Frictional resistance of randomly oscillating surfaces
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Abstract

The frictional resistance of randomly oscillating surfaces is studied by assuming that the oscillatory motion is a stationary Gaussian narrow-band random process. The approach is also based on simple explicit frictional resistance formulas for harmonic oscillations. The probability distribution functions of the maximum surface shear stress for laminar flow as well as smooth turbulent and rough turbulent flow are presented. The maximum surface shear stress follows the Rayleigh distribution for laminar flow and the Weibull distribution for smooth turbulent and rough turbulent flow. Examples of the friction coefficients based on the mean values of the maximum surface shear stress for the three flow regimes are also given.

1 Introduction

The motion of ships and structures is difficult to estimate from an analytical point of view. A major difficulty is the prediction of the damping of the system. This is an important part of the problem for systems that can be excited to move near resonance, since the damping is the only mechanism that can set any limit to the motion. Examples are roll damping of ships, as well as damping of large-volume structures and wave-power devices. Although the contribution from the frictional damping is small, it is important because, e.g. for ordinary ships, the frictional damping leads to significant scale effects in model tests (Myrhaug and Sand [1]). The frictional damping is normally negligible for full-scale ships. However, in the case of fouling by the growth of shells (which is a common roughness on ships that remain some days in a harbour with warm and infected water) the frictional damping is important even for full-scale ships (Myrhaug [2]). For large-volume structures where the
amplitude of oscillation is small relative to the diameter of the structure (<4),
eddies are not generated and therefore the frictional damping is negligible. 
Here the friction between the wall and the water contributes to the loss of 
energy. In most of these cases the flow is in the smooth turbulent flow regime. 
The frictional resistance of oscillating surfaces has been treated by Myrhaug 
[3], and was extended to cover the case of oscillating surfaces in steady 
currents in Myrhaug and Slaattelid [4]. A rational approach to the latter 
problem was presented by Myrhaug and Slaattelid [5].

This paper presents the frictional resistance of randomly oscillating 
surfaces. The oscillations are described as a stationary Gaussian narrow-band 
random process. Further, the approach is based on simple explicit friction 
coefficient formulas for harmonic oscillations. The probability distributions 
of the maximum surface shear stress for laminar flow as well as smooth turbulent 
and rough turbulent flow are presented.

2 Explicit frictional resistance formulas for harmonic 
oscillations

The maximum surface shear stress for harmonic oscillations is given as

$$\frac{\tau_m}{\rho} = \frac{1}{2} f_w U^2$$

(1)

where U is the oscillatory velocity amplitude, $f_w$ is the friction coefficient, and 
$\rho$ is the density of the fluid.

For laminar flow (Stokes’ second problem; Schlichting [6]) the friction 
coefficient is given by

$$f_w = r \ Re^{-s}$$

(2)

where

$$r = 2 \quad \text{and} \quad s = 0.5$$

(3)

and

$$Re = \frac{UA}{v}$$

(4)

is the Reynolds number associated with the oscillatory motion, A is the 
oscillatory displacement amplitude, and $v$ is the kinematic viscosity of the 
fluid, see Fig.1.

For smooth turbulent flow the friction coefficient can also be represented 
by Eq. (2), but with other values of r and s. Jonsson [9] proposed to use
Figure 1: Friction coefficient vs. Reynolds number for laminar and smooth turbulent flow. Laminar flow: —— Eqs. (2) and (3) for harmonic oscillations; - - - Eq. (17) for random oscillations. Smooth turbulent flow: —— Eqs. (2) and (5) for harmonic oscillations; - - - Eq. (18) for random oscillations. All the data are for sinusoidal waves/oscillations: x Kamphuis [7]; ▲ Jensen et al. [8].

Figure 2: Friction coefficient vs. amplitude to roughness ratio for rough turbulent flow: —— Eq. (5) (Soulsby et al. [11]) for sinusoidal oscillations; - - - Eq. (19) for random oscillations. All the data are for sinusoidal waves/oscillations: + Bagnold [12]; x Kamphuis [7]; □ Jonsson and Carlsen [13]; Δ Sumer et al. [14]; ○ Sleath [15].
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r = 0.0465 and s = 0.1, but since then new data for smooth turbulent flow have been published (Jensen et al. [8]). They also found that the flow becomes fully turbulent after Re reaches the value of approximately \(3 \cdot 10^6\), see Fig. 1. Based on these results Myrhaug [10] proposed to use Eq. (2) with

\[
r = 0.0450 \quad \text{and} \quad s = 0.175
\]

which is shown in Fig. 1.

