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Optimum Design of a Planing Boat's Hull Form

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Abstract

This paper is concerned with an approach to obtain the minimum resistance hull form parameter set **x** (X_1, X_2 ...., X_{11}) to a planing boat model from many towing test data. About ten X_i s, which are necessary to form a hull, have been transformed into the dimensionless numbers for the law of similarity. Total resistance coefficient at a planing condition is selected as objective function and its minimum is searched under restrictive conditions. X_i 's correlations in a hull form, X_i being within experimental data and near their mean values, and other requirements are taken into the conditions. Penalty terms are added to make modified objective function. The terms which are made from the above conditions and work to gain a larger number when the search is done far from the optimum. Against an initial **x** in the above feasible region, the modified objective function is successively minimized to get the optimum **x**, which means successive reformation of hull. A changed principal dimension affects the other ones because of their requiring correlations in a hull form, which will perplex designers. How to keep the correlation conditions in satisfaction will be proposed as a successful example.

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1 Introduction

Theoretical approach of a hull form design or development is now difficult because of changing floating positions of a craft in the semi-planing and planing condition with a tub on transom . To develop hull forms, a series towing tank test will be done (see F igure 1). Measured C t, $\Delta\theta$ and ΔD of a model on still water give resistance and floating position at a Froude number F_∇ (=V/($\nabla^{1/3}$ g) $^{1/2}$). C t is total resistance coefficient (resistance/ {(1/2) ρV^2 $\nabla^{2/3}$ }). $\Delta\theta$ (minutes) is trim angle change as positive for bow up from initial trim angle θ . ΔD is non-dimensional draft change at the deepest point of transom (draft change / b_1) as positive for decreasing draft. Hull form parameters etc. are set as follows.

X_1	: chine height at ord. 10.	(stern)	$X_1 = h_1 / b_1$
X_2	: chine height at ord. 5.	(midship)	$X_2 = h_2 / b_1$
X_3	: chine height at ord. 2.		$X_3 = h_3 / b_1$
X_4	: keel height at ord. 2.		$X_4 = H_3 / b_1$
X_5	: keel height at ord. 0.	(bow)	$X_5 = H_4 / b_1$
X_6	: total length.		$X_6 = L / b_1$
X_7	: half width at ord.5.	(midship)	$X_7 = b_2 / b_1$
X_8	: half width at ord.2.		$X_8 = b_3 / b_1$
X_9	: curvature of transom.		$X_9 = \ell / b_1$
X_{10}	: initial trim angle.		$X_{10} = \theta$.
X_{11}	: displacement volume.		$X_{11} = \nabla / b_1{}^3$
X_{12}	: $X_6{}^2$'s non-negative term by eqn (6).		
X_{13}	: $X_6 \; X_{10}$'s non-negative term by eqn (6).		
X_{14}	: $X_{10}{}^2$'s non-negative term by eqn (6).		

(see Figure 2)

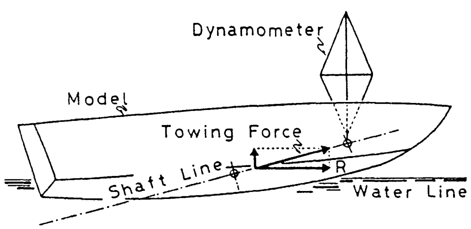

Figure 1 : Towing Tank Test

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2 Data Summarization

Empirical formlae from model test data and classification of these models will be tne best for data summarization.

2.1 Empirical Formlae

$$Ct = a_0 + a_1 X_1 + a_2 X_2 + \ldots\ldots\ldots\ldots + a_{11} X_{11} \tag{1}$$

with the regression coefficient vector $\{a\}$ that is given

$$\{a\} = \{ [X]^T \ [X]\}^{-1} [X]^T \ \{Ct\}$$

Matrix [X] consists of n sets of $[1, X_1, X_2 \ldots\ldots\ldots, X_{11}]$. C t at a F_∇ have n data. n means number of towing tests and n = 78 in this paper.

$\Delta\theta$ and Δ D have an equation form similar to this, which give floating position.

