24.40	41.40	8.451	25.71	43.52
10^6	7.751	22.71	40.21	8.31	24.62	43.32	8.803	26.73	46.32
10^7	9.052	22.42	42.74	9.45	24.22	45.61	9.703	24.92	47.32

Table 3 Natural frequencies of beam with intermediate elastic support

η	k (N/m²)	K_{es}=10³ N/m			K_{es}=10⁵ N/m			K_{es}=10⁷ N/m		
		ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.6	0.1	3.49	13.56	31.24	4.22	13.45	31.22	13.95	17.97	42.52
	10^3	3.49	13.56	31.26	4.32	13.61	31.29	13.97	17.99	42.54
	10^5	3.54	14.05	31.52	4.36	14.11	31.78	13.99	18.01	42.73
	10^6	3.85	14.36	31.70	4.61	14.39	31.98	14.37	18.17	43.37
	10^7	4.53	16.76	35.29	5.02	16.77	35.79	16.80	18.85	48.53
1.0	0.1	3.03	16.11	36.08	4.52	16.37	36.36	16.13	19.55	46.33
	10^3	4.04	16.12	36.14	4.54	13.39	36.40	16.14	19.58	46.36
	10^5	4.13	16.15	36.23	4.61	16.45	36.46	16.16	19.68	47.01
	10^6	4.33	16.67	36.93	5.04	16.47	38.02	16.48	19.71	47.49
	10^7	5.49	18.06	39.48	5.97	18.28	40.33	18.34	19.99	48.62

frequencies increase as k increased, which is the normal behavior. This may justify because for low values of k, the moment M_t plays no part in calculating the field matrix of the beam element. At hinge H, when we assume that M_t= -M_c a sudden change in M_t is introduced which causes anomalous change in the values of the natural frequencies. While for large values of shear stiffness both M_t and M_c play their normal part in the calculations and things go back to their normal behavior. So, for the case of elastic composite beams with intermediate hinge, it is better to use the assumption of M_t= M_c=0.0 for small values of shear stiffness.

Conclusion

The model presented in the paper has been used to numerically calculate natural frequencies of elastic composite beams. The dependence of the natural frequencies on geometrical and material parameters of beam, such as k, $(EI)_t$ and $(EI)_c$ has been introduced in the model. Two intermediate cases have been considered, intermediate hinge and elastic support. The results show that the effect of k, $(EI)_t$ and $(EI)_c$ is very important and must be taken into account when calculating the natural frequencies of elastic composite beams. Two assumptions have been considered for analyzing the case of intermediate hinge. The results obtained using both assumptions are converted for higher values of shear stiffness k and diverted for small values of k.

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