For rough turbulent flow Soulsby et al. [11] proposed the following friction coefficient formula, obtained as the best fit to the data in Fig. 2,

\[
f_w = 1.39 \left( \frac{A}{z_0} \right)^{0.52}
\]

where \(z_0\) is the roughness parameter related to the Nikuradse sand roughness parameter \(k\) by \(z_0 = k/30\) (Schlichting [6]).

3 Frictional resistance for random oscillations

3.1 Probability distribution of the maximum surface shear stress

The basis for the present approach is that the maximum surface shear stress for harmonic oscillations given in Eq. (1) combined with Eqs. (2) to (6), is valid for random oscillations as well. Consequently it is assumed that each harmonic component can be treated individually. The accuracy of this assumption should be validated by using a full boundary layer model to calculate the shear stress for random oscillations. However, although the goodness of this approximation is questionable, it should be adequate as a first approximation. Further it is assumed that the oscillatory displacement \(a(t)\) is a stationary Gaussian narrow-band random process with zero expectation and with the one-sided spectral density \(S_{aa}(\omega)\), where \(\omega\) is the cyclic frequency of oscillation. Then the oscillatory velocity \(u(t) = da(t)/dt\) will also be a stationary Gaussian narrow-band random process with zero expectation and with the one-sided spectral density \(S_{uu}(\omega) = \omega^2 S_{aa}(\omega)\). The accuracy of the narrow-band assumption will be discussed subsequently.

For a narrow-band process the oscillations are specified as a "harmonic" oscillation with cyclic frequency \(\omega\) and with slowly varying amplitude and phase. Then the oscillatory displacement is given as (see e.g. Sveshnikov [16])

\[a(t) = A(\varepsilon t) \cos [\omega t + \Phi(\varepsilon t)], \quad \varepsilon << 1\]

is introduced to indicate that the oscillatory displacement amplitude \(A\) and the phase \(\Phi\) are slowly varying with \(t\). Then the oscillatory velocity is given as \(u(t) = \omega A(\varepsilon t) \sin [\omega t + \Phi(\varepsilon t) - \pi/2] + O(\varepsilon)\), where the term \(O(\varepsilon)\) represents terms of order \(\varepsilon\). As a first approximation, which is consistent with the narrow-band assumption, the terms of \(O(\varepsilon)\) are neglected, and accordingly the oscillatory velocity amplitude is related to the oscillatory displacement amplitude by \(U = \omega A\), where \(U\) is
slowly varying with \( t \) as well.

The accuracy of the approximate relation for \( u(t) \) obtained by neglecting the terms of \( O(\varepsilon) \) is discussed in Sveshnikov [16]. A test of the accuracy is the error in the variance of the derivative of the random function \( a(t) \). It appears that this error is small in the case of a narrow-band spectrum. Overall some of the main features are covered by using the narrow-band approximation.

Now \( A \) and \( U \) will both be Rayleigh-distributed. The distribution function for a Rayleigh-distributed random variable \( x \) is given as

\[
P(x) = 1 - \exp\left(-\frac{x^2}{x_{\text{rms}}^2}\right); \quad x \geq 0
\]  

where \( x_{\text{rms}} \) is the root-mean-square (rms) value of \( x \). \( A_{\text{rms}} \) and \( U_{\text{rms}} \) are related to the zeroth moments \( m_{0aa} \) and \( m_{0uu} \) of the displacement and velocity spectral densities, respectively, or corresponding to the variances of the displacement \( (\sigma_{aa}^2) \) and the velocity \( (\sigma_{uu}^2) \), by

\[
A_{\text{rms}}^2 = 2m_{0aa} = 2\sigma_{aa}^2 = 2\int_0^\infty S_{aa}(\omega)d\omega
\]  

and

\[
U_{\text{rms}}^2 = 2m_{0uu} = 2\sigma_{uu}^2 = 2\int_0^\infty S_{uu}(\omega)d\omega = 2\int_0^\infty \omega^2 S_{aa}(\omega)d\omega
\]  

From Eq. (9) it also appears that \( m_{0uu} = m_{2aa} \), where \( m_{2aa} \) is the second moment of the oscillatory displacement spectral density. A reasonable choice for \( \omega \) is the mean zero-crossing frequency, which is obtained from the spectral moments of \( a(t) \) as

\[
\omega = \omega_{m2} = \left(\frac{m_{2aa}}{m_{0aa}}\right)^{1/2} = \left(\frac{m_{0uu}}{m_{0aa}}\right)^{1/2} = \frac{U_{\text{rms}}}{A_{\text{rms}}}
\]  

where Eqs. (8) and (9) have been used.

