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Methods to assign the safe manoeuvre and trajectory avoiding collision at sea

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Abstract

The paper presents methods of assigning and results of simulating safe manoeuvre and trajectory to avoid a collision at sea. The process of avoiding a collision has been resolved in three ways: with defined CTPA (Collision Threat Parameters) and indispensable manoeuvre, assigning trajectory with using TAM (Table of Admissible Manoeuvres) and the third as a non-linear programming task with the use of a maximum principle. The results have proved the ability to estimate an optimal and save manoeuvre, trajectory of the own ship to avoid a collision when meeting a number of moving targets.

1 Introduction

At the time of development of anti-collision systems (ARPA) and their introduction on board of the merchant fleet a profound part of navigators' work connected with ship manoeuvring in the encounter situation has been transferred to the systems. This raised the safety of sailing and lead to a better utilisation of ships' resources. A proper usage of an anti-collision system besides a good staff training requires also an introduction of adequate algorithms that will help the navigator undertake the right decision.
With the development of modern electronic digital computers and digital techniques the contemporary trends of ship steering automation lead to an automatic prescription of the anti-collision manoeuvre together with a quantitative estimation of collision risk made on the basis of data supplied by the anti-collision system on board. It seems that the best solution would be the construction of a separate equipment that would constitute an expansion of the conventional anti-collision system. Such an equipment while not interfering with the basic anti-collision system on board should automatically determine a

collision avoiding manoeuvre or trajectory, supply the navigator with a legible display of the manoeuvre and in a most expanded version it should generate commands to the rudder and the main engine, that is steer the ship along a calculated, safe trajectory. An approach of this kind will allow to implant the anti-collision system into an integrated ship's navigating system. Up to date papers listed in the expert literature do not treat the above elements jointly but as separate problems.
The first research dedicated to the mathematical determination of a collision avoiding manoeuvre in a one-ship encounter was made in the late fifties and the beginning of the sixties (it had a connection with the problem of the triangle of velocities). Research in the seventies concentrated on respecting the „International Regulations to Prevent Collisions at Sea". The papers written at that time based on the so called manoeuvre diagrams that gave the possibility of choosing a save manoeuvre while respecting the sea rules in relation to the course angle and the actual distance between the ships. In the seventies and eighties first steps were undertaken to utilise the modern theory of steering (optimisation, non-linear programming methods, game theory, expert systems, fuzzy sets) to solve the collision avoidance problems. Contemporarily however, not many authors propose practical solving of the algorithms of automatic determination of the safe manoeuvre or the safe trajectory [1].
The article presents three methods of automatic determination of an anti-collision manoeuvre and trajectory using a microcomputer equipment enhancing the efficiency of an anti-collision system.

2 Determination of the collision avoiding manoeuvre utilising CTPA (Collision Threat Parameters Area) [3,5]

The method of specifying CTPA was created by A.Lenart [3]. In the conjugate system of co-ordinates of the position (X, Y) and movement (V_x, V_y) (Figure 1) A.Lenart [3] drew a relation,

$$Y = A_j X - B_j \tau \quad \text{where } A_j = \frac{X_j Y_j \pm D_{CPAj}\sqrt{D_j^2 - D_{CPAj}^2}}{X_j^2 - D_{CPAj}^2}, \quad B_j = A_j V_{Xj} - V_{Yj} \qquad (1)$$

in which Xj and Yj are the relative co-ordinates of j-th target and

$D_j^2 = X_j^2 + Y_j^2$,

D_{CPAj} - the distance value of the closest contact,

V_{Xj}, V_{Yj} - X and Y components of the velocity vector of the j-th target,

τ - conjugate time (e.g. 12 minutes).

Eq. 1 describes the locus of points for which D_{CPAj}= const in the conjugate system of co-ordinates of position (X, Y) and motion (V_x, V_y). On the other hand the locus of points in the conjugate system of co-ordinates for which the time of reaching the distance of the closest contact is constant (T_{CPAj}= const) can be determined on the basis of the circle equation (2):

