MODAL PARAMETER ESTIMATION OF A SMALL SCALE MODEL OF AN OFFSHORE PLATFORM: A COMPARISON OF TWO DIFFERENT ALGORITHMS

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Abstract - A modal parameter estimation algorithm using "Goal Programming", a nonlinear optimization technique, is presented. Some numerical simulations that reproduce complex practical applications were performed and the results estimated through Goal Programming algorithm are compared with those estimated through a classical modal estimation method - the Orthogonal Polynomials Method. Experimental tests were carried on a small scale model of a fixed offshore platform and the Frequency Response Functions (FRFs) obtained are analyzed using the Orthogonal Polynomials Method and Goal Programming algorithm. The Modal Assurance Coefficient (MAC) matrix is used to correlate the experimentally estimated mode shapes and the numerical ones, obtained through a finite element code. It is shown that the developed algorithm estimates modal parameters more accurately than the classical method does.

NOMENCLATURE

j : response (or output) degree of freedom
k : reference (or input) degree of freedom
H_{jk}^{THE}(\omega) : theoretical value of the FRF
H_{jk}^{EXP}(\omega) : experimental (measured) value of the FRF
r : mode number
N : total number of modes
In experimental Modal Analysis, the accurate estimation of modal parameters (natural frequencies, damping ratios and modal shapes) is very important, once these parameters will be used for updating a finite element model of the structure. Errors on the estimation of modal parameters will strongly affect the updated model. An updated model that matches the measured dynamic characteristics of the actual structure (experimental model) will allow the prediction of its behavior when any kind of load is applied. Besides, it may be useful for identifying damages that may occur during the structure lifetime.

This work presents a modal parameter estimation algorithm using “Goal Programming” nonlinear optimization technique. The application of Goal Programming technique for estimating modal parameters is proposed by the authors aiming the improvement of modal parameter accuracy, mainly in complex cases. Some numerical simulations are performed and the results estimated through Goal Programming algorithm are compared with those estimated through a classical modal parameter estimation method - the Orthogonal Polynomials Method.
Modal parameters of a small scale model of a fixed offshore platform are then estimated through these two algorithms. The experimental tests were performed exciting the model through two non simultaneous input forces and the responses were measured by a set of 32 accelerometers.

It is shown that the developed algorithm estimates the modal parameters of the model more accurately than the other method does. A comparison between the modal shapes estimated through these two algorithms and the numerical ones is performed by computing the MAC (Modal Assurance Coefficient)\(^3\) matrix.

**GOAL PROGRAMMING OPTIMIZATION TECHNIQUE**

In Modal Analysis, after performing an experimental test and obtaining the FRFs, the next stage is the estimation of modal parameters: natural frequencies, damping ratios and residues (modal shapes). The different existent modal parameter estimation methods have advantages and limitations. Some of the most complex practical cases are structures of high modal density, heavily damped structures and experimental signals contaminated with noise. In these cases, most of the classical methods do not present satisfactory accuracy.

Based on its success in other areas\(^4-7\), Goal Programming was used by the authors for estimating modal parameters. Goal Programming is an optimization algorithm whose purpose is to minimize one or more functions subject to constraints. In the case of modal estimation, this function is the squared error between the experimental and the theoretical FRFs. It is an iterative algorithm, so an initial point is required.

The Goal Programming optimization algorithm, developed and presented by Ignizio\(^1\), enables the search for multiple goals under constraints.

For the conventional formulation of a nonlinear programming problem, it is necessary to minimize an explicit scalar objective function of the problem variables. However, in the Goal Programming optimization technique the objective function does not need to be explicit.

In the modal estimation problem, the goal is to obtain a group of variables (modal parameters) that minimize the total squared error between experimental and theoretical values of the FRFs. The total squared error is defined by Equation (1).
\[ E_{\text{TOTAL}} = \sum_{j=1}^{N_{\psi}} \sum_{k=1}^{N_{e}} E_{jk} \] (1)

where

\[ E_{jk} = \sum_{r=1}^{F_i} e_{jk}(\omega_r) \ e^*_{jk}(\omega_F) \] (2)

\[ e_{jk}(\omega) = W(\omega) (H_{jk}^{\text{EXP}}(\omega) - H_{jk}^{\text{THE}}(\omega)) \] (3.a)

\[ e^*_{jk}(\omega) = W(\omega) (H_{jk}^{\text{EXP}}(\omega) - H_{jk}^{\text{THE}}(\omega)) \] (3.b)

\[ F_i < f < F_f : \text{frequency range of the measured FRF} \]

\[ W(\omega) : \text{weighting function} \]

