Vertical ice loads on offshore structures due to changes in water levels
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1 Introduction

In snowy cold regions, vertical ice forces act on offshore structures, including bridge piers and intake towers, with ice sheets adfrozen to them as the water level changes (Figure 1): when the water level rises, vertical forces act upward to uproot the structure; when the water level falls, vertical forces act downward to buckle the structure. Therefore, the effects of vertical forces should be considered when designing structures upon which ice forces are exerted.

In this report, we explain mathematically these ice forces, and we give tables and figures for the practical computation of the ice forces with attention to destructions by adfreezing and bending. Although all the tables and the figures cannot be shown due to page limitation, we have selected the important ones to discuss the procedure to calculate vertical ice forces.

In addition, the interaction between the ice and the structure made of multiple piles was theoretically produced.
2 Procedure to Calculate Vertical Ice Forces

(1) Mathematical Explanation of the Vertical Ice Forces
Assuming that an ice sheet has an infinite expansion, Kerr\(^{(1)}\) used the theory of elasticity of flat plates to calculate the axial forces exerted upon the pile, as shown below in this section.

The equation to calculate the deflection of the plate on the elastic disc can be by:

\[ D \nabla^4 W + \omega_0 W = 0 \]  
\[ (1) \]

Where,

\[ \nabla^4 = \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \right)^2 \]  
\[ (2) \]

If \( D \) represents the rigidity of the flat plate, the following equation can be produced:

\[ D = \frac{Eh^3}{12(1 - \nu^2)} \]  
\[ (3) \]

Where,

\( E \); elastic modulus of ice sheet  
\( a \); radius of pile  
\( h \); thickness of ice  
\( \nu \); Poisson's ratio

After the general solution of equation (1) is obtained by Kelvin's function, the boundary conditions, (4 - a), (4 - b) and (4 - c), are used to determine the arbitrary constants of equation (1) in order to formulate equation (5).

\[ r \to \infty, \quad W = 0 \]  
\[ (4-a) \]

\[ r \to a, \quad W = \Delta \]  
\[ (4-b) \]

\[ r \to a, \quad dW/dr = 0 \]  
\[ (4-c) \]

\[ W(r) = \Delta \left\{ -\text{kei}'(\lambda a) \text{ker}(\lambda r) + \text{ker}'(\lambda a) \text{kei}(\lambda r) \right\} / K_1 \]  
\[ (5) \]

Shearing forces per unit width is given by equation (6), and vertical ice forces acting on the pile are given by equation (7).

\[ Q = -D \frac{d}{dr} (\nabla^2 W) \]  
\[ (6) \]

\[ P = -2\pi a Q(a) = 2\pi a D \lambda^3 \Delta \frac{\{\text{kei}'(\lambda a)\}^2 + \{\text{ker}'(\lambda a)\}^2}{K_1} \]  
\[ (7) \]

Where

\( \Delta \); change in the water level  
\( \lambda = \left( \frac{w_0}{D} \right)^{0.25} \)

\( K_1 = \text{kei}(\lambda a) \text{ker}'(\lambda a) - \text{kei}'(\lambda a) \text{ker}(\lambda a) \)

\( W_0 \); the unit weight of sea water
However, as the change in the water level increases, it is believed that the bending moment causes the ice sheet to be destroyed by bending, or that the ice sheet is destroyed by adfreezing because separation takes place at the point of contact between the ice and the structure\(^{(2)}\). Thus, in order to estimate the vertical ice forces due to the change in the water level, the following two factors should be considered: (1) when adfreezing causes destruction of the ice sheet and (2) when bending causes destruction of the ice sheet.

\textbf{(2) When Adfreezing Causes Destruction}

The adfreeze bond force (vertical ice force) between the ice sheet and the structure can be obtained from equation (8), and the boundary, where the ice sheet is destroyed by adfreezing due to the change in the water level, is given by \( P = P_B \).

\[ P_B = 2\pi a h \pi_B \]  
\[ \tau_B = D\lambda^3 \Delta \left[ \left\{ ker' (\lambda a) \right\}^2 + \left\{ kei'(\lambda a) \right\}^2 \right] / h K \]  
where \( \tau_B \) = bond strength. 