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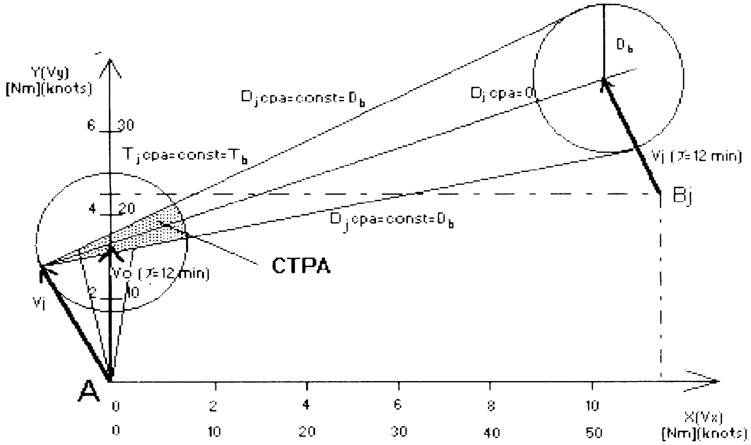

Figure 1: A CTPA area display.

$$\left[X-(V_{xj}+\frac{X_j}{2T_{CPAj}})\tau\right]^2+\left[Y-(V_{yj}+\frac{Y_j}{2T_{CPAj}})\tau\right]^2=(\frac{D_j\tau}{2T_{CPAj}})^2 \quad (2)$$

where the locus of points for the circle centre points lie on a straight line:

$$Y=\frac{Y_j}{X_j}X-(\frac{Y_j}{X_j}V_{xj}-V_{yj})\tau . \quad (3)$$

It is assumed that an target B_j is dangerous, when at the moment of observation t , assuming that $D_{CPAj}=D_b$ and $T_{CPAj}=T_b$ respectively we have:

$$D_{CPAj}<D_b \quad \text{and} \quad T_{CPAj}<T_b \quad (4)$$

where: the values D_b of the safe distance and T_b - the time to reach that distance are set by the system operator (at say D_b =1Nm, T_b =20 min).

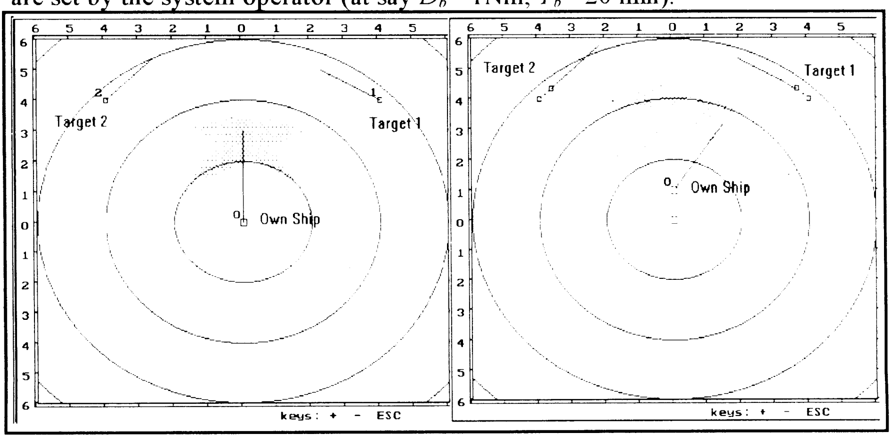

Figure 2: The practical realisation of CTPA area determined for two targets with the proposal of the safe manoeuvre.

Geometrically the above condition is satisfied when the end of the vector of own ship A positioned at (V_x, V_y) in the conjugate system of co-ordinates of position and motion is found outside of the CTPA danger area. In the calculation algorithm the time of overtaking the manoeuvre of own ship and of targets was also respected. A practical realisation of visualisation of the total CTPA area determined for two targets with the proposal of the safe manoeuvre is pictured on figure 2. One of the drawbacks of the method presented here is that although the algorithm prescribes the manoeuvre leading out of the situation of collision created at a given time, it doesn't however assume the possibility of the appearance of new targets and does not state the moment of return of own ship to its planned course.

3 Prescribing a safe ship trajectory basing on TAM (Table of Admissible Manoeuvres) [5]

Inserting a subroutine "indispensable manoeuvre" inside a program loop gives the possibility of describing the safe trajectory as a series of manoeuvres. The time of lasting of the individual manoeuvres is determined by the time of realisation of the real manoeuvre considering the dynamical characteristics of the ship. The "indispensable manoeuvre" is a relatively simple algorithm that lets us prescribe the kinematics manoeuvre of avoiding collision in a multi-target encounter. Prescription of necessary changes of course or/and velocity of own ship in a situation of an alien target encounter is realised through a geometrical analysis of the vectors of velocity of the ships. On this basis a relation is derived for the minimal change of the course of own ship to the left (*LB*) and to the right (*RB*) side of the ship at a given velocity (Eq.5) and at a given safe distance D_{CPAj}=Db.

