Lay-up optimization for the hull of a competition sailing yacht

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Abstract

Deformability and buckling load of yacht in fibers reinforced plastic sandwich depend on the skins stacked sequence. In this work an optimization of the laminae fibers direction of a competition yacht existing design is proposed. This procedure has been divided into three parts (i.e. material characterization; surface model definition; lay-up optimization). First of all a set of unidirectional specimens has been realized, by using the same fibers and matrix (carbon/epoxy) used for the hull as well as the same procedure and workers, in order to characterize the material according to the ASTM D3039, employing strain gage technique. In the second part, by means of an original software in Turbo-Pascal (which uses the half-width value matrix as an input) linked to Pro/ENGINEER, has been possible to obtain the body plan and the surface and finite element models of the sailing yacht for the subsequent analyses. In the third step, an optimization procedure, that uses the results of FE structural analyses in three different navigation configurations, is performed, with the aim to obtain the fibers direction able to minimize the yacht deformability, taking in account buckling loads also. An approximated analytical model has been used in conjunction with a sweep technique in order to evaluate the best of the solutions.

1 Introduction

In the latest years composite materials have had a very high diffusion in sailing yacht building. Particularly regarding the competition field where, generally, no budget limit is provided. It is necessary to remind that the old IOR has been substituted by the IMS which has rerouted the design towards light and stiff
hulls. In fact the rating value, that is a kind of penalty that every sailing yacht has to pay during a regatta, results to be lower for such kind of yacht. In order to have a low rating value composite materials can be used, since they give to the structure lightness and a relatively high stiffness. Since the bending is the prevalent stress, generally the yachts are built by using glass or carbon epoxy sandwich with a PVC core. However, under generic load conditions, except tension, thin sandwich easy buckle; this is the reason why buckling behavior has been take in account in this paper.

In order to minimize the yacht deformations, in this work an optimization procedure, integrated by FEM structural analyses performed by using ANSYS code, has been developed; this procedure regards the lay-up of the sandwich (in terms of fibers orientation) taking in account buckling loads also. Three load conditions of the yacht have been examined. The procedure has been set in reference, where however buckling has not been considered.

2 Problem analysis

The yacht has been drawn and designed in a Shipyard of Palermo (ITALY) by Albeggiani & Inzerillo (numerical data have been kindly granted by themselves and computed by the authors) and it has the following characteristics:

\[ L_{o.a.} = 7.60 \text{ m} \quad \text{Displacement} = 1.100 \text{ kg} \]
\[ l_{\text{max}} = 2.60 \text{ m} \quad \text{Ballast} = 450 \text{ kg} \]
\[ \text{Draft} = 1.70 \text{ m} \quad \text{Tot. Sail Area} = 36 \text{ m}^2 \]

The yacht has been realized to take part to regatta in the IMS Class. The structure is a carbon/epoxy sandwich with a PVC core. The total thickness of the sandwich is 20 mm; the core is 15 mm thick, while the 6 laminae 0.83 mm.

2.1 Material properties

The sandwich has been tested according to the ASTM D3039 by using strain gage technique, in order to characterize the material which will be used for the yacht building. For the tests, 0° and 90° unidirectional specimens have been realized, by using the same fibers and matrix used for the hull as well as the same procedure and workers. A summary of the test results is shown:

\[ \nu_f = 38\% \quad E_{11} = 85.500 \text{ MPa} \quad E_{22} = 4.200 \text{ MPa} \quad G_{12} = 5.170 \text{ MPa} \]
\[ X_f = 800 \text{ MPa} \quad Y_f = 20 \text{ MPa} \quad S = 60 \text{ MPa} \]

Where \( \nu_f \) is the volume fiber percentage, \( E_{11} \) and \( E_{22} \) are the Young’s moduli parallel and normal to the fibers, \( X_f \) and \( Y_f \) are the respective tensile strengths, \( G_{12} \) is the shear modulus and \( S \) is the shear strength.

2.2 Surface and finite element models

The yacht as a whole is defined both by the hull and the superstructures. The mathematical definition of the hull has been a great problem (the so called hull equation) by centuries. However, new CAD software capabilities have partially
solved this problem. In fact, for example, it is possible to define a model by means of different parametric surfaces (i.e. blended in one or two directions, swept). In this work a blended surface in two directions has been used. The procedure can be divided into three consecutive and linked steps: a) Sections construction; b) Surface hull construction; c) Superstructures construction.