The theoretical expression of a FRF is given by

\[ H_{jk}(\omega) = \sum_{r=1}^{N_r} \left( \frac{A_{jk}}{\omega - \lambda_r} + \frac{A^*_{jk}}{\omega - \lambda^*_r} \right) \] (4)

where the residue expression is

\[ r_{A_{jk}} = -\frac{\phi_p^T \phi_q r}{2m_i \omega_{d,r}} \] (5)

and the pole expression is

\[ \lambda_r = \sigma_r + j\omega_{d,r} = -\xi \omega_{a,r} + j\omega_{a,r} \sqrt{1 - \xi^2} \] (6)

For each vibration mode, the variables are: natural frequency, damping ratio and some residues (one for each measured FRF). The effects of out-of-band modes can be taken into account by means of residual terms (residual mass and residual flexibility), increasing, in this case, the number of variables of the problem.
The variables must be limited within lower and upper bounds. This is considered by the algorithm as a goal of higher priority than the squared error minimization. So, the constraints that must be applied are as follows:
- lower and upper bounds for each natural frequency;
- lower and upper bounds for each damping ratio;
- no constraints related to residues and residual terms.

To start the iterative process, each variable must have an initial estimate and an initial increment. The initial estimate may be obtained through one of the alternative methods available to the user. In other words, Goal Programming improves the accuracy of the modal parameters estimated through a modal parameter estimation method. The user must define the number of FRFs and the number of modes to be identified within the frequency range of analysis. In some cases, especially when the system presents high modal density, the user must use some tool to choose the correct number of modes. Two other possibilities are to give different weights to different parts of the FRFs and to exclude some parts of the FRFs. A tolerance value may be given, such that the program stops running if all parameter differences between two consecutive iterations are lower than this tolerance.

NUMERICAL SIMULATIONS

In order to study the behavior of the developed technique for estimating modal parameters, some numerical tests were done with a set of simulated FRF, each one with three vibration modes. To allow a comparison, the parameters were also estimated through the Orthogonal Polynomials Method (OP). The FRFs were selected to represent two complex practical cases:
- A: Lightly damped structure, with high modal density;
- B: Heavily damped structure, with high modal density.

Table 1 shows the values of modal parameters for these two cases. The FRFs were synthesized according to their theoretical expression (Eq. 4). In order to simulate experimental functions, two levels of noise (5 and 10%) were added to the theoretical FRFs. The level of noise is the relation between the RMS value of the noise and the RMS value of the theoretical FRF.

The accuracy of the estimation is expressed through the relative error between the estimated modal parameter values and the real ones (for natural frequencies, damping ratios and residues). In tables 2 and 3 these errors are shown for the results obtained through OP and through Goal Programming.
(GP) algorithm respectively. The OP results were used as starting values for the GP optimization algorithm.

In case A without noise, the parameters estimated through the two algorithms are exactly equal to the real ones. When some noise is added, the accuracy of the results obtained through GP is higher than that of OP, mainly with respect to damping and residues.

Case B represents a heavily damped system with high modal density. Therefore, it is more difficult than case A, and the accuracy of damping and residues estimated through GP was always greater than that of OP. When 10% noise was added, OP was not able to identify the three existent modes.

TABLE 1 - Value of Modal Parameters of the Studied Cases.

<table>
<thead>
<tr>
<th></th>
<th>(f_1) (Hz)</th>
<th>(f_2) (Hz)</th>
<th>(f_3) (Hz)</th>
<th>(\xi_1) (%)</th>
<th>(\xi_2) (%)</th>
<th>(\xi_3) (%)</th>
<th>(\Delta A/s/kg)</th>
<th>(\Delta A/s/kg)</th>
<th>(\Delta A/s/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20.</td>
<td>20.2</td>
<td>20.5</td>
<td>0.7</td>
<td>0.5</td>
<td>0.8</td>
<td>(-4.e-4)i</td>
<td>(-1.e-3)i</td>
<td>(-1.e-3)i</td>
</tr>
<tr>
<td>B</td>
<td>20.</td>
<td>20.4</td>
<td>21.</td>
<td>2.</td>
<td>5.</td>
<td>3.</td>
<td>(-4.e-4)i</td>
<td>(-1.e-3)i</td>
<td>(-1.e-3)i</td>
</tr>
</tbody>
</table>

TABLE 2 - Relative Errors of Modal Parameters Estimated Through OP (%).

<table>
<thead>
<tr>
<th>LEVEL OF NOISE</th>
<th>0 %</th>
<th>5 %</th>
<th>10 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE</td>
<td>(f_1)</td>
<td>(\xi_1)</td>
<td>(\Delta A)</td>
</tr>
<tr>
<td>A</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>B</td>
<td>0.11</td>
<td>12.41</td>
<td>31.75</td>
</tr>
</tbody>
</table>

* the method was not able to identify the three existent modes.

