Ultimate capacity of ships’ hulls in combined bending

Dominique Bégin\(^{(1)}\), Tadeusz Jastrzębski\(^{(2)}\), Maciej Taczała\(^{(2)}\)

\(^{(1)}\)Bureau Veritas, France
\(^{(2)}\)Technical University of Szczecin, Faculty of Maritime Technology, Poland

Abstract

The paper deals with the problems of ultimate longitudinal strength. A method is presented for evaluation of the ultimate capacity. Theoretical considerations are given concerning interaction of vertical and horizontal bending moments. The paper is concluded with the results of numerical calculations of the ultimate bending moments for various combinations of horizontal and vertical curvatures and moments.

1. Introduction

The conventional assessment of the ship hull girder longitudinal strength is based on comparison of maximum elastic stress in the hull section with allowable stress, specified as a fraction of yield stress. Thus the elastic section modulus calculated for horizontal axis is treated as a measure of longitudinal bending strength. This attitude, though widely applied, does not provide with information concerning resistance of the ship hull in extreme conditions. This can be achieved by evaluation of ultimate capacity thus becoming an important point in ship structural rational design.

Beginning of research work on the ship hull ultimate capacity dates back to 1965 when Caldwell [3] considered collapse of the ship hull modelling structural elements as panels and determining ultimate load for each panel based on either analytical method or experiment. In his approach post-buckling behaviour was not accounted for. This is a serious drawback as the structural elements fail sequentially and now of ship collapse some are in pre-buckling and others in post-buckling range. Smith [19] considered a panel as a number of beams with...
attached plating and took into account beam-column and plate failure modes. Rutherford and Caldwell [18] performed a thorough analysis of Energy Concentration, a tanker which was broken during discharging oil in the port. They performed finite element analysis of each stiffener and applied the results to evaluation of the ultimate bending moment. They studied influence of various factors on the ultimate capacity; material properties, fabrication factors, corrosion and lateral pressure. Similar investigations were also made by other authors [1,4,5,9,17]. The ultimate load is thoroughly discussed in [10,11] where available analytical techniques are presented and obtained results are compared with those from experimental investigation.

Ships’ hulls in combined bending were investigated in [6,13]. Gordo and Soares [8] proposed an interaction equation to account for interaction between horizontal and vertical bending moments.

Since the ship structure is complex direct calculations using the finite element method are hardly applicable as being too costly and in most cases unprofitable due to a very large model in terms of number of finite elements. Some efforts have been undertaken; examples include papers by Kutt et al. [12] where finite element non-linear calculations were presented for various hull configurations, and by Bendiksen [2] who performed simplified finite element calculations employing plastic node method.

Only few papers refer to experimental investigation. Nishihara made tests using models corresponding to typical cross-sections of a tankers and bulk-carriers [16]. Experimental investigation was performed also by Mansour et al. [15] and Thayamballi et al. [20].

The most important load effect considering the longitudinal strength is vertical bending moment. For many types of ships, however, the combined effect of the vertical and horizontal bending moments is important.

2. Calculation procedure

The problem of determination of the ultimate capacity calls for numerical method taking into consideration both geometrical non-linearity (buckling) and plasticity. Full finite element analysis is practically difficult for performance due to excessive number of finite elements which must be used for idealisation of the structure. A model of the beam subject to bending is considered instead and the structure is decomposed into a number of components - stiffened panels and unstiffened plates which are treated as independent of each other (Fig. 1).
The behaviour of the components is easy to determine in the case of components subject to tension; their behaviour directly follows the material model, usually elastic-perfectly plastic. For components subject to compression also buckling must be taken into account. The behaviour of the compressed components is approximately evaluated according to simplified formulae based on theoretical investigation supported by results of the nonlinear finite element analysis of stiffened plates. Plate, beam-column, flexural-torsional and local web modes of failure are taken into account (Fig. 2).