The first step has been solved starting from the *half width matrix* and then modelling a specified number of sections by using Bezier's curves. This procedure gives the bidimensional hull sketch.

An output file (in ASCII format) written by the software, is the input for Pro/ENGINEER. It treats the curves as one feature and, by blending these, it gives the hull parametric surface. Many information have been extracted from the surface analysis such as slope and tangency. These data have permitted little changes in the bidimensional sketch in order to increase the hull fairing. Once the hull has been defined has been possible to insert the superstructures composed by deck, home-deck and cockpit. The used technique employs boundary curves definition. Figure 1 shows the generation of the home-deck. It is important to note that with respect to the model used in reference, in this case it has been possible to define a true 3D model of the yacht.

The FE mesh generation has been carried out by using the Pro/FEM modeller. The used strategy has been that to set a specified number of divisions on the boundary curves like keel, wash-board, transom in order to have quasi-rectangular elements with a side practically coincident with the water lines, at least. This is not true only for few elements near the bow, but this approximation is not important due to the stiffeners of this part in the real configuration. The above fact is very important especially if (as foreseen) the yacht is built by using composite materials; in this paper, in fact, this side has been assumed as the origin of a local reference, in order to define the composite mechanical properties simplifying both design and realization procedures.

For yacht and sails quadrilateral isoparametric layered shell elements with 8 nodes (4 nodes for sails) and 6 degrees of freedom for each node have been used. Several different layers are permitted for this element. Different thickness, fibers orientation and orthotropic material properties can be defined for each layer. The element formulation is based on Mindlin's theory. The ANSYS code reduces the

![Figure 1: Blended surface with boundary curves in two directions.](image-url)
“locking” effect. Note that the sails have been modelled without taking in account their effective shape, but simply like plates to which apply the loads, stressing the structure by means of the shrouds and the mast. The latter has been modelled with beam elements. The shrouds, the aft stay and the forestay, modelled using truss elements, have been prestrained at $0.003 + 0.005$, while the bulb has been assumed to be a concentrated mass attached to the keel. Particular care has been spent in finding a suitable mesh for the yacht, that could assure reliable results and reasonable computer times. The final mesh shown in Figure 2, has been reached after performing the appropriate convergence analyses and it is composed from about $5,700$ elements. With respect to reference a channel along the keel has been considered for a more realistic determination of buckling loads. The stiffness of the sails has been opportunistically increased in order to reduce the eventuality that buckling appears in them. Since they are linked to the structure of the yacht by bars, they should not influence in remarkable way the results in the hull.

2.3 Buckling analyses

Generally two techniques are available to predict the buckling load with the finite element method: nonlinear and linear buckling analyses. Nonlinear analysis is the most accurate approach: it employs a non linear static analysis increasing loads or displacements; with this technique nonlinear effects (plastic behavior, large deflection response, etc.) can be included. Linear buckling analysis predicts the buckling strength of an ideal linear elastic structure. Large displacements could be also considered, but with the following restrictions: (a) the structure behavior during the prebuckling phase is characterized by negligible displacements and the buckling condition is reached
in improvised way, showing a deformed state of nature completely different from that agent in the prebuckling, (b) they change linearly with the load; this happens, for example, in beams and in slabs subjected to compression, in pipes and spherical shells subjected to external pressure. In this paper only linear buckling analyses are performed, considering that the last cases are very close to that examined and that in a sandwich panel the plastic strain, that could develop up to the failure, is generally negligible; such a choice is justified also by the fact that this method requires shorter computer time, this is not negligible considering the high number of analyses to perform and that it is interesting to compare the behavior of the different analyzed configurations more than calculate the absolute values.

Buckling loads so calculated will be upper bounds to the exact ones. The differences decrease as more as the previous hypotheses are true, as in the present case, where, however, the presence of stiffeners (strings, spar-frame, keelson, etc.), increasing the buckling loads, have been neglected in the calculations.

