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An attempt to ship propulsion control system optimization under random input

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Abstract

Random wave resistance to the ship motion has been taken into account. In such circumstances any optimization of ship propulsion control system requires a knowledge of wave resistance stochastic characteristics. A method of such an optimization of architecture and parameters of the ship propulsion controller has been discussed.

1 Introduction

A wave resistance to the ship motion depends on the ship speed and on the sea state. Among others it is a function of the wave amplitude second power. In the same time the wave amplitude is of random nature.

A ship propulsion task is to overcome the resistance of the sea. In particular the task of a ship propulsion control is to satisfy some control criteria. Let us take into account a simplified linear model of ship propulsion control. Its block diagram is represented in Fig.1 in which P_T denotes thrust horse-power, z - sea resistance power, ω - engine shaft angular velocity, σ - e.g. stresses. The sea resistance power, z, is a stochastic process. In such circumstances any design of the ship propulsion control system should take into consideration this fact . To design, and to optimize in particular, a ship propulsion control system it requires among others the knowledge of sea resistance stochastic characteristics. On the basis of assumed wave resistance stochastic characteristics it is possible to optimize a ship propulsion control system. As an appropriate mathematical criterion it could be the optimality necessary condition in shape of a functional representing the engine output signal variance, e.g. Domachowski [1].

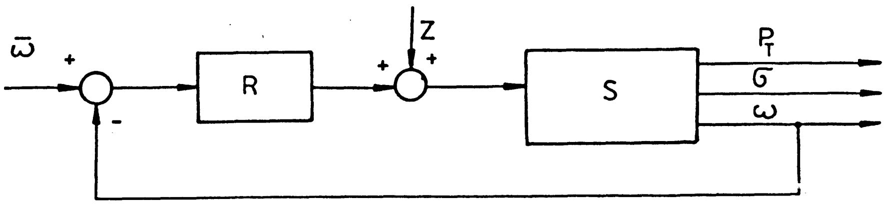

Figure 1: Block diagram of the ship propulsion control.

2 Optimality necessary condition of ship propulsion control system

Assuming a linear ship propulsion control system let us denote

$$y = [\, P_{Tz}, \omega_z, \sigma_z \,]^T \tag{1}$$

as a component of engine output response to the sea resistance power, z, see Fig. 1. In such circumstances there is

$$\Delta[\, P_{Tz}, \omega_z, \sigma_z \,]^T = [g_{Pz}, g_{\omega z}, g_{\sigma z}\,] * \Delta z \tag{2}$$

where $g_{Pz}, g_{\omega z}, g_{\sigma z}$ are components of ship propulsion control system impulse response to the sea resistance power, z, and * denotes the convolution.

Assume Δz is a difference between the random sea resistance power and its statistical average, and ΔP_{Tz} , $\Delta\omega_z$, $\Delta\sigma_z$ are changes, respectively, of thrust horse - power, of engine shaft angular velocity, of stresses corresponding to the change of the sea resistance power equal Δz. Then as a criterion function of a ship propulsion control system it could be e.g. a linear combination of variances of above mentioned signals. This is, according to the expression (2),

$$l_{Pz}\, E\,(\Delta P_{Tz})^2 + l_{\omega z}\, E\,(\Delta\omega)^2 + l_{\tau z}\, E\,(\Delta\sigma)^2 =$$

$$\sum_{i\, \varepsilon [\, P_Z,\, \omega_Z,\, \sigma_Z\,]} l_i\, E\,(g_i * \Delta z\,)^2 = F\,[\, g_i\,(t)\,] = \text{minimum} \tag{3}$$

where E means statistical average (expectation value), l_i denotes weight factor, F - square functional depending on ship propulsion control system impulse responses, g_i (t). Each function g_i (t) depends among others on the architecture and parameters of ship propulsion governor. An optimization of the governor transfer function (or impulse response) is the design task in the considered circumstances. However in farther considerations, for to simplify them, let us assume that an impulse response of the whole control system should by optimized.

As a next simplification let us limit the considerations to the variance of only one control signal of the ship propulsion control system. This is reasonable because of that only a general method of ship propulsion governor optimization is an item of interest.

The necessary condition of the functional $F[g_i(t)]$ minimization is given as follows

$$\delta F = 0\,, \tag{4}$$

where dF denotes a variation of functional F. It turnus out, e.g. Domachowski [1], that in the considered circumstances the necessary condition (4) takes on the following form:

$$\int_{-\infty}^{+\infty} g_i(\tau)\, r_{\Delta z}(\mu - \tau)d\tau = 0 \qquad \text{for } \mu > 0, \tag{5}$$

where

$$r_{\Delta z}(\tau) = \lim_{T \to \infty} (1/2T) \int_{-T}^{T} \Delta z(t)\, \Delta z(t+\tau)\, dt \tag{6}$$

is the correlation function of Δz stochastic process.