The limit of application for regression equations should be set. About 95% data under normal distribution are included in the 11 dimensional ellipse as follows:-

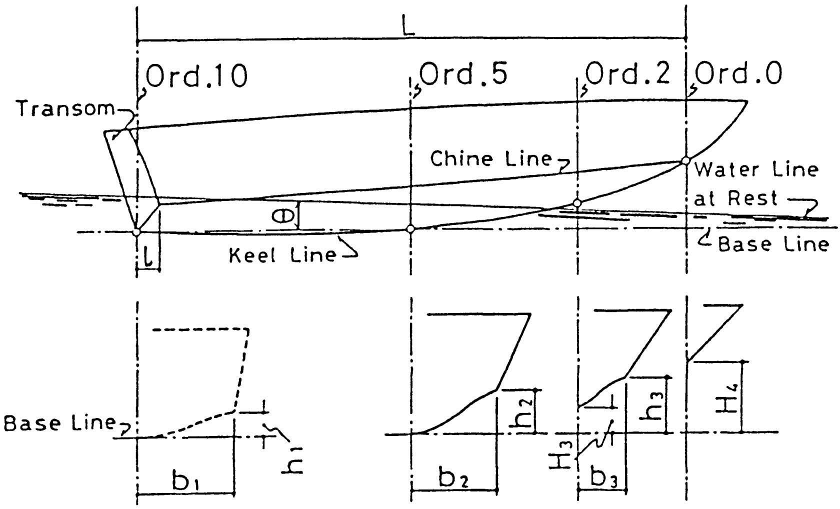

Figure 2 : Principal Dimension

$$\{\Delta X\}^{T} \ \mathrm{B}^{-1} \ \{\Delta X\} \leqq x^{2} \ \ (\mathrm{f},\ \mathrm{a}) \tag{2}$$

f =11, a=0.05, then x^2 (f , a) = 19.68. **B** is variance-covariance matrix made from hull form parameters. ΔX is deviation vector composed by element X_i - $\overline{X}_i$.

$\overline{X}_i$ means mean value of X_i. As another limitation, f is used instead of x^2 distribution. Then about 75% data are included under free distribution [1]. The correlation conditions between hull form parameters are kept in satisfaction by eqn (2) because of B. Let us classify hull form data on visible coordinates system by applying principal component analysis to know multi-dimensional data-characteristics.

2.2 Classif ication

Figure 3 shows the classification of models on coordinates Z_1 - Z_2 of principal component analysis. The first principal component axis Z_1 has the largest distribution of the data. The second one Z_2, which is perpendicular to Z_1 axis, has second large distribution and so on. Each model's score on this coordinates system can be calculated by:-

$$Zi = \sum_{\nu=1}^{11} \ \beta i\nu \ . \ \frac{X\nu - \overline{X\nu}}{\sigma\nu}, \ \mathrm{i}=1,2, \text{etc..} \tag{3}$$

$\beta\ i\nu$ is ν-th element of i-th eigenvector of [R] that is correlation matrix made from n sets of $[X_1\ ,\ X_2\ , \ldots\ldots, X_{11}]$ data. $X\nu$ and σ_ν are ν-th mean value and standard deviation of n $X\nu$ -data respectively. The calculations are done for major principal components Z_1, Z_2 etc. by eqn (3) to get classi f ication of the data on Z_1 - Z_2 plane etc., which enables the data characteristics to show distinguishably on such lower dimensional coordinates like Figure 3. Thirteen kinds of tested models are put onto the thirteen groups of points which are composed of near six ones respectively (see Figure 4). The center of gravity and displacement of a 2.5m length model is set six ways. We call the model group No.1 to 6 the first cluster (1), No.7 to 12 the second cluster (2) and so on. A hull form is pictured on its cluster and thirteen kinds of clusters or models form a shape like ellipse.

Transom's half width b_1 equals to 1, which is selected as the unit of its hull form dimensions. Good examples showing hull form's characteristics are
(6) 6th cluster containing hull forms having smaller transom or longer hull,
(12) 12th cluster containing ones having larger transom or shorter hull,
(1) 1st cluster containing ones having larger initial trim,

(10) 10th cluster containing ones having smaller initial trim,
(9) 9th cluster containing ones having higher chine height and
(11) 11th cluster containing ones having lower chine height.

On the next section the optimun hull form parameter set (X_1 , X_2, ..., X_{11}) which performs the minimum resistance will be searched by SUMT (Sequencial Unconstrained Minimization Technique).