The results of calculating equation (9) are shown in Figure 12. In the area where \( \tau_{b/\Delta} \) is below the lines, adfreezing causes destruction.

\textbf{(3) When Bending Causes Destruction}

According to the theory of elasticity of flat plates, the bending moment can be shown by equation (10):

\[ M_r = -D \left\{ \frac{d^2 w}{dr^2} + \nu \frac{1}{r} \frac{dw}{dr} \right\} \]  

The bending moment of the ice sheet at the point of contact between the structure and the ice sheet is given by equation (11).

\[ M_r (r = a) = -D\lambda^2 K_2 / K_1 \]  

This equation can be used to formulate equation (12), which gives the change in the water level when (the bending stress is equal to the flexural strength).

\[ \Delta_{r, max} = \sigma_f h^2 K_1 / 6 D\lambda^2 K_2 \]  

Where

\( \sigma_f \) = flexural strength

\[ K_2 = kei'(\lambda a)kei(\lambda a) + ker'(\lambda a)ker(\lambda a) \]

The vertical ice forces exerted upon the structure when the ice sheet is destroyed by bending is given by equation (13), obtained from equations (7) and (12).
The results calculated from equations (12) and equation (13) are shown in Figures 13 and 14, respectively. In the area where $\frac{\sigma}{W/\Delta}$ is below the lines, bending causes destruction.

(4) Conditions Where Both Adfreezing and Bending Simultaneously Causes Destruction

Equation (14) can be produced from equations (9) and (12).

$$\sigma_f = \frac{6K_2}{TB} h\left[\left\{\text{ker}'(\lambda a)\right\}^2 + \left\{\text{kei}'(\lambda a)\right\}^2\right]$$  \hspace{1cm} (14)

The results of calculating equation (14) are shown in Figure 15. In the area above the curve, adfreezing is the cause of destruction before bending, while, in the area below the curve, the opposite is true.

The procedure to calculate vertical ice forces is:

![Flowchart](image)

Figure 3. Flowchart to calculate the vertical ice forces.

1) Determine adfreeze bond strength between the ice sheet and the structure ($T_B$), the flexural strength of the ice sheet ($\sigma_f$), the radius of the structure ($a$), the thickness of
the ice sheet (h), Poisson's ratio (ν), Young's modulus of the ice sheet (E), and the anticipated change in the water level (Δ).

2) Calculate \( \sigma f / \tau_B \) and use its result to determine by Figure 15 whether the ice sheet is initially destroyed by adfreezing or by bending.

3) When the adfreezing causes destruction, calculate \( \tau_B / \Delta \). Use the result to determine by Figure 12 if adfreezing actually causes destruction. If it does, calculate the vertical ice forces, \( P \), by equation (8); if not, calculate \( P \) by equation (7).

4) If the result of 2) indicates that bending causes destruction before adfreezing, calculate \( \sigma f / \Delta \). Use this result to determine by Figure 13 if bending actually causes destruction. If it does, calculate \( P \) by equation (13); if not, calculate \( P \) by equation (7).

3 A Brief Discussion on Adfreeze Bond Strength and Flexural Strength

(1) Adfreeze Bond Strength

Although we have the results of experiments made on adfreeze bond strength, including those by Saeki et al, we are unable to include all of them because of page limitation. Therefore, we will only discuss the significant ones here.

The adfreeze bond strength depends upon various factors. As the ice temperature falls, the strength increases. However, it decreases with an increase in the diameter of the pile, and becomes constant when \( \pi \phi /D_{gr} \) is more than or equal to 80 (where \( \phi \) stands for the diameter of the pile, and D_{gr} represents the grain size of the sea ice, which is equivalent to the diameter of the plane circle whose area is equal to that of a single crystal of the sea ice).