$$\Delta RB \setminus LB = \pm\delta_{jb} + q_o^j - \arcsin\left(\frac{V_j}{V}\sin\left(\pm\delta_{jb} + N_j - \psi_j\right)\right) \tag{5}$$

The values of velocities V_+ and V_- limiting the range of admissible values at a given own ship course are derived from (6):

$$\Delta V_{-\setminus+} = \frac{\sin\left(\pm\delta_{jb} + N_j - \psi_j\right)}{\sin(\pm\delta_{jb} + N_j - \psi)} \quad \text{where:} \quad \delta_{jb} = \arcsin\frac{D_B}{D_j} \tag{6}$$

It is interesting to make an insight into relation (6) with variable ψ being treated as a non-related variable. The relation will then look generally as:

$$V_{-/+} = \frac{A_{-/+}}{\sin(B_{-/+} - \psi)} \tag{7}$$

where the values of $A_{-/+}$ and $B_{-/+}$ are treated as constant before a potential manoeuvre, and where we also have:

$$A_{-\setminus+} = V_j \sin\left(N_j - \psi_j \pm \delta_{jb}\right), \quad B_{-\setminus+} = N_j \pm \delta_{jb} \tag{8}$$

From relations (7) and (8) we can draw a conclusion that both functions V_+ and V_- are of the form 1/sinx, and that they have a phase shift in relation to each other of the value of 2 arcsin D_b/D_j, and also that the minimum of the functions are not equal ($A_{-} \neq A_{+}$). Analysis of relationship (6) leads to the description of the range of forbidden velocities, i.e. velocities leading to collision for every own course f. It is being suggested to display such data in the form of the so called "manoeuvre table". The horizontal co-ordinate of such a table constitute the values of ψ made discrete every 2 or 5 degrees, whilst the vertical co-ordinate is the set of values of own velocity V quantified every 0,5 or 1 knots. The table elements would be 0 or 1. Zero is related to a collision manoeuvre, while 1 means a safe manoeuvre. The logical product of such tables prescribed for all watched targets would constitute an answer table of safe manoeuvres. An example of a table of manoeuvres for two targets is shown in figure 3.

Set course 000° — New course 055°

Speed \ Course	300	305	310	315	320	325	330	335	340	345	350	355	000	005	010	015	020	025	030	035	040	045	050	055	060
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
V= 18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
16	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
10	0	0	0	0	0	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
4	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Time = 10:24 Course = 55 [Deg] & Speed = 18 [Kts]

Figure 3: The table of manoeuvres for two targets.

Prescription of a trajectory takes into account the process of own ship returning to the planned course. Also the manoeuvring style is such that other targets will read the own ship's intentions as clear. This effect is obtained by running the procedure of respecting sea rules only at the time of choosing the first manoeuvre. A simulation of solving a collision situation using the table of manoeuvres and dynamics of own ship is shown in figure 4.

Values: CPA and TCPA		
Targ 1	3.35	-29.42
Targ 2	2.13	10.37
Targ 3	3.11	-19.71
Targ 4	2.15	-13.79
Targ 5	4.02	-20.39
Targ 6	1.57	-13.02
Targ 7	3.17	-9.35
Targ 8	1.12	-4.52
Targ 9	0.52	4.02
Targ 10	3.27	-13.03
Targ 11	2.35	-12.89
Targ 12	0.52	-10.45
Targ 13	4.78	-16.22
Targ 14	4.02	-20.51
Targ 15	3.33	-25.45
Targ 16	1.04	-10.55
Targ 17	1.34	-123.8
Targ 18	0.34	-40.40
Targ 19	2.01	-23.55
Targ 20	2.68	-20.00
OWN SHIP Ψ,V:	-5	18
y,x:	-0.41	8.04

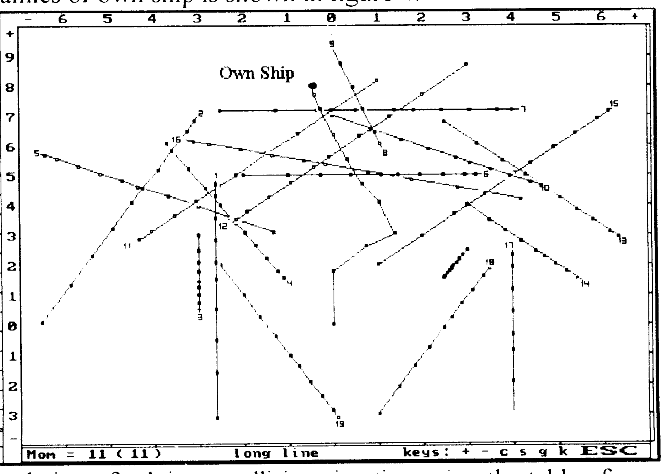

Figure 4: The simulation of solving a collision situation using the table of manoeuvres for 20 targets and dynamics of own ship.