It follows from the equation (5) that there are no poles in the left half-plane of the function

$$Q(\lambda) = G_i(\lambda)\, S_{\Delta z}(\lambda) \tag{7}$$

where $S_{\Delta z}(\lambda)$ denotes the spectral density function of Δz stochastic process, $G_i(\lambda)$ - the transfer function of the ship propulsion control system which in the same time is the Fourier's as well as Laplace's transform of the function $g_i(t)$. This is the other form of the optimality necessary criterion of the considered function $g_i(t)$. Let us remark that $Q(\lambda)$ is the Fourier's transform of the left side of the equation (5).

As it has been mentioned above, $G_i(\lambda)$ is the transfer function of the ship propulsion control system. As such a transfer function $G_i(\lambda)$ should finally satisfy all the following requirements:

i) no poles in the left half-plane,

ii) $G_i(0) = \varepsilon_{s,z}$, where $\varepsilon_{s,z}$ denotes a maximum acceptable steady state error of the unit step response,
iii) $G_i(\infty) = 0$ which is the control system realizability condition,
iv) no poles in the left half-plane of the corresponding function $Q(\lambda)$.

Summarizing, according to (7), the poles of the transfer function $G_i(\lambda)$ should belong to the set of zeros of the function $S_{\Delta z}(\lambda)$, situated in the left half-plane. Similarly the set of zeros of the transfer function $G_i(\lambda)$ should contain the poles of the function $S_{\Delta z}(\lambda)$, situated in the left half-plane. The amplification coefficient of the transfer function $G_i(\lambda)$ should be equal to $\varepsilon_{s,z}$.

3 Wave resistance stochastic characteristics

An optimization of a ship propulsion control system requires the knowledge of the spectral density function of ship resistance power, $S_{\Delta z}(\lambda)$ - see (7). Let us remark

$$z = Rv \tag{8}$$

where R denotes the ship sea resistance power, v - the ship speed. The last one has been assumed constant.

The ship wave resistance depends among others on the wind velocity, and on the sea wave amplitude. The influence of the sea wave amplitude prevails. The consecutive part of ship sea resistance is maximal in the case of strictly opposite wave. Such a case has been assumed in the following considerations.

In the case of reqular wave the ship wave resistance is defined as follows, e.g. Dudziak [2]

$$R_w = \rho_w \ \xi^2 \ \rho \ g \ B^2 / L \tag{9}$$

where B denates the ship width, L - ship length, R_w - ship wave resistance, g - accelerati of gravity, ξ - wave amplitude, ρ - water density, ρ_w - wave resistance factor. While assuming as constant the ship speed the knowledge of stochastic characteristics of squared sea wave amplitude is sufficient to determine the ship resistance power stochastic characteristics, see (8) and (9). Unfortunately there is no data on the squared sea wave amplitude stochastic characteristics. They can`t be theoretically determined, and no experimental characteristics are known. In the same time several attempts have been done to determine the spectral density function of the sea wave amplitude. For that reason in the following attempt to ship propulsion control system optimization under random sea wave disturbances the spectral density function of the ship wave resistance has been approximated.

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Aiming to approximate the ship wave resistance spectral density function let us represent the relationship (9) in the following form:

$$R_w = a\,\xi^2 \tag{10}$$

where $a = \rho_w\, \rho\, g\, B^2/L$. Assume there is known the probability density function of the sea wave amplitude. Suppose F(x), G(y), and f(x), g(y) denote distribution function and probality density function of respectively the sea wave amplitude, ξ, and the ship wave resistance, R_w. For $y > 0$ it is

$$G(y) = P(R_w < y) = P(a\xi^2 < y) = P(-\sqrt{y/a} < \xi < \sqrt{y/a}) =$$

$$= F(\sqrt{y/a}) - F(-\sqrt{y/a}) \tag{11}$$

Let us remark $F(x) = 0$ for $x \leqslant 0$, because of that $\xi > 0$.
Finally

$$G(y) = \begin{cases} 0 & \text{for } y \leqslant 0 \\ F(\sqrt{y/a}) & \text{for } y > 0 \end{cases} \tag{12}$$

and

$$g(y) = \begin{cases} 0 & \text{for } y \leqslant 0 \\ F(\sqrt{y/a})/\,2\sqrt{ay} & \text{for } y > 0 \end{cases} \tag{13}$$

If only the sea wave amplitude probability density function, F(x), was known then it would be possible to determine the ship wave resistance probability density function, see (13). As a consequence as well the correlation function as the spectral density function of the ship wave resistance could be determined. Unfortunately a sea wave amplitude probability density function is so far unknown.