Furthermore, the adfreeze bond strength largely depends upon the material of the structure. The degree of dependance of the strength on the ice temperature based on the material is shown in Figure 4, which shows that the adfreeze bond strength increases with fall in ice temperature. Also, Figure 5 shows the degree of dependence of the adfreeze bond strength upon the surface roughness of the material. \( R_z \) demonstrates the surface roughness of the material. This is evidence that an increase in the surface roughness of the material leads to an increase in adfreeze bond strength.

![Figure 4. Relationship between adfreeze bond strength and temperature of ice.](image1)

![Figure 5. Relationship between adfreeze bond strength and surface roughness.](image2)
Therefore, when the adfreeze bond strength is examined, the surface roughness of the material needs to be analyzed. Provided that adfreeze bond forces are the resultant of shearing forces and cohesion of the ice, we consider that the shearing forces transcend the cohesion when the surface is rough, and that the cohesion transcends the shearing forces when the surface is smooth.

(2) Flexural Strength
The flexural strength depends upon the temperature, the density, and the crystals of the ice. According to the experiment by Saeki et al (4), the flexural strength at Katsurazawa Lake was reported to be 5-10 kgf/cm² and 10-15 kgf/cm² at Shinotsu Lake.

4 Structures Made of Multiple Piles
In reality most structures are composed of multiple piles. Granted that the deflection at an arbitrary point can be given by the superimposed deflection of all the piles, interaction is generated by ice forces. The addition of the interaction forces to ice forces might create a very dangerous situation. Thus, the effect of the interaction also has to be considered. We have formulated an equation to calculate the vertical ice forces when there are two piles with a radius of \( a \), which are separated from each other by the distance of \( \ell \) (Figure 6).

![Figure 6. Plane view in which two piles are separated from each other by the distance of.](image_url)

Figure 6. Plane view in which two piles are separated from each other by the distance of.

![Figure 7. Three dimensional general view.](image_url)

Equation (15) is true according to a geometrical relationship.

\[
    r' = \sqrt{r^2 + \ell^2 - 2r\ell \cos \theta} \quad (15)
\]

Also, the deflection at an arbitrary point, \( N \), can be given by equation (16) if the deflection between the two piles can be superimposed.

\[
    W_{N} = W(r) + W(r') \quad (16)
\]

Therefore, the resultant force of the ice forces can be given by equation (17) if equation (16) is true.
\[ P = P_0 + P' \quad (17) \]

\[ P_0 \] is originally the ice force of one pile, and \( P' \) is the interaction with others. Since the ice forces exerted upon the pile can be given by the line integration of the shearing forces per unit width along the contour, equation (17) can be transformed into equation (18).

\[ P = -2\pi a Q(a) - 2 \int_C Q(r) \, ds \quad (18) \]

This shearing force can be shown by the following vector:

\[ Q = -\text{grad}(D\nabla^2 W) \quad (19) \]

Because the deflection, \( W \), is not restricted by the angle, \( \theta \), the scalar quantity of this vector results in equation (6).

Now the contour, \( \zeta \), in Figure 8 is divided into semi-circle \( C \) and an arbitrary shape of \( C' \). In the simply connected domain, \( R \), since \( Q(r) \) is continuous, its line integration can be transformed into the surface integration shown below. As long as the contour passes through \( a \) (the lower limit) and \( b \) (the upper limit), \( \int_C Q(r) \, ds \) is independent of the path of the integral. (Actually, it is impossible to define the domain \( R \) because a pile actually exists there, but for convenience, ice sheets are assumed to exist in \( R \)).

\[ \oint \mathbf{Q} \, ds = \iint_R \text{rot} \mathbf{Q} \, d\mathbf{R} = 0 \]

\[ \therefore \int_C Q(r) \, ds = \int_{C'} Q(r) \, ds' \quad (20) \]

Using the integration of the straight line \( s \), the line integration of equation (18), \( \int_C Q(r) \, ds \), can be calculated easily.