Figure 1 shows, for case A with 10% noise, a comparison between the theoretical FRF and the FRF synthesized through OP. Figure 2 shows, for the same case, a comparison between the theoretical FRF and the FRF synthesized through GP. One can see in these figures that the curve fitted using GP is more accurate than the one fitted using OP.
TABLE 3 - Relative Errors of Modal Parameters Estimated Through GP (%).

<table>
<thead>
<tr>
<th>LEVEL OF NOISE</th>
<th>CASE</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>f</td>
<td>ξ</td>
<td>,A</td>
</tr>
<tr>
<td>0%</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>0.36</td>
<td>8.73</td>
<td>21.17</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>0.17</td>
<td>4.33</td>
<td>11.03</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>0.07</td>
<td>1.32</td>
<td>4.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>2.56</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.36</td>
<td>8.73</td>
<td>21.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>3.37</td>
<td>5.32</td>
</tr>
</tbody>
</table>

Based on the comparisons above, the authors concluded that the accuracy of the modal estimation using Goal Programming technique in complex cases is higher than that of OP, mainly with respect to damping ratios and residues. Concerning natural frequencies, both algorithms yield good results in all cases.

![Theoretical FRF and FRF synthesized through OP - case A with 10% noise.](image-url)
An important topic involving modal parameter estimation using GP is the starting point, which includes initial estimates for modal parameters and their initial increments. Some tests were performed to study the influence of the initial point on the final results. The authors concluded that in difficult cases, such as those presented above, the results may be dependent on the initial point. In these cases the initial increments for damping ratios and residues must be high in relation to their initial estimates to avoid the iterative procedure being bound to a local minimum.

FRFs of fixed offshore platforms usually present low modal density. However, in some cases this may not be true. For example, some fixed offshore platforms have local modes near to global modes, increasing its modal density. Besides, if the structure is damaged, it will lose symmetry and its FRFs may also present high modal density\(^9\). In these situations, estimates through GP will probably be much more accurate than others classical methods ones.

**EXPERIMENTAL TESTS**

Experimental tests were carried on a small scale model of a fixed offshore platform, and the measured FRFs were analyzed through OP and GP. The small scale model was built with PVC and ABS tubes and designed according to the Similitude Theory in a 1/45 geometric scale factor.
The main geometric characteristics of the model are presented in figure 3, which shows its frontal view and a typical cross section.

The experimental tests were performed in air on the structure by means of two non simultaneous random forces applied in point 13 in x-direction and in point 11 in y-direction, both in the second level of the structure (EL - 255.56 mm) (see figure 3). The response of the structure were measured by 32 accelerometers located in the nodal points of the four platform legs (16 in the x-direction and 16 in the y-direction). So, 64 FRFs were measured from the tests. The FRFs were obtained through a developed system which uses LabVIEW.

The analyzed frequency range was from 0 to 60 Hz, and six global vibration modes were identified (two flexural modes in x-direction, two flexural modes in y-direction and two torsional modes). Then, the modal parameters were estimated through OP and through GP, allowing a comparison of their results.

Fig. 3 - Frontal view and a typical cross section of the platform model, with its geometric characteristics.
Table 4 presents this comparison for natural frequencies and damping ratios.

**TABLE 4 - Comparison of Natural Frequencies and Damping Ratios Estimated Through OP and GP.**

<table>
<thead>
<tr>
<th>Vibration Modes</th>
<th>OP Results</th>
<th>GP results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Natural Freq. (Hz)</td>
<td>Damping Ratio (%)</td>
</tr>
<tr>
<td>Ist Flexural X</td>
<td>8.14</td>
<td>1.26</td>
</tr>
<tr>
<td>Ist Flexural Y</td>
<td>8.58</td>
<td>1.51</td>
</tr>
<tr>
<td>Ist Torsional</td>
<td>13.08</td>
<td>1.40</td>
</tr>
<tr>
<td>2nd Flex. X</td>
<td>27.35</td>
<td>2.05</td>
</tr>
<tr>
<td>2nd Flex. Y</td>
<td>29.09</td>
<td>2.49</td>
</tr>
<tr>
<td>2nd Torsional</td>
<td>34.04</td>
<td>1.48</td>
</tr>
</tbody>
</table>

One can see from Table 4 that the results are very similar. The maximum variation of natural frequencies was 0.25% and the maximum variation of damping ratio was 14.2%. Once the experimental test was performed in air, its damping ratios are low. If the tests were performed in sea, these damping ratios would be higher (4% - 6%) due to the fluid-structure and the soil-structure interaction. In this case GP would probably estimate the modal parameters much more accurately than OP. It is important to mention that flexural modes in orthogonal directions do not appear in the same FRF. For example, the three y-direction flexural modes do not appear in a FRF which response is measured in x-direction.