For the same purpose let us apply the ergodicity hypothesis to the ship wave resistance. Taking into account the relationship (10) we get

$$r_R(\tau) = \lim_{T\to\infty} (1/T) \int_{-T/2}^{T/2} R_w(t)\, R_w(t+\tau)\,dt =$$

$$= \lim_{T \to \infty} (1/T)\, a^2 \int_{-T/2}^{T/2} \xi^2(t)\ \xi^2(t+\tau)dt \tag{14}$$

The last term in the relationship (14) is proportional to the sea wave amplitude correlation function. Unfortunately no data are up to now known to this paper's author on the form of the considered correlation function.

4 Example

For at least to illustrate the discussed method of ship propulsion control system optimization let us linearize the relationship (10)

$$R \approx 2a\,\xi \tag{15}$$

In such a case

$$r_R(\tau) \approx \lim_{T \to \infty} (1/T) \int_{-T/2}^{T/2} 2a\xi(t)2a\xi(t+\tau)dt = 4a^2\, r_\xi\,(t), \tag{16}$$

and as a result

$$S_R(\omega) \approx 4a^2\, S_\xi(\omega) \tag{17}$$

A generalized spectral density function of the sea resistance power takes then the following form

$$S_{\Delta z}(\lambda) \approx 4a^2\, S_\xi(\lambda) \tag{18}$$

Let us suppose

$$S_\xi\,(\lambda) = a\,\lambda^{-4} \exp(-B\lambda^{-4}) \tag{19}$$

where A and B depend on the sea wave parameters, e.g. Dudziak [2]. For the purpose of that example let us approximate

$$\exp(-B\lambda^{-4}) \approx 1 - (B\lambda^{-4})/1! + (B\lambda^{-4})^2/2! \tag{20}$$

then

$$S_\xi\,(l) \approx A\lambda^{-4}\,(1 - B\lambda^{-4} + 1/2\, B^2\lambda^{-8}) = A/\lambda^{12}\ (\lambda + j\sqrt[4]{\lambda_1}\,)$$

$$(\lambda - j\sqrt[4]{\lambda_1})(\lambda + \sqrt[4]{\lambda_1})(\lambda - \sqrt[4]{\lambda_1})(\lambda + j\sqrt[4]{\bar{\lambda}_1})$$

$$(\lambda - j\sqrt[4]{\bar{\lambda}_1})(\lambda + \sqrt[4]{\bar{\lambda}_1})(\lambda - \sqrt[4]{\bar{\lambda}_1}) \qquad (21)$$

where

$$\lambda_1 = (B/2)e^{j\pi/4}, \qquad \bar{\lambda}_1 = (B/2)e^{-j\pi/4} \qquad (22)$$

Three forms of the considered transfer function $G_i(s)$ are satisfying the optimality necessary condition, see chapter 2. They are as follows

i) $$G_i(s) = \frac{\sqrt{B/2}\ \varepsilon_{s,z}}{(s + \sqrt[4]{\lambda_1}\ s)(s + \sqrt[4]{\bar{\lambda}_1}\ s)} \qquad (23)$$

ii) $$G_i(s) = \frac{\sqrt{B/2}\ \varepsilon_{s,z}}{(s + j\sqrt[4]{\lambda_1}\ s)(s + j\sqrt[4]{\bar{\lambda}_1}\ s)} \qquad (24)$$

iii) $$G_i(s) = \frac{\sqrt{B/2}\ \varepsilon_{s,z}}{(s + \sqrt[4]{\lambda_1}\ s)(s + \sqrt[4]{\bar{\lambda}_1}\ s)(s + j\sqrt[4]{\lambda_1}\ s)(s + j\sqrt[4]{\bar{\lambda}_1}\ s)} \qquad (25)$$

One of the above mentioned forms of transfer function should also satisfy the sufficient condition of the ship propulsion control system optimality.

5 Final remarks

An analysis of a ship propulsion control system under random circumstances requires the knowledge of the stochastic characteristics of ship sea resistance. The last ones depend among others on the squared sea wave amplitude. The knowledge of the spectral density function of the squared sea wave amplitude would enable a statistical optimization of the ship propulsion control system.

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