In the case of vertical or horizontal bending the procedure is applied based on increasing the curvature of the beam. In each load increment using the Newton-Raphson method the equilibrium state (position of the neutral axis) is established as well as the actual value of the bending moment is evaluated by integration over the cross-sectional area. After yielding and/or buckling of the considerable number of components the moment vs. curvature curve is no longer ascending and the value of the ultimate bending moment is obtained. Calculations are performed in hogging and sagging loading conditions to obtain complete
knowledge of the overall strength. Interpretation of the loading condition - hogging or sagging - is easy in the case of the vertical bending. In combined bending the sign of curvature should rather be considered to avoid ambiguity. The position of the neutral axis – in terms of the inclination angle with respect to the base line is kept constant throughout the analysis. Effects of shear and torsion are neglected in the present analysis.

As earlier mentioned there are poor data available on the damage of real structures due to the loss of the ultimate strength. This is why the verification of the computational procedure is very restricted and it is necessary to make comparisons to complex numerical calculations or to laboratory tests. The computer code used in the present analysis was verified against the actual value of the ultimate bending moment for the Energy Concentration and reference cases presented by Nishihara [1].

In the case of combined bending two different approaches have been advocated Gordo and Soares [6] proposed the method in which the total curvature is decomposed into horizontal and vertical parts according to the formulation:

\[ C_x = C \cos \theta \quad \text{and} \quad C_y = C \sin \theta \quad (1) \]

where

\[ C = \sqrt{C_x^2 + C_y^2} \quad (2) \]

is the total curvature and \( \theta \) is the angle between neutral axis and horizontal axis. Ratio between them is kept constant throughout the analysis. On the other hand Mansour et al. [13] presented the results of calculations of ship hulls ultimate strength obtained applying the assumption that the two components of the total bending moment increase with respect to a certain parameter. The proposed the following relationship between vertical and horizontal bending moments, after a similar equation by Mansour and Thayamballi [14],

\[ m_v + km_H = 1 \quad \text{or} \quad m_H + km_v = 1 \quad (3) \]

dependant on the ratio between \( m_v \) and \( m_H \). In eqn (3) \( m_v \) and \( m_H \) have the following definitions

\[ m_v = \frac{M_v}{M_{vult}}, \quad m_H = \frac{M_H}{M_{Hult}} \quad (4) \]

where \( M_v \) is vertical bending moment, \( M_H \) – horizontal bending moment, \( M_{vult} \) – vertical ultimate bending moment and \( M_{Hult} \) – horizontal ultimate bending moment and \( k \) is a factor dependant on cross-sectional areas of deck, bottom and sides.

Using their approach additional iteration process must be performed to establish the position of the neutral axis not only in terms of shift from original (or previous) position but also in terms of rotation. Necessary condition is formed by enforcing either the defined horizontal/vertical bending moments ratio or keeping the either moment constant throughout the analysis.
3. Models for analysis

For the analysis the following bulk carrier with typical cross-section has been taken. The main particulars of the ship are: length - 256.57 m, breadth - 43.00 m, height - 23.90 m. The cross section of the analysed ship together with axis of the coordinate system and denotations of the bending moments is shown in Fig. 3.

![Fig. 3. Cross section of bulk carrier](image)

4. Results of calculation

4.1. Ultimate capacity in vertical and horizontal bending

Analysis of results begins with consideration of the simple cases of vertical and horizontal bending. In Fig. 4 and 5 results are presented in the form of bending moment vs. curvature diagram for vertical and horizontal bending, respectively. It can be seen that the horizontal component is equal to 0 in case of the vertical bending as the cross section is symmetrical with respect to the vertical axis. For horizontal bending a non-zero value of the vertical component appears due to lack of symmetry with respect to the horizontal axis. The presented solutions have been obtained applying the procedure, where the total curvature is increased and the appropriate ratio between horizontal and vertical curvature is maintained ($c_H/c_V=0$ for vertical and $c_V/c_H=0$ for horizontal bending). Solution in the case of constant ratio of horizontal to vertical bending moment ($M_H/M_V=0$) is the same as for $c_H/c_V=0$ - Fig. 4, whereas obtaining solution for $M_V/M_H=0$ is impossible as there always must appear a non-zero value of vertical moment.
4.2. Ultimate capacity in combined bending for constant ratio of curvatures

A series of diagrams is presented in Fig. 6-9 to illustrate the behaviour and the ultimate capacity of the analysed ship’s hull in the case of combined bending for ratios of horizontal to vertical curvatures ranging from 0.1 to 0.9. This type of calculation corresponds to the procedure proposed in [6].