In linear buckling the limit condition of elastic stability exists when:

\[ [K] = 0 \]  

(1)

where \([K]\) is the stiffness matrix. In the case of large displacements and elastic material behavior, nonlinearity exists between strain and displacement (geometric non-linearity), for which the \([K]\) matrix could be written as the sum of a \([K_0]\) constant matrix and of a \([K_\sigma]\) load dependent matrix:

\[ [K] = ([K_0] + [K_\sigma]) = 0 \]  

(2)

If \([K_\sigma]\) depends linearly on the load it can be written as:

\[ ([K_0] + \lambda [K_\sigma]) = 0 \]  

(3)

where \([K_\sigma]\) is evaluated at some arbitrarily chosen level of loading (the exercise loads in this paper) and \(\lambda\) is a load factor, that, multiplied for the loads which generated \([K_\sigma]\), furnish the looked for critical load intensity. It is possible then to write:

\[ ([K_0] + \lambda [K_\sigma]) \{u\} = 0 \]  

(4)

This relationship defines a typical eigenvalue problem and therefore it allows to calculate the eigenvalues \(\lambda\) and the corresponding eigenvectors \(\{u\}\). In this case the load factor is equal to the minimum value of \(\lambda\).

In this paper buckling behavior of the whole sandwich is analyzed, assuming that core material has adequate compressive and shear moduli to prevent the laminate skins from local buckling.

2.4 Optimization procedure

In an optimization procedure design variables are independent quantities varied in order to achieve the optimum design; upper and lower limits are specified to serve as constraints. State variables are dependent variables used to constraint the design; they are response quantities, function of the design variables, and may have a maximum and/or minimum limit. In this way design and state variables
define a region of feasible designs. Objective function is the dependent variable to minimize in the feasible design region. A procedure based on the construction of approximated analytical models and on the application of a sweep technique, has been employed in order to find the optimal fibers orientation in each layer of the laminate.

The analytical model approximate the dependence of both the objective function and state variables by the design variables. The optimization problem is a constrained one by limits on design variables. It is converted to an unconstrained one in order to use more efficient algorithms for the minimum research\textsuperscript{12}; the conversion is made by adding penalties terms to the objective function approximation to take in account the imposed constraints. The procedure is carried out by applying an iterative technique and halts when the convergence is achieved or if the number of specified loops has been performed. The approximated model is obtained by calculating the objective function and the state variables for several sets of design variable values and performing least squares fits between the data points\textsuperscript{13}. In order to speed up convergence, feasible values of design variables have been initially assigned. Each optimization loop generates a new data point close to the minimum of actual analytical model and the approximations are updated. The approximated objective function is minimized instead of the real one. At the end of the procedure the set of data corresponding to the minimum value of the calculated objective functions, is set as the optimum design. Convergence does not necessarily indicate that a true global minimum has been obtained; therefore it is often opportune to repeat the analyses, starting from different initial values. In this work the approximated analytical models have taken on the form of a fully quadratic representation with cross terms:

$$F = a + \sum_{n=1}^{N} b_{n} x_{n} + \sum_{n=1}^{N} c_{n} (x_{n})^{2} + \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} d_{nm} (x_{n})(x_{m})$$ (5)

In this relationship the quantities $a$, $b_{n}$, $c_{n}$, $d_{nm}$ are calculated with the least squares method, $x$ is the set of design variables and $N$ is the total set number.

With sweep technique actual optimum of each design variable is joint subsequently to values of the other ones, uniformly swept in the space of design, and the corresponding values of the objective function are calculated.

The optimization analyses are developed in three phases. In the first, using a sufficient number of iterations, an approximated analytical model is found, with the aim to locate the design variables corresponding to a local optimum. In the second, in order to analyze other regions of the feasible domain not explored before, a sweep generation is used. Finally in the third, a new approximated analytical model is developed, starting from the best sets of the design variables calculated in the preceding two phases.

2.5 Load conditions

Three load conditions have been studied; for all of these weight has been considered, by assigning appropriate densities to the elements and gravity acceleration. The first (marked with \#1) considers the yacht plunged into water
with 30 degrees of heel and sailing close-hauled with a true wind of 25 knots. The second configuration (#2) considers the yacht in the same hypothesis, but supported bow and stern by two consecutive waves; for this purpose some nodes of the extreme bow and stern belonging on the plane of longitudinal symmetry are constrained. In the last one (#3) the yacht is considered in a symmetrical configuration (without heel) always supported by two consecutive waves, but subjected to the weight only. A fourth configuration, corresponding to the yacht supported by one wave has not been considered.

To the purposes of the optimization the following variables have been considered:

- The design variables: given by the fibers orientations, $\beta_1$ and $\beta_2$, in two layers of the laminate; since a third layer with fibers orientation of $0^\circ$ is employed (water lines directions), the global sandwich composition is therefore $[0/\beta_1/\beta_2/core/\beta_2/0]$; the quantities $\beta_1$ and $\beta_2$ are variable in the range $-90^\circ$ to $90^\circ$.