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4 The process avoiding collision as a task non-linear programming [2,4,6,8]

It will be assumed that in the process of the motion target Bj keeps on its course ψ_j and speed ϑ_j while own ship A is undertaking safety precautions and trying to change its course ψ. The speed of ship A while manoeuvring is constant. Calculations are based on the kinematics model of real motion of the own ship A and moving target Bj, where trajectories of state **x** and steering vector **u** are defined as follows:

$$\mathbf{x} = \left[x_1, x_2, \ldots, x_{2j+1}, x_{2j+2}, \ldots, x_{2n+1}, x_{2n+2}\right] = \left[x_0, y_0, \ldots, x_j, y_j, \ldots, x_n, y_n\right]^T \quad (9)$$

$$\mathbf{u} = |u| = \psi \quad \text{where:} \quad (10)$$

For time $\mathbf{T} = [t_p, t_k]$, where t_p t_k with initial state $\mathbf{x}(t_p)$ the ships' motion model is been defined as the system (11):

$$\begin{aligned} \dot{x}_1 &= \vartheta_A \sin u \\ \dot{x}_2 &= \vartheta_A \cos u \\ &\;\cdot \\ \dot{x}_{2n+1} &= \vartheta_n \sin \psi_n \\ \dot{x}_{2n+2} &= \vartheta_n \cos \psi_n \end{aligned} \quad (11)$$

Dynamical constraints for trajectory **x** follow from the safe pass condition requiring a distance from B_j that is greater than or equal to a safe distance D_b CPA. This is pictured by eq. (12) where g_j is the function of constraints.

$$g_j(\mathbf{x}(t), t) = D_b^2 - (D_{AB_j}^t(t))^2 \le 0 \quad \forall\, t \in [t_p, t_k], \text{ where } D_b \in R^1 \quad (12)$$

Optimisation of safe trajectory has to be based on the obvious criterion cost of own ship's. The deviation from a planned trajectory ψ_z *(set course)* is criterion of the cost. The problem is therefore defined as seeking an optimal trajectory $\hat{\mathbf{x}}$

such that $$\mathbf{J}(\hat{\mathbf{x}}(t), \hat{\mathbf{u}}(t), t) = \min_{U_o, X_o} \int_{t_0}^{t_k} f_o(1/2(\psi_Z - u(t))^2, \quad (13)$$

where: ψ_z planned trajectory as the set course of own ship.

Minimisation of criterion **J**, for the state (9) defines optimal safety trajectory of own ship's. There is a deviation between the actual course $u(t)$ ψ and set course ψ_z of own ship. The problem of safe trajectory optimisation can be handled as the problem of dynamic optimisation with a defined termination time t of process. The state variables' **x** are influenced by dynamical trajectory constraint's $g_j(x)$. A method of solving [8] is to seek optimal solutions in control space **U** basing on the maximum principle. A modified quality functional (13) was constructed, where the space of control **U** was approximated with factors a_j and basic function φ_j set of partially constant control $\mathbf{u}(\mathbf{a})(t)$ (14):

$$\mathbf{u}(\mathbf{a})(t) = \sum_{j=0}^{M-1} a_j\, \varphi_j(t) \quad (14)$$

where: $a_j \in R^1$, factors $\varphi_j = \begin{cases} 1 \text{ for } t \in (t_j, t_{j+1}) \\ 0 \text{ for others } t \end{cases}$, M is number of basic functions.