Therefore, the interaction is given by equation (21). By the same logic, equation (22) can be used to calculate ice forces exerted upon \( n \) piles (\( n = \text{number of piles} \)). By setting dimensionless parameters, \( \eta = a/\ell \) and \( \varepsilon = \lambda \ell \), respectively, and using the dimensionless value of \( P'/P_0 \), equation (23) can be produced. However, the domain of \( \eta \) is between 0 and 0.5.
\[ P' = -\frac{2D\Delta\lambda^2}{K_1} [DL\text{ker}'\lambda a + DR\text{ker}'\lambda a] \quad (21) \]

\[ \text{DI} = \text{ker}\lambda(\ell + a) - \text{ker}\lambda(\ell - a), \quad \text{DR} = \text{ker}\lambda(\ell + a) - \text{ker}\lambda(\ell - a) \]

\[ P = -2\pi aQ(a) - 2 \sum_{i}^{n} \int_{si} Q(r)dr \quad (22) \]

\[ \frac{P'}{P_0} = \frac{\text{kei}\epsilon\eta\{\text{kei}\epsilon/(1 + \eta) - \text{kei}\epsilon/(1 - \eta)\} + \text{ker}'\epsilon\eta\{\text{ker}\epsilon/(1 + \eta) - \text{ker}\epsilon/(1 - \eta)\}}{\pi\epsilon\eta[s\{\text{kei}'(\epsilon\eta)\}^2 + \{\text{ker}'(\epsilon\eta)\}^2]} \quad (23) \]

Using equation (23), the relationships between \( \epsilon \) and \( \eta \) when the influence value is 1%, 5%, and 10% (Figure 9). If \( \ell \) increases, the degree of influence decreases. Conversely, if the thickness of the ice, Young's modulus and the radius of the ice increase, the degree of influence increases.

Figure 8 also shows that domain A is \( P' / P_0 < 0.01 \), domain B is \( 0.01 < P' / P_0 < 0.05 \), and domain C is \( 0.1 < P' / P_0 \).

As an example, Figure 10 shows the relationship between \( \ell \) and \( P' / P_0 \) when \( a \) (the radius of the piles), \( E \) (Young's modulus) and \( h \) (the thickness of the ice) are equal to 50 cm, 20000 kg/cm\(^2\) and 50 cm, respectively.

When the distance between the two piles are close to each other at 100 cm, the influence value is 35%. In order to keep the value below 1%, the distance between the two piles has to be more than 12 m.

When all the necessary values are given, the influence value can be calculated by equation (23), and the distance between the two piles and the cross-section of the pile can be determined by back calculation.}

![Figure 9. The relationship between \( \epsilon \) and \( \eta \).](image)

![Figure 10. An example demonstrating the relationship between the distance between the piles and the influence values.](image)
Figure 11. The results of calculating eq. (7).

Figure 12. The results of calculating eq. (9).

Figure 13. The results of calculating eq. (12).

Figure 14. The results of calculating eq. (13).

Figure 15. The results of calculating eq. (14).
5 CONCLUSIONS

1. To estimate vertical ice forces due to the change in the water level, values of the bending strength and the adfreeze bond strength of the ice sheet must be given.
2. As the radius of the pile decreases, bending and adfreezing cause destruction more easily because shearing forces per unit width increase.
3. The thickness of the ice is occasionally increased by a high degree of heat conduction at the point of contact between the ice and the structure. As a result, forces increase at the boundary where adfreezing causes destruction. In this case, the value of \( \tau_B / \Delta \) that can be estimated by Figure 12 or by equation (9) should be multiplied by the rate of increase of the ice thickness.
4. The interaction of multiple piles decreases as the distance between the piles increases, but the interaction increases as the rigidity of the flat plate increases.
5. If the measured thickness of the ice sheet, Poisson's ratio and Young's modulus are given, the relationship between the cross-sectional shape of the pile, the distance between the piles, and the interaction can be found using Figure 9 or equation (23), which will be helpful for the safe design of structures.

References