- The objective function: represented by the relative displacement in absolute value in the $z$ direction, $u_z$, in the configuration #1 between the edge of the stern and the root of the mast. This function assures the selection of a fibers orientation that guarantees the greatest flexural stiffness in longitudinal direction, compatibly with the other conditions.

- Three state variables given by the relative displacements in absolute value in $y$ direction, $u_y$, of the two points of the wash-board belonging in the section of the mast (points A and B in Figure 3), calculated in the three configurations. These variables assure that the deformations of the main section do not overcome certain values, that by the experience have been set equal to:

  $0 \leq u_y(1) \leq 7 \text{ mm}$  $0 \leq u_y(2) \leq 7 \text{ mm}$  $0 \leq u_y(3) \leq 7 \text{ mm}$

- Other two state variables given by the relative displacements similar to the objective function, but evaluated in the configurations #2 and #3, with limits:

  $0 \leq u_y(2) \leq 10 \text{ mm}$  $0 \leq u_y(3) \leq 10 \text{ mm}$

- Other two state variables, that furnish limits on the torsional deformation for the configurations #1 and #2. They are evaluated as relative rotations, $\alpha$, between the sections containing the points C and D and the points E and F (see Figure 3). The following limitations are assigned to them:

  $0 \leq \alpha(1) \leq 0.0006 \text{ rad}$  $0 \leq \alpha(2) \leq 0.0006 \text{ rad}$

- Other three state variables given by the linear buckling load factors. Limits of 4 are fixed, according to reference $^3$:

  $4 \leq \lambda(1)$  $4 \leq \lambda(2)$  $4 \leq \lambda(3)$

![Figure 3: Points location used for the state variables evaluation.](image-url)
2.5.1 Loads in bowline

The #1 load condition is represented in Figure 4. In equilibrium, the aerodynamic load on the sails, \( F_s \), is balanced by an equal and opposite load, \( F_h \), that is produced by the immersed appendixes of the hull. The component \( F_R \) (balanced by \( D_h \)) is responsible of the progression of the hull. The \( F_H \) load does not give contribution to the speed, but only to his sideslip, arranging with \( L_h \) an upsetting couple. This couple must be balanced by the moment produced between the weight \( W \) (including the crew disposed windward), and the hydrostatics lift \( B \).

Loads \( F_R \) and \( F_H \) could be estimated by the expressions\(^\text{14}\):

\[
F_R = C_R 0,6 A v^2 \quad F_H = C_H 0,6 A v^2
\]

where \( A \) is the sail area (equal to 23m\(^2\)), \( v \) the speed (25knots) and \( C_H \) and \( C_R \) are the lift and drag coefficients. These coefficients can be taken equal to\(^\text{14}\):

\[
C_H = 0,8 \quad C_R = 0,3
\]

Finally it is obtained:

\[
F_H = L_h = 2.782 \text{ N} \quad F_R = D_h = 1.043 \text{ N}
\]

\( F_H \) has been distributed as pressure on the sails. Remaining loads have been applied as concentrated forces to the centers of gravity of the keel (\( L_h, D_h \)) and of the sails \( F_R \). The draft has been iteratively calculated by imposing the equilibrium between weight (including the crew) and hydrostatic lift both in terms of forces and moments.

3 Analysis of results

In the following the more interesting results are reported. The remaining those could be qualitatively deduced in reference\(^3\). Each system of applied loads determines different kinds of deformations, that are added in a way dependent by the navigation conditions.