3 Optimum Hull Form

Search for the hull form parameter set vector **x** (X_1 , X_2, ..., X_{11}) which causes hull to have the minimum resistane is an optimum problem. A problem "minimization of objective function f (**x**) under restrictions of gi (**x**) $\geqq 0$, i = 1,2, ..., N" is transformed to the one "minimization of p(**x**) without restriction". p (**x**) is the modified objective function. N means number of restrictions.

$$p(\mathbf{x}) = f(\mathbf{x}) + r_k \cdot \sum_{i=1}^{N} \frac{1}{gi(\mathrm{x})} \tag{4}$$

r_k is purturbation coefficient at k step. $r_1 \geqq r_2 \geqq r_3$ .... > 0. At the final step, p (**x**) 's minimum value nearly equals to f (**x**) 's one because of the least r_k 's value. A set of N restrictions composes the feasible region's barrier by sum of $1/g_i$ (x) and the second term of eqn (4) works as the penalty to gain a larger value when the search is done far from the optimum. Initial **x** in the feasible region have many approachs to the minimum p (**x**) such as the steepest descent method, conjugate gradient one, etc. [2] . In this paper, Zangwill's conjugate gradient method without differential calculus is used, which enables the calculation steps to be stable [3] . $|\Delta\ p/p\ (\mathbf{x}, r_k)| \leqq 10^{-4}$ is set for converge coefficient. Initial value of r_k is set at 10^{-3} and decreases 10^{-1} a step until k's fifth step.

Each term of eqn (4) are set as follows.

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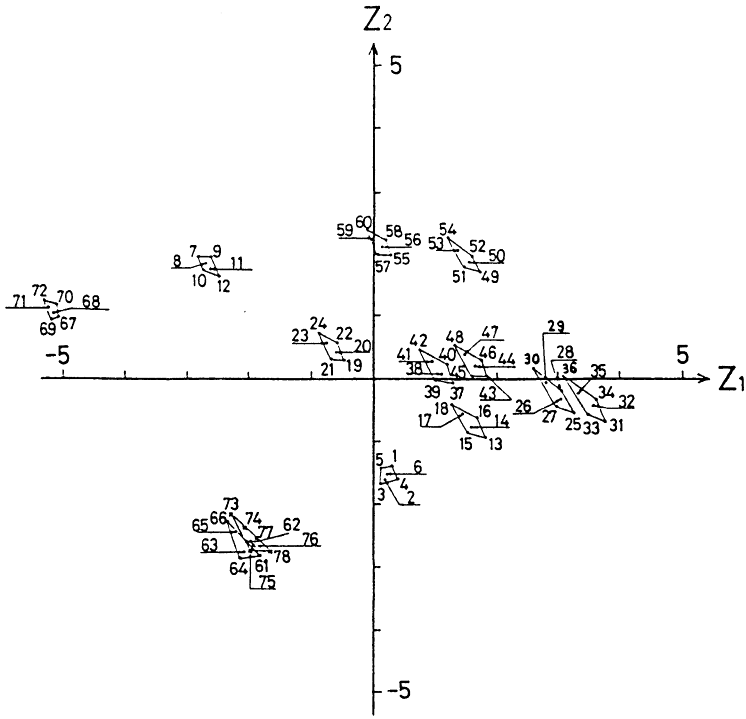