In order to compare the accuracy of the used algorithms, the total squared error between the FRFs synthesized through each algorithm and the experimentally measured FRFs were computed, and their relation is shown below.

\[
\frac{\text{Total Squared Error With OP}}{\text{Total Squared Error With GP}} = 1.15
\]

This value proves that GP estimation was a little better than that of OP.

The Modal Assurance Coefficient (MAC) was computed to compare the mode shapes estimated through the two algorithms and the numerical mode shapes (obtained through a finite element code). The MAC correlates two set of mode shapes. The main diagonal of the MAC matrix indicates how similar the two sets of mode shapes are. The closer one main diagonal value of MAC matrix is to the unity, the more similar the two compared modes are. The values out of the main diagonal of MAC matrix correlate shapes of
different vibration modes. If these modes are really orthogonal, their MAC values must be close to zero.

There are some techniques that use MAC matrix for detecting structural damages\(^9-10\). One must have an experimental-numerical MAC matrix which main diagonal values are close to the unity and the other elements are close to zero, before damage. After damage, the MAC matrix will present different values which may allow the damage detection\(^9-10\).

The MAC matrix correlating numerical mode shapes and mode shapes estimated through OP and the MAC matrix correlating numerical mode shapes and mode shapes estimated through GP are shown in (7) and (8) respectively.

\[
\text{MAC MATRIX (Numerical x OP)} = \begin{bmatrix}
0.979 & 0.002 & 0.000 & 0.245 & 0.004 & 0.004 \\
0.009 & 0.991 & 0.001 & 0.002 & 0.243 & 0.000 \\
0.002 & 0.000 & 0.986 & 0.001 & 0.003 & 0.277 \\
0.228 & 0.004 & 0.000 & 0.968 & 0.000 & 0.017 \\
0.001 & 0.303 & 0.002 & 0.004 & 0.974 & 0.000 \\
0.001 & 0.000 & 0.403 & 0.001 & 0.000 & 0.951 \\
\end{bmatrix}
\] (7)

\[
\text{MAC MATRIX (Numerical x GP)} = \begin{bmatrix}
0.985 & 0.001 & 0.011 & 0.241 & 0.001 & 0.004 \\
0.005 & 0.995 & 0.001 & 0.002 & 0.248 & 0.000 \\
0.003 & 0.003 & 0.974 & 0.001 & 0.001 & 0.271 \\
0.180 & 0.000 & 0.002 & 0.969 & 0.001 & 0.017 \\
0.000 & 0.303 & 0.000 & 0.003 & 0.986 & 0.000 \\
0.002 & 0.001 & 0.389 & 0.004 & 0.004 & 0.946 \\
\end{bmatrix}
\] (8)

One can note that these matrixes are very similar. Values in the main diagonal are close to the unity and among all elements out of the main diagonal, only six are not close to zero. These elements correlate the first x-flexural mode with the second x-flexural mode, the first y-flexural mode with the second y-flexural mode and the first torsional mode with the second torsional mode. However, only two of the total number of degrees of freedom (six) were measured (displacement in x-direction and displacement in y-direction) in the experimental test. To study the influence of the omission of the other four degrees of freedom in the MAC values, the MAC matrix correlating numerical-numerical mode shapes, using only displacements in x-direction and y-direction was computed and presented in (9).
The analysis of the numerical-numerical MAC matrix shows that the same elements are not close enough to zero. So, the numerical-numerical MAC matrix using all six degrees of freedom was computed and all out-of-main-diagonal elements were very close to zero. This proves the great importance of the consideration of all degrees of freedom when comparing experimentally estimated mode shapes with numerical mode shapes. However, in practice the measurement of rotational degrees of freedom is very difficult, and in some cases even impossible. The solution is to perform the expansion of the measured mode shapes using numerical results. There are several expansion algorithms and the complement of this work, in a near future, will be the use of some of them before computing the MAC matrix.

CONCLUSIONS

The estimation of modal parameters using the Goal Programming optimization technique was developed to improve the accuracy of modal parameter estimation. Numerical simulations representing complex practical cases were performed and the modal parameters estimated using Goal Programming algorithm were more accurate than those estimated through the Orthogonal Polynomials Method.

Modal parameters estimated from an experimental test in a small scale model of a fixed offshore platform proved once more the high accuracy of the developed algorithm.

A comparison between experimentally estimated and theoretical mode shapes through the Modal Assurance Coefficient (MAC) shows the prime importance of considering rotational degrees of freedom. An expansion algorithm must be applied to the experimentally measured mode shapes for allowing a better correlation between these ones and numerical mode shapes.
REFERENCES