It can be seen that the ratio of the horizontal to vertical component of the total bending moment depends on the ratio of curvatures which is an obvious conclusion. The ratio of the moments is not constant, however, nor even a regular tendency can be found.

In Fig. 10 distribution of the ultimate bending moment with respect to $c_H/c_V$ is shown. A minimum of the function can be observed around $c_H/c_V=0.4$. 
4.3. Ultimate capacity in combined bending for constant ratio of bending moments

Similarly to the previous case a series of computations was done for the case of combined bending for moment ratios ranging from 0.1 to 0.9. This type of calculation corresponds to the procedure proposed in [13]. Results are given in Fig. 11-14.

Fig. 8. Moment vs. curvature curve for $c_H/c_V=0.6$

Fig. 9. Moment vs. curvature curve for $c_H/c_V=0.9$

Fig. 10. Variation of ultimate bending moment with respect to $C_H/C_V$

Fig. 11. Moment vs. curvature curve for $M_H/M_V=0.1$

Fig. 12. Moment vs. curvature curve for $M_H/M_V=0.3$
4.4. Comparison of the two computational procedures

For comparison of the results obtained applying the two described computational procedures, the cross section was evaluated for both of them. To find the correspondence between them the ratio between bending moments \( r_M \) was set to

\[
r_M = \frac{M_{HW}}{M_{YS} + M_{YW}}
\]

where \( M_{HW} \) is a horizontal wave bending moment, \( M_{YS} \) - vertical still water bending moment and \( M_{YW} \) - vertical wave bending moment. To keep the same bending moment ratio at least in the linear range when computing the cross section for constant ratio of horizontal to vertical curvatures, the initial position of the neutral axis in terms of rotation with respect to position of coordinate axes is evaluated based on

\[
r_C = \tan \theta = \frac{M_{HW}}{M_{YS} + M_{YW}} \frac{I_y}{I_z}
\]

where \( I_y \) is a moment of inertia with respect to the horizontal neutral axis and \( I_z \) - moment of inertia with respect to the vertical neutral axis.

The values for the analysed cross section are as following: \( M_{HW}=1269000 \text{ kNm}, \ M_{YS}=1305000 \text{ kNm}, \ M_{YW}=2198000 \text{ kNm}, \ I_y=204.63 \text{ m}^4, \ I_z=501.65 \text{ m}^4. \) With
these values we have \( r_M = 0.36 \) and \( r_c = 0.15 \). Ultimate bending moments are the following: for constant curvature ratio: \( M_{ul} = 5298000 \) kNm and for constant curvature ratio \( M_{ul} = 5113000 \) kNm. The difference is 3.5% which is not very large but may be meaningful. Worth noting is also that the position of the neutral axis at collapse is different than initial one what is presented in Fig. 16.

5. Summary and conclusions

Two different solution procedures for analysis of ship hulls in combined bending are presented, discussed and applied for the case of a bulk carrier. The two approaches use different loading schemes resulting in different kinematics of deformations and, consequently, stress distribution over the cross section and the value of the ultimate bending moment. For the demonstrated case the difference was not large, however, for other cases the difference can be greater. Rotation of the neutral axis when increasing loading should be taken into consideration investigating the problem of ultimate capacity in combined bending. Moreover, the simplifying assumption was made after [13] that the ratio of the horizontal to vertical bending moments remains constant throughout the analysis. It would be worth considering how the components of the bending moment change when increasing and what effect it has on the ultimate bending moment.

References