Figure 3 : Clusters

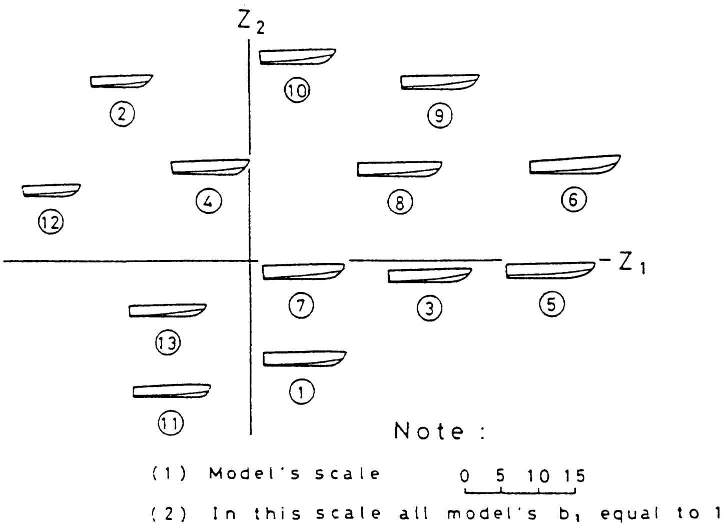

Figure 4 : Models

3.1 Objective Function

As one of the examples, f (x) = C t at F_∇ = 3.5 expressed by a quadratic expression as eqn (5). F_∇ = 3.5 is such a number in planing condition as velocity V=46kt under the initial displacement volume ∇=100 m^3 .

$$f(x) = a_o + \sum_{i=1}^{14} a_i \; x_i \; . \qquad (5)$$

The variables x_1 through x_{11} are modified ones to take positive values like ones of linear programming problem.

$$x_i = X_i - \overline{X_i} + a_i \; \sigma_{xi} \; , \qquad I=1,2, \ldots\ldots., 11 \qquad (6)$$

The mean value $\overline{X_i}$ and standard deviation σ_{xi} in eqn (6) are made by n X_i s of hull form data. a_i is the constant that is the multiplier to give the inferior and superior limits of x_i. . a_i =2. 0 except 12th cluster's a_7 and a_{10} . a_7 = 2.5 and a_{10} = 3. 0 in 12th cluster. The quadratic variables x_{12} , x_{13} and x_{14} are also taken positive values as eqn (6)

3.2 Restrictive Conditions

(1) Applicable range of the equation:

$$\{\Delta x\}^T \; \mathbf{b}^{-1} \; \{\Delta x\} \leqq x^2 \; (f, a). \qquad (7)$$

f = 14 and a = 0. 05. Under the transformed bull form parameters by eqn (6) and additional three quadratic terms, deviation vector Δx and variance-covariance matrix **b** are set in eqn (7). **b** keeps correlations between hull form parameters.

(2) Each hull form parameter etc. keeps near mean value.

$$0 \leqq x_i \leqq 2 \; a_i \; \sigma_{xi} \, , \qquad I= 1,2 \ldots\ldots\ldots.14. \qquad (8)$$

(3) Hull form has the same characteristics with the initial hull form's ones
For example, ⑥6th cluster's similar hull forms are conditioned by the outward planes that contain No.32 test model at center (see Figure 3). Each plane is recognised by calculating its intercepts on axes Z_1, Z_2 and Z_3. c and d are the intercept points expressed by (c, 0) on Z_i axis and (0, d) on Z_j axis on Z_i - Z_j plane. Then, six data from No.31 through 36 form a hexahedron around No.32 point on these coordinates. The planes limiting the hexahedron are expressed by eqn (9).

$$1 - \left(\frac{Zi}{c} + \frac{Zj}{d}\right) \geqq 0,\ i, j=1,2 \text{ and } 3. \tag{9}$$

Scores of Z_4 are limited to the ones from the lowest value e to the highest one f.

$$e \leqq Z_4 \leqq f. \tag{10}$$

4) Maximum trim angle change level
Dynamic lift appears remarkably beyond about F_∇ = 2.5. As increase in F_∇ from about 2.5 up to 3.5, the stern rises back to the point of initial draft at rest and goes further up. $\Delta\theta$ in planing condition decreases than that of semi-planing condition and keeps ups and downs about 300 and 150 minutes. This causes both wetted surface area and Ct to become small. $\Delta\theta$ at F_∇ = 2.5 is selected for controlling the maximum $\Delta\theta$ level. The correlation coefficients among C t at F_∇ = 3.5 and $\Delta\theta$ at F_∇ = 2.5, 3. 0 and 3.5 equal to -0.76, -0.71 and -0. 60 respectively. $\Delta\theta$ at F_∇ = 2.5 has not only the strongest relation with C t but also shows the maximum in $\Delta\theta$ - C t curve.

The maximum trim angle change level is taken at intervals of 30 minutes such as 270, 240, 210 minutes and so on, by using the quadratic expression as eqn (5) to $\Delta\theta$ at F_∇ = 2.5.