The structure and crew weight, when the hull is immersed, cause the longitudinal yacht inflexion with concavity toward the lower part and his stretching in crosswise direction. Instead, the shrouds (subject to preload and to load deriving from the sails) and the weight with emerged hull, cause opposite deformations. Result analysis seems to show that the analyzed configurations significantly represent different behaviors. The optimization procedure has required relatively few iterations in order to locate with sufficient reliability an optimum configuration. In figure 5 diagrams of objective function and design variables vs. number of set are reported. It is observed that the algorithm locates a region of local minimum that begins after eight sets and finishes at the end of the first optimization phase. Afterward the sweep technique for complessively twenty
iterations is applied. At the beginning and at the end of the second sweep step, in correspondence of the design variable values of $\beta_1 \equiv \beta_2 \equiv 90^\circ$, lower values of $u_2$ are found. In the third phase, starting from the sixteen best sets of design variables relieved in the previous phases, a new approximated analytical model is developed, giving the same previous least values of the objective function, with the same values of $\beta_1$ and $\beta_2$. Therefore these values define the optimum design of the yacht and the resultant sequence of the sandwich is: [0/90/90/core/90/90/0]. Such result could be legitimated considering that the relative displacement $u_2$ depends on the deformability of the yacht in longitudinal and in transversal direction. In general, stiffness in longitudinal direction should grow by increasing fibers at $0^\circ$, while those in transversal direction should grow by increasing fibers at $90^\circ$. However the closed shape of the yacht and its height already confer it an elevated flexural stiffness in longitudinal direction, that the introduction of other layers at $0^\circ$ is not able to clearly elevate: in fact configurations with layers near to $0^\circ$ produce decidedly worse values of the objective function. The results seem to show, instead, that the longitudinal stiffness improves much if the strain of the generic transversal section is reduced, by introducing layers close to $90^\circ$. Regarding to buckling load factors the calculations have given values in the range 5-6; these values are practically few influenced by the change of fibers angles and the normalized displacements are higher in the sails. Moreover it has been verified that increasing the sails stiffness the buckling load factors increase. The above fact permits to say that the results are valid only from a mathematical point of view and that the corresponding load factors on the hull must be considered higher than those calculated. Buckling load factors of the isolate hull (including superstructures), subject to the loads of the configuration #3, with $\beta_1$ and $\beta_2$ varying between $0^\circ$ and $90^\circ$, are also calculated. The corresponding extreme buckling loads are close to seven ($\beta_1 = \beta_2 = 0^\circ$) and seventeen times ($\beta_1 = \beta_2 = 90^\circ$) the loads on the hull in the real configurations and confirm that the

![Figure 5: Objective function [mm] and design variables (degrees) vs. set number](image-url)
imposed limit on the load factors in the optimization procedure are respected; moreover the configurations with $\beta_1$ and $\beta_2$ tending both toward $90^\circ$ are also the more stable. In figure 6 the buckled shape of the hull with $\beta_1=\beta_2=90^\circ$ is shown. With respect to this, for $\beta_1=\beta_2=0^\circ$ the higher normalized displacements are located closer to the bow.

Effectively the used optimization procedure fits very well to the purpose and it achieve to find a combination of stiffness in the different directions, in order to limit the deformability of the yacht. Another advantage is to have analyzed the yacht as a whole and not isolate parts of it, taking in account the mutual influences between all the regions.

The found minimum seems to be a global minimum and so other optimization cycles starting from different initial points have not been performed. Moreover it has been verified that far from the discontinuities the stress state is in safety. The check of resistance of regions with hard discontinuities has not been performed since it has not been considered of interest for the paper.

4 Conclusions

The use of Pro/ENGINEER has been a powerful instrument to model the surface of the yacht. It has permitted an easy yacht generation starting from the sections. Result analysis shows that the three different analyzed load configurations significantly represent different behaviors. Deformability and buckling loads are influenced by the change of fibers angles, showing that higher values can be achieved only if a proper orientation of the fibers is found.

The optimization method based on the creation of an approximated analytical model, followed by the application of a sweep on the design variables of the actual optimum and again by the creation of another approximated analytical model, starting from the previous best sets of design variables, has been demonstrated suitable to resolve the problem of the research of the optimal fibers disposition, as well as of shape optimization. The minimum value of the objective function has been located in the second step of the sweep phase and has been found again in the following phase, demonstrating that is sometimes opportune to employ the sweep technique in order to explore all the field of the feasible domain. Probably analogous results could be achieved repeating the procedure, starting from different imposed initial values.

The resultant sequence from the used optimization procedure results to be $[0/90/90/core/90/90/0]$. Laminae with $\beta_1$ and/or $\beta_2$ close to $0^\circ$ could foresee, but
the latter determines in the hull a behavior more flexible than the correspondent one with fibers close to 90°. This is probably because the closed shape of the hull and its height already confer it a high flexural stiffness in longitudinal direction, that the introduction of other laminae at 0° is not able to increase clearly, as on the contrary is able to determine the introduction of fibers at 90°, increasing the stiffness in crosswise direction and reducing the relative displacements in z direction of the control points.

It is found also that the limits of 4 on the buckling load factors are not violated for whatever fibers disposition and that the configurations with \( \beta_1 \) and \( \beta_2 \) tending both toward 90° are also the more stable.

References