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After introducing the penalty function k for breaking the dynamical trajectory constraints $g_j(x)$ and partially constant function $\mathbf{u}(\mathbf{a})(t)$, the quality factor **J** attains the shape:

$$\mathbf{J}(\mathbf{u}(\mathbf{a})(t),k)=\sum_{1}^{N}0.5(\psi_{Set}-\psi)^2+0.5\sum_{1}^{N_g}kM_k(g_j)^2 \qquad (15)$$

where: k - are the penalty coefficient, N_g - the number of trajectory constraints. Such modified quality factor **J** (15) can be solved relative to steering **u** expansion coefficients a_j and penalty k and can reduce the problem to a finite dimension problem without constraints. The maximum principle as one of the forms of obligatory conditions for local optimality in problems of dynamical optimisation is utilised here. It is assumed that the right hand side equations of state are continuous as well as their derivatives and that the function integrated function's differentiable. In every iteration of the penalty function method a substitute problem is solved, where the solution of the finite dimensional problem is the vector of the optimal coefficients of the control **u(a)** expansion. First control **u(a)**, the starting to account in algorithm is appointed with regard target which has the biggest risk factor of collision Rj and first direction of the move own ship is depended of The Sea Rules [7]. The factor Rj due to a given target varies in time depending on the navigation situation and is a function of D_b - CPA and T_b - $TCPA$.

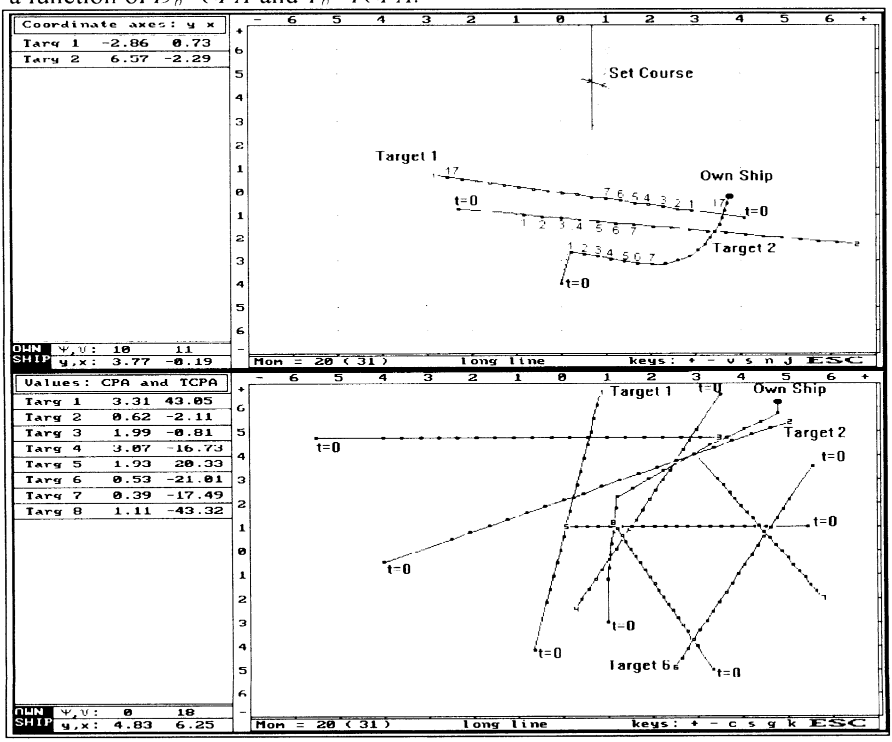

Figure 5: The trajectory of own ship to avoid collision with moving 2 targets and 8 targets.

Search for the best coefficients minimising the quality factor (15) is done with the help of the gradient method without constraints [7]. Control $\mathbf{u}^d$ which reduces the value of functional $\mathbf{J}(\mathbf{u})$ is presented by:

$$\mathbf{u}^d = \mathbf{u} + d\frac{\partial \mathbf{H}}{\partial \mathbf{u}} \text{ where: d is the factor of step length.} \qquad (16)$$

Problem (13) was analysed for moving targets' cases where constraints moved simultaneously with target B_j. It was be assumed that the safe passing radius was D_b=*CPA*. D_b was considered in (12). State **x** was made desecrate in N=30 points. The initial state **x** of the process (details of own ship and collision situation) was assumed as in figure 5.

5 Conclusion

In this paper , we presented an overview of the main results of three methods of avoiding collision at sea. The safe manoeuvre and trajectory proposals can be simulated on the display of ARPA anti-collision system as an additional feature of the system. Such an expansion of the ARPA system functions can be of significant help to the operator undertaking a decision in a situation of the danger of collision. A drawback of the proposed methods is their ignorance of the alien targets' strategy. Constant monitoring however gives the chance to notice changes in the course or the speed of the targets which leads to the recalculation of the proposed motion parameters of own ship making them suitable for the new situation.

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