The probability distribution function of \( \tau_m/\rho \) can now be obtained by transformation of the variable \( A \) to \( \tau_m/\rho \). By using \( U = \omega A \) in Eqs. (1) to (7), and using Eqs. (8) to (10), the probability distribution function of the normalized maximum surface shear stress for the three flow regimes are given by
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\[ P(\dot{t}) = 1 - \exp(-\dot{t}^\beta) \; ; \; \dot{t} \geq 0 \]  
(11)

where

\text{laminar: } \beta = 2 \; ; \; \dot{t} = \frac{\tau_m}{\rho u_{*L}^2} \; ; \; u_{*L}^2 = \frac{1}{2} \cdot 2Re_{rms}^{-0.5} U_{rms}^2 \]  
(12)

\text{smooth: } \beta = 1.212 \; ; \; \dot{t} = \frac{\tau_m}{\rho u_{*S}^2} \; ; \; u_{*S}^2 = \frac{1}{2} \cdot 0.0450 Re_{rms}^{-0.175} U_{rms}^2 \]  
(13)

\text{rough: } \beta = 1.35 \; ; \; \dot{t} = \frac{\tau_m}{\rho u_{*R}^2} \; ; \; u_{*R}^2 = \frac{1}{2} \cdot 1.39 \left( \frac{A_{rms}}{z_0} \right)^{-0.52} U_{rms}^2 \]  
(14)

\[ Re_{rms} = \frac{U_{rms} A_{rms}}{v} \]  
(15)

It appears that the maximum surface shear stress is Rayleigh-distributed for laminar flow, while it is Weibull-distributed for smooth turbulent and rough turbulent flow.

3.2 Friction coefficient

The friction coefficient for random oscillations is defined in Eq. (1) as

\[ f_w = 2 \frac{\tau_m/\rho}{U^2} \]  
(16)

where \( \tau_m/\rho \) is represented by an appropriate characteristic statistical value, which can be obtained when the probability distribution of the maximum surface shear stress for random oscillations is known.

As an example the friction coefficients based on the expected (mean) value of the maximum surface shear stress, \( E[\tau_m/\rho] \), will be calculated. For a Weibull-distributed random variable \( x \) with the distribution function \( P(x) = 1 - \exp(-x^\beta) \) where \( x \geq 0 \) and \( \beta > 0 \), the expected value of the random variable is given by \( E[x] = \Gamma(1 + 1/\beta) \), where \( \Gamma \) is the Gamma function (see e.g. Bury [17]). Then, by representing \( U \) by \( U_{rms} \), the friction coefficients associated with \( E[\tau_m/\rho] \) for the three flow regimes are given by
\[ f_w = 1.772 \, Re_{rms}^{-0.5}, \text{ laminar} \]  (17)

\[ f_w = 0.0422 \, Re_{rms}^{-0.175}, \text{ smooth} \]  (18)

\[ f_w = 1.27 \left( \frac{A_{rms}}{z_0} \right)^{-0.52}, \text{ rough} \]  (19)

It should be noted that use of the rms oscillatory velocity amplitude ensures that the variance of the displacement of sinusoidal oscillations matches that due to the displacement spectrum for random oscillations. Thus the results in Eqs. (17) to (19) show that the effect of random oscillations is to reduce the maximum surface shear stress by a factor of 0.89, 0.94 and 0.91 for the three flow regimes, respectively, compared with the results for equivalent harmonic oscillations given in Eqs. (1) to (6). These results are shown in Figs. 1 and 2 marked as "random; rms".

Other characteristic statistical values of \( \frac{\tau_m}{\rho} \) are given in Myrhaug [10]. However, the most appropriate statistical value of the maximum surface shear stress to use in an actual calculation of the damping of a marine system undergoing random oscillations is not yet clear, but will depend on the problem dealt with. Further, the present approach should be verified with measurements and by simulations using a full boundary layer model. However, although the present formulas are simple, it is believed to be adequate as a first approximation to represent the frictional resistance for random oscillations.

4 Conclusions

The paper presents the frictional resistance of randomly oscillating surfaces by assuming that (1) the oscillations are described as a stationary Gaussian narrow-band random process, and (2) simple explicit friction coefficient formulas for harmonic oscillations are valid for random oscillations as well. The probability distribution functions of the maximum surface shear stress for laminar flow as well as smooth turbulent and rough turbulent flow are presented. The maximum surface shear stress follows the Rayleigh distribution for laminar flow and the Weibull distribution for smooth turbulent and rough turbulent flow. Examples of the friction coefficients based on the expected values of the maximum surface shear stress for the three flow regimes are also given, showing that the effect of random oscillations is to reduce the maximum surface shear by approximately 10% compared with the results for equivalent harmonic oscillations. Although the present formulas are simple, they should be adequate as a first approximation to represent the frictional resistance for random oscillations.
References