4 Result

4.1 Resistance Performance

Under controlling the maximum trim angle change $\Delta\theta$ at F_∇ = 2.5 two resistance performances of the optimum hull forms from ⑥6th cluster are obtained as shown in Figure 5. The performance for controlled $\Delta\theta$ at $F_\nabla = 2.5$ less than 180 minutes by solid line, while the one less than 150 minutes by dotted line. F_∇ - C t curves shows that the maximum trim angle change levels 180 and 150 minutes are kept in the range of $F_\nabla \geqq 2.5$. The former initial trim angle θ =-52 minutes compared with the later one -58 minutes. The similar discussions about all clusters show that the hump in F_∇ - C t curve takes lower value while higher one at $F_\nabla = 3.5$ with the stronger limitation of the levels. The hump is got at $F_\nabla = 1.0$ at which the maximum wavemaking resistance is obtained.

4. 2 Limitation of maximum trim angle change

After presuming the resistance performances against the optimum hull forms of clusters from (1) through (13), quadratic interpolations were done. The obtained thirteen curves are shown in Figure 6. The abscissa is taken as $\Delta\theta$ at $F_\nabla = 2.5$, the ordinate C t at $F_\nabla = 3.5$. The C t values from the first through 78th test models ones are plotted by "O" marks. "X" mark shows the mean value of them.

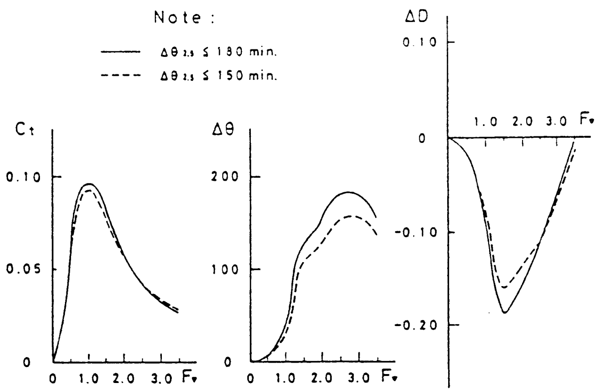

Figure 5 : Resistance Performance

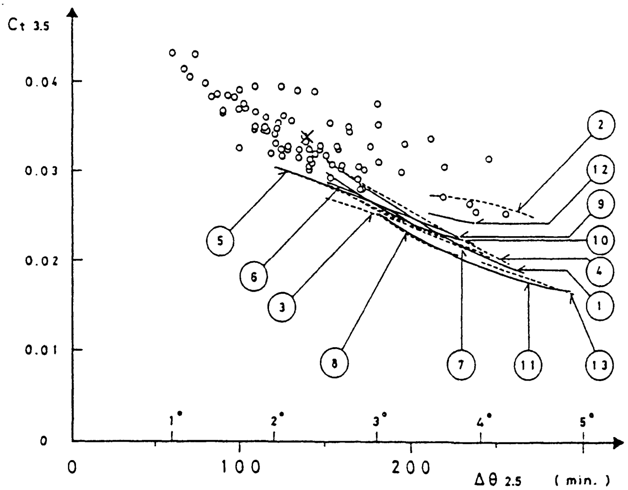

Figure 6 : $\Delta\theta - C\,t$

They have the general trend of higher values than the curves' values through the optimization technique. The right edges of the curves are the points in the case of $\Delta\theta$ free [4]. Curves show that stronger controlled $\Delta\theta$ causes C t to increase. The mean value of C t's increment becomes 0.0045/degree through linear interpolations to each curves' points.

4. 3 **Hull form**

With lowering the maximum trim angle changes such as 270, 240, 210 minutes and so on, the minimum resistance hull forms become smaller in displacement volume, longer in total length, higher in cross point of chine line with keel line at bow, and higher in chine line.

5 **Conclusions**

A method how to improve hull form's principal dimensions until getting the minimum resistance under required conditions is presented. Under successive decreases in maximum trim angle change level against the improved hull forms, the final hull forms become smaller in displacement volume, longer in total length, higher in cross point of chine line with keel line at bow and higher in chine line. With the stronger limitation of the maximum trim angle change, the

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hump in a F_{∇} - C t curve shows lower value and the total resistance coefficient in high speed range shows higher one. The mean value of the total resistance coefficient's increment becomes 0.00